Range Straggling in Nuclear Track Emulsion

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The contributions to the range variance in nuclear track emulsion have been studied and the magnitude of the various effects calculated. Bohr straggling is found to be the most important effect when the energy is high, but for slow particles, a number of others combine to dominate the straggling. Range errors caused by distortion are also studied. Measurements are reported on the straggling of eighteen particle groups consisting of protons, pions, and muons. With the particle mass and velocity as variable parameters, the theoretical estimates are tested and a detailed accounting of the contributions to the straggling is made.

I. INTRODUCTION

THE range is that feature of a charged particle track which is most convenient for measurement and which usually has the most significance. The range straggling determines the resolution obtainable and limits the accuracy with which mean ranges can be determined. Emulsion is one of the most important instruments of nuclear physics; consequently, this limitation on its quantitative application merits close study.

The straggling can be measured more satisfactorily in emulsion than in other types of matter because the individual trajectories are visible, and the true track length or rectified range, rather than merely the depth of penetration, may be determined. On the other hand, several different effects contribute to the straggling in emulsion, and each must be understood before the range variance of a particle group becomes predictable.

In this paper, we summarize the results of the theory of straggling in homogeneous media. Then we calculate a number of additional effects that contribute to the straggling in emulsion. Finally, we report measurements, made under well-defined conditions, of the straggling of protons, pions, and muons. These measurements provide data for detailed comparisons with the theory.

II. ANALYSIS OF STRAGGLING EFFECTS

A. Straggling in Rigid, Homogeneous Media

The magnitude of the range straggling in homogeneous materials was calculated first by Bohr.[1] Refinements of the calculations have not greatly altered his estimate of the range straggling. According to Bohr, the range variance is

$$\Sigma_B^2 \equiv \beta_2 \equiv \langle R^2\rangle - \langle R\rangle^2 = 4\pi n_e z^2 e^4 \int_0^T \frac{dT}{\langle dT/dR\rangle^3}, \quad (1)$$

where n_e is the electron density in the stopping material, R is the residual range, T is the kinetic energy of the particle, $\langle dT/dR\rangle$ is the mean rate of energy loss, and ze is the particle charge in esu. The relativistic form of this equation has been given by Lindhard and Scharff[2]:

$$\Sigma_B^2 = 4\pi n_e z^2 e^4 \int_0^T \frac{dT(1-\beta^2/2)}{\langle dT/dR\rangle^3(1-\beta^2)}, \quad (2)$$

where βc is the particle velocity.

The individual energy losses in collisions with electrons are distributed in a highly unsymmetrical manner, but the distribution of ranges, in accord with the central limit theorem,[3] tends toward normality when many separate collisions are required to stop the particle. Lewis[4] has recently re-examined the range straggling of nonrelativistic particles and evaluated the moments of the range distribution in a rather satisfactory manner. He made allowance for the electronic binding which is not included in Bohr's calculation. Lewis finds that the second moment of the range distribution is unaffected, at least in good approximation. He also found explicit expressions for the mean range and the higher moments of the range distribution.

The mean range is usually taken to be

$$\langle R\rangle = \int_0^T \frac{dT}{\langle dT/dR\rangle}. \quad (3)$$

Lewis showed that this expression must be corrected, because in it the mean path per unit energy loss has been identified incorrectly with the reciprocal of the mean rate of energy loss. In addition, he does not make the usual slight approximation that $\langle dT/dR\rangle$ is independent of the particle mass. The two corrections together we call the *Lewis effect*. Allowing for this effect increases the mean range by the factor $1+e_1$, where $e_1 \approx 0.41/M\tau\iota$ for emulsion. In the expression for e_1, which is valid for all charged particles except electrons, M is the particle mass in units of the proton mass, $\tau = T/M$ is in Mev, and $\iota \equiv (1/z^2)\langle dT/dR\rangle$ is in Mev per cm.

The quantity $\sigma = z^2\Sigma_B/M^{\frac{1}{2}}$ is a measure of the range straggling which depends only on the particle velocity.

[1] N. Bohr, Phil. Mag. **30**, 581 (1915).

[2] J. Lindhard and M. Scharff, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 15 (1953).

[3] H. Cramer, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, 1946).

[4] H. W. Lewis, Phys. Rev. **85**, 20 (1952).

TABLE I. Percentage range straggling of protons in a rigid homogeneous medium having the composition of nuclear track emulsion. The percentage range straggling for particles other than protons may be found from this table if one utilizes the fact that at a given velocity, the percentage straggling varies inversely with the square root of the particle mass and does not depend on its charge.

τ(Mev)	1	2	5	10	20	50	100	200	500	1000	2000	5000	10 000
$\phi=\dfrac{100M^{\frac{1}{2}}\Sigma_B}{\langle R\rangle}$	2.11	1.94	1.66	1.53	1.42	1.29	1.21	1.13	1.02	0.952	0.904	0.963	1.11

Likewise, $\lambda=z^2\langle R\rangle/M$ ($\langle R\rangle$ being the mean range) is a measure of the range which depends only on the particle velocity. The ratio σ/λ is a slowly varying function[5] which describes the Bohr straggling for any particle. In Table I, "the percentage straggling," $\phi=100\sigma/\lambda=100M^{\frac{1}{2}}\Sigma_B/\langle R\rangle$, in an homogeneous medium of the same composition as emulsion is given. It may be utilized directly for protons of the energy τ, or interpreted simply for any other particle of energy $M\tau$.

For a measure of the asymmetry of the range distribution, we take the third moment, $\beta_3\equiv\langle(R-\langle R\rangle)^3\rangle$. Using the methods of Lewis, and evaluating quantities for emulsion, we obtain an expression for the third moment in asymptotic form:

$$\omega^3\equiv z^6\beta_3/M$$
$$=-5\times10^{-5}\frac{\tau^2}{\iota^4}\left[1-\frac{14}{3A}-\frac{70}{9A^2}-\frac{140}{9A^3}+\cdots\right]. \quad (4)$$

In emulsion, one takes

$$A=\iota\tau/125$$

and[5]

$$\iota=(0.5325/\beta^2)\ln 3077\beta^2\gamma^2\exp(-\beta^2)\ \text{Mev/cm}$$

[where $\gamma=(1-\beta^2)^{-\frac{1}{2}}$], except at very high or very low energies. The range asymmetry is so small that it has not as yet been successfully measured.

B. Additional Range Straggling Effects

1. *Distortion Straggling*

Nuclear track emulsion tends to suffer distortions in processing, which alter the lengths of charged-particle tracks, thus causing systematic range errors and additional straggling of the measured ranges.

We recognize two types of distortion: (a) macroscopic distortion, which is described by the translation of points $\mathbf{r}_0$ into points $\mathbf{r}=\mathbf{r}_0+\mathbf{n}$, where $\mathbf{n}$ is a continuous vector function of position. This type of distortion results from the change in volume of the emulsion on dissolving out the silver halide and from strains introduced in processing and drying the emulsion. (b) microscopic distortion, which occurs because silver halide crystals dissolve in processing, leaving voids. On drying, the collapse of the gelatin is accompanied by random local displacements of developed grains.

(*a*) *Macroscopic distortion straggling.*—Assuming that the emulsion has large-scale uniformity of composition, each unit volume of it shrinks to the same volume, $1/S_0$, after processing. This imposes a condition on $\mathbf{n}$. Its components, η_x, η_y, and η_z, must satisfy the partial differential equation:

$$S_0\begin{vmatrix}1+\partial\eta_x/\partial x_0 & \partial\eta_y/\partial x_0 & \partial\eta_z/\partial x_0\\ \partial\eta_x/\partial y_0 & 1+\partial\eta_y/\partial y_0 & \partial\eta_z/\partial y_0\\ \partial\eta_x/\partial z_0 & \partial\eta_y/\partial z_0 & 1+\partial\eta_z/\partial z_0\end{vmatrix}=1. \quad (5)$$

In addition, the vector $\mathbf{n}$ must vanish on the plane $z=0$ (the glass-emulsion interface). Now the components of distortion η_x and η_y are produced by components of stress in the xy plane that result, for example, from uneven drying. Both components of shearing stress must vanish at free surface of the emulsion. Therefore, we may take $(\partial\eta_x/\partial z)_S=(\partial\eta_y/\partial z)_S=0$, where the subscript S refers to the free surface of the emulsion.

The simplest nontrivial solution for the distortion vector is

$$\begin{aligned}\eta_x&=(\Delta/T^2)(2z_0T-z_0^2)\cos\theta,\\ \eta_y&=(\Delta/T^2)(2z_0T-z_0^2)\sin\theta,\\ \eta_z&=z_0[(1/S_0)-1],\end{aligned} \quad (6)$$

where T is the original emulsion thickness. We find that such equations[6] usually describe the distortion fairly well when θ and Δ are considered constants in limited volumes of the emulsion. The quantity Δ has the significance that it is the tangential displacement of a point on the surface of the emulsion, and θ defines the direction of the displacement.

For a track element of length dR_0, we assume the following relations and definitions:

$$dR_0^2=dx_0^2+dy_0^2+dz_0^2;\quad dR^2=dx^2+dy^2+S_0^2dz^2;$$
$$\sin\delta=dz_0/dR_0;\quad \tan\phi=dy_0/dx_0;\quad \tan\theta=\eta_y/\eta_x;$$
$$c=\cos\delta\cos(\phi-\theta);\quad \mu^2=\eta_x^2+\eta_y^2.$$

Here dR is the distorted length of the track element corrected only for shrinkage. The effect of the macroscopic emulsion distortion may be calculated in terms of these quantities. Neglecting the terms of second degree and higher in $(\sin\delta)d\mu/dz_0$, we have for the mean range $\langle R\rangle$ of a group of particles that had an undistorted mean range R_0:

$$\langle R\rangle=R_0+\langle\bar{c}(\mu_2-\mu_1)\rangle. \quad (7)$$

[5] W. H. Barkas and D. M. Young, University of California Radiation Laboratory Report UCRL-2579 Rev. (unpublished).

[6] Lal, Pal, and Peters, Proc. Indian Acad. Sci. **38A**, 398 (1953).

Here $\bar{c}$ is the mean value of $\cos\delta\cos(\phi-\theta)$ taken along one track. The subscript 2 refers to the point where the track stops, and the subscript 1 refers to the point at which it starts. The brackets $\langle\ \rangle$ indicate a mean value for more than one track.

Similarly, the range variance, Σ_a^2, arising from the macroscopic distortion is given by

$$\Sigma_a^2=\langle R^2\rangle-\langle R\rangle^2=\langle\bar{c}^2(\mu_2-\mu_1)^2\rangle-\langle\bar{c}(\mu_2-\mu_1)\rangle^2. \quad (8)$$

If the distortion is of the usual type given by Eq. (6), we can put $\mu=(\Delta/T^2)(2z_0T-z_0^2)$ or $\Delta(1-d^2/T^2)$, where $d(=T-z_0)$ is measured from the surface of the emulsion.

In connection with the measurements being reported here, the macroscopic distortion was determined for each plate. The quantities Δ and θ were measured in the part of the plate where the ranges were measured by observing the x and y coordinates of steep tracks at several points separated by equal intervals of depth in the emulsion. The second differences of the measured x and y coordinates in most cases were statistically constant. Some clear deviations from constancy were observed. Such failures of the simple distortion law given by Eq. (6) usually seem to occur near the glass-emulsion interface. These experiments were performed with relatively thin 200-micron emulsion, which limits the distortion effect and keeps the calculations simple. Unsystematic observations on emulsions of 600 microns thickness revealed no distortion effects of a different character from those observed in 200-micron emulsion.

In the calculation of the range, a correction is always made for the shrinkage of emulsion, but an error in the mean range and an apparent range straggling will be introduced when the shrinkage factor is chosen incorrectly. If the correct shrinkage factor is S_0, and a factor $S=S_0+\Delta S$ is chosen, then the mean value of the range found will be

$$\langle R\rangle=R_0[1+(\Delta S/S_0)\langle\overline{\sin^2\delta}\rangle+\cdots], \quad (9)$$

where R_0 is the correct mean value, and the bar indicates an average value for one track. Also, as a result of an incorrect choice of shrinkage factor, a group of tracks all of the same true range R_0 will acquire an apparent range variance Σ_S^2, given by

$$\Sigma_S^2\approx R_0^2\frac{\Delta S^2}{S_0^2}\{\langle\overline{\sin^2\delta}^2\rangle-\langle\overline{\sin^2\delta}\rangle^2\}. \quad (10)$$

This is usually small and is negligible in our experiments.

The magnitude of the emulsion distortion as well as its effect on the range groups measured in this study is compiled in Table II. While for groups in which the tracks are closely parallel the distortion straggling is generally negligible, for randomly oriented tracks the effect may be important. The reverse is the situation with regard to the mean range. Here the parallel tracks may suffer from a systematic range error if they dip in the emulsion so as to stop at a z coordinate that is systematically different from the one at which they started.

(*b*) *Microscopic distortion straggling.*—Assume that spherical cavities are left in the gelatin on fixing the emulsion. For points in a sphere of diameter d, $\langle x^2\rangle-\langle x\rangle^2=d^2/10$. On drying, the collapse of the sphere adds an additional variance of about this magnitude to the distribution of distances between points corresponding to former voids. Therefore range straggling Σ_b^2 of magnitude about

$$\Sigma_b^2=\frac{3(S_0-1)}{20S_0}d\langle R\rangle \quad (11)$$

is introduced. For Ilford C.2 emulsion this is about $0.02\langle R\rangle$ microns^2 where $\langle R\rangle$ is measured in microns. (Winand[7] has studied C.2 emulsion and has found $d\approx0.2\mu$. We have made a similar investigation of the grains in Ilford G5 emulsion. Under the electron microscope they appear spherical and have a mean diameter of 0.27 micron.)

2. *Observer Error*

Even a careful observer will introduce a range variance term, Σ_0^2, which usually is about

$$\Sigma_0^2=0.01\langle R\rangle \text{ microns}^2. \quad (12)$$

The error increases when the tracks are badly scattered, when they are steeply dipping, and when they extend beyond one field of view of the microscope. The observer error was evaluated by analyzing measurements taken at different times on the same tracks. Apparent range straggling introduced by errors in recording readings or by faulty arithmetic is not described by Eq. (12).

TABLE II. Effects of macroscopic emulsion distortion on the ranges of measured particle groups. All lengths are in microns.

Particle	Δ	$\langle R\rangle$	$\langle R\rangle-R_0$	$100\Sigma_a/\langle R\rangle$
Proton	13.0	4513	−3.2	0.05
Proton	3.1	4556	+0.4	0.00
Proton	9.3	4618	+1.2	0.02
Proton	13.5	4605	−2.2	0.03
Proton	23.0	4680	+9.4	0.16
Proton	35.3	4535	−19.6	0.08
π^+	8.8	732	−0.9	0.04
π^+	3.1	738	+0.1	0.01
π^+	9.3	762	+0.3	0.05
π^+	13.5	702	−0.4	0.04
π^+	28.4	727	+2.0	0.22
π^+	18.9	728	+1.7	0.18
π^-	8.8	766	+0.9	0.05
π^-	3.1	751	−0.1	0.02
π^-	9.3	765	−0.3	0.06
μ^+	13.5	599	0.0	0.6
μ^+	28.4	606	0.0	1.3
μ^+	18.9	593	0.0	0.9

[7] L. Winand, *Photographic Sensitivity* (Butterworths, London, 1951), p. 288.

3. Heterogeneity Straggling

The emulsion consists of bodies of silver halide embedded in gelatin. Since the stopping powers of these two materials are not the same, the particle ranges tend to fluctuate. Clearly, if more than the normal part of a path happens to be in gelatin, the range is extended, and conversely. For the purpose of estimating the heterogeneity effect, one may assume that the silver halide is in the form of spheres of diameter d distributed with uniform average density throughout the emulsion. One may also assume that the number of spheres traversed by the moving particle in a fixed element of path is given by a binomial distribution. The variance of a binomial distribution is somewhat too high to be correct for concentrated emulsion where randomness is inhibited by contact between adjacent silver halide crystals. A closer approximation probably is obtained on dividing the estimated variance by the shrinkage factor. A calculation employing the binomial distribution with this correction gives for the range variance, Σ_h^2, arising from the heterogeneity effect,

$$\Sigma_h^2 \approx \frac{(r-1)^2(S_0-1)(S_0+8)d}{12S_0[1+r(S_0-1)]^2}\langle R\rangle, \qquad (13)$$

where $\langle R\rangle$ is the mean range, S_0 is the ratio of emulsion volume to gelatin volume (approximately the shrinkage factor), r is the ratio of the ranges that would be found in gelatin and silver bromide separately, and d is the mean grain diameter. If we assume $r=4$ and $S_0=2$, then

$$\Sigma_h^2 \approx 0.03\langle R\rangle(\text{microns})^2.$$

4. Momentum Straggling

Range straggling, of course, refers only to the distribution of ranges of particles all of which have the same initial momentum. With finite source and detector dimensions, it is not possible to obtain groups of particles all of the same momentum, but momentum distributions are involved. In a limited momentum interval, the mean range is given precisely[8] by $\langle R\rangle = cp_1^q$, where c and q are constants and p_1 is the momentum. Therefore, if the quantity $k=Rp_1^{-q}$ is measured, its percentage standard deviation will be the same as that of R. It is, however, often impossible to relate individual ranges and momenta, but only the momentum distribution is known. From a finite source, particles with a distribution of true momenta p_1 contribute to the group with apparent momentum p.

In the experiments made by us, protons scattered out of the 184-inch cyclotron beam by a cylindrical rod target were bent through 180° in the cyclotron magnetic field and detected in emulsion. Because the target had a radius of only $\frac{1}{16}$ inch and was some 46 inches from the plate, the uncertainty of the momentum of any particular proton was only about one part in a thousand. Mesons were produced in a smaller target, but at a distance of only seven inches, so that the uncertainty in individual particle momenta was about four parts in a thousand. The apparent momentum p of each particle was calculated, under the assumption that it came from the center of the target. Then, by knowing the distributions of points from which it could have come, the distribution of momenta p_1 contributing to the group of particles of apparent momentum p was determined. Quantities ω_n were calculated[8] so that $\langle p_1^n\rangle = p^n(1+\omega_n)$, where the brackets indicate the expectation value of the random variable p_1^n. If Σ_m^2 is the range variance term introduced by the momentum uncertainty, then $\Sigma_m/\langle R\rangle \approx (\omega_{2q}-2\omega_q)^{\frac{1}{2}}$. In Table III these contributions to the range straggling have been listed.

The range distribution of mesons arising from the decay of pions stopped in the emulsion is affected somewhat by the inner bremsstrahlung, but a calculation based on the probability function given by Eguchi[9] confirmed the belief that the effect is negligible if all ranges deviating from the mean range by more than four standard deviations are discarded. One event of this sort was found.[10] A similar statement applies to the pi-mu decays that are abnormal because the pion decayed before it came to rest.

5. Finite Grain Spacing and Grain Size

If the particle is not strongly ionizing on entering the emulsion, the grains rendered developable may be separated by distances that are not negligible. If the mean grain spacing is a, then the probability[8] of not producing a grain in a distance x is $\approx e^{-x/a}$. The tracks appear shortened systematically by an amount a, and an additional range variance of a^2 is introduced. At the end of the track where the particle stops, the grains are usually dense, and this correction may be disregarded. When the particle starts from a well-defined point such as the terminus of a pi meson or in a star, the correction should not be made.

The finite grain size causes an additional uncertainty at the beginning and end of the track, but since again the diameter of C.2 grains is only 0.2μ, the correction for this effect is negligible in our measurements.

6. Surface Roughness

If the surface of the emulsion is rough, a further error is introduced in determining the point of entry. To study this, optical tests have been made on the surface. When an optically flat piece of glass was laid on emulsion surfaces, interference fringes were visible and usually widely spaced. Therefore, no correction for roughness effects seems necessary unless the angle of dip is extremely small.

[8] W. H. Barkas, University of California Radiation Laboratory Report UCRL-2327 (unpublished).

[9] T. Eguchi, Phys. Rev. **85**, 943 (1952).

[10] W. H. Barkas, Am. J. Phys. **20**, 5 (1952).

C. Geometrical Straggling

Two frequently employed measures of the particle range are affected by the scattering of the particle. The systematic shortening of the range and the additional straggling observed when these methods are used should be mentioned. In the first method, only the projection R_p of the particle range on the original direction of the particle motion is observed. In the second method, the straight-line distance R_c between the first and last point of the track is measured. The distribution of these measures of the particle range are separate topics of study, the investigation of which has been undertaken in another study.

D. Recapitulation of Straggling Effects in Emulsion

The observed variance of the rectified range, Σ^2, may be expected to consist of the sum of the following terms:

1. The Bohr straggling, Σ_B^2.—This is predictable from well-known theory, given the velocity, charge, and mass of the particle.

2. The macroscopic distortion straggling, Σ_a^2.—This is predictable from a knowledge of the mean range, the distribution of particle trajectories, and the distortion as a vector function of position.

3. The proportional straggling, $\Sigma_p^2=\Sigma_b^2+\Sigma_0^2+\Sigma_h^2$.—This straggling term really consists of three terms, each proportional to the mean range. The magnitudes of these terms do not depend on the mass or charge of the particles producing the tracks, but do depend on the inhomogeneity of the emulsion and somewhat on the observer. For C.2 emulsion and a skilled observer one estimates $\Sigma_p^2=0.06\langle R\rangle$ microns^2, where $\langle R\rangle$ is in microns.

4. The end straggling, Σ_e^2.—This arises from small end effects. It is practically independent of the particle range for short ranges, and its influence is negligible except when the range is small. For alpha particles in C.2 emulsion, for example, Σ_e is less than one-half micron.[11]

5. The momentum straggling, Σ_m^2.—This is not a true range straggling effect, but it is a component of the measured straggling when the group of tracks measured is not monoenergetic.

The straggling arising from each of the effects *1* to *5* is listed in Table III. The anticipated resultant straggling, Σ, is also calculated and compared with the corresponding measurements. The measurements themselves are described in Part III. We conclude that: (*a*) only extremely short ranges should suffer straggling in which end effects play the dominant part, (*b*) for somewhat greater ranges the proportional straggling should dominate unless large macroscopic distortion effects are present, and (*c*) the straggling of long ranges should be well described by the Bohr effect alone.

[11] F. C. Gilbert, Phys. Rev. **93**, 499 (1954).

TABLE III. Range straggling effects. Standard deviations in percent of particle mean range.

Particle group	Number of tracks	$\langle R\rangle$ microns	$100\frac{\Sigma_B}{\langle R\rangle}$	$100\frac{\Sigma_a}{\langle R\rangle}$	$100\frac{\Sigma_p}{\langle R\rangle}$	$100\frac{\Sigma_m}{\langle R\rangle}$	$100\frac{\Sigma}{\langle R\rangle}$	Observed straggling
μ^+	50	599	4.0	0.6	1.0	0.0	4.1	3.6±0.3
μ^+	51	606	4.0	1.3	1.0	0.0	4.3	4.6±0.4
μ^+	52	593	4.0	0.9	1.0	0.0	4.2	4.5±0.4
π^-	54	766	3.5	0.1	0.9	0.7	3.7	3.7±0.4
π^-	49	751	3.5	0.0	0.9	0.7	3.7	4.7±0.5
π^-	44	765	3.5	0.1	0.9	0.7	3.7	4.0±0.4
π^+	56	732	3.5	0.0	0.9	0.7	3.7	4.5±0.4
π^+	49	738	3.5	0.0	0.9	0.7	3.7	3.7±0.4
π^+	51	762	3.5	0.1	0.9	0.7	3.7	4.2±0.4
π^+	65	702	3.5	0.0	0.9	0.7	3.7	3.8±0.2
π^+	69	727	3.5	0.2	0.9	0.7	3.7	3.7±0.2
π^+	78	728	3.5	0.2	0.9	0.7	3.7	4.5±0.2
Proton	10	4513	1.35	0.05	0.36	0.24	1.42	1.2±0.2
Proton	10	4556	1.35	0.00	0.36	0.24	1.42	1.2±0.2
Proton	10	4618	1.35	0.02	0.36	0.24	1.42	1.2±0.2
Proton	10	4605	1.35	0.03	0.36	0.24	1.42	1.1±0.2
Proton	10	4680	1.35	0.16	0.36	0.24	1.42	1.5±0.3
Proton	10	4538	1.35	0.08	0.36	0.24	1.42	1.0±0.2
Proton	···	40	1.94	···	2.58	···	3.22	3.2[a]
Proton	···	100	1.74	···	2.44	···	2.98	3.0[a]
Proton	···	200	1.65	···	1.73	···	2.40	2.4[a]
Proton	···	450	1.55	···	1.16	···	1.94	2.2[a]

[a] From the curve through the measurements of Han and Endt (see reference 13).

III. STRAGGLING MEASUREMENTS

A. Measurement of the Range

The range of a charged particle track in these experiments is taken to be the length of the rectified track; that is, the distance along the track between the extremities of the first and last grains as seen by using a microscope with an oil-immersion objective. The process of measurement is to break up the track into substantially straight segments, to measure the horizontal and vertical components of each segment (allowing for the vertical shrinkage), to compute the length of each segment, and to add the lengths together. All the events on one plate were measured with a single microscope so that the relative ranges did not depend on the microscope calibration. The absolute calibrations of the microscopes were also known, however, to about 0.1 percent. The shrinkage factor was measured in another series of experiments and was 2.3±0.1 for the plates used.

B. Straggling of Muons

A range histogram of 558 positive mu mesons observed as decay products of pi mesons that stopped in 200-micron emulsion is presented in Fig. 1. As discussed in Sec. II.B.4, these muons may be considered monoenergetic. The measurements were made in several plates. Since the stopping power of each plate probably is different from that of the others, each range was multiplied by a "plate factor" before combining the data, so that the average range for each plate was the same. It has been shown[12] that this adjustment affects

[12] F. M. Smith, University of California Radiation Laboratory Report UCRL-2371 (unpublished).

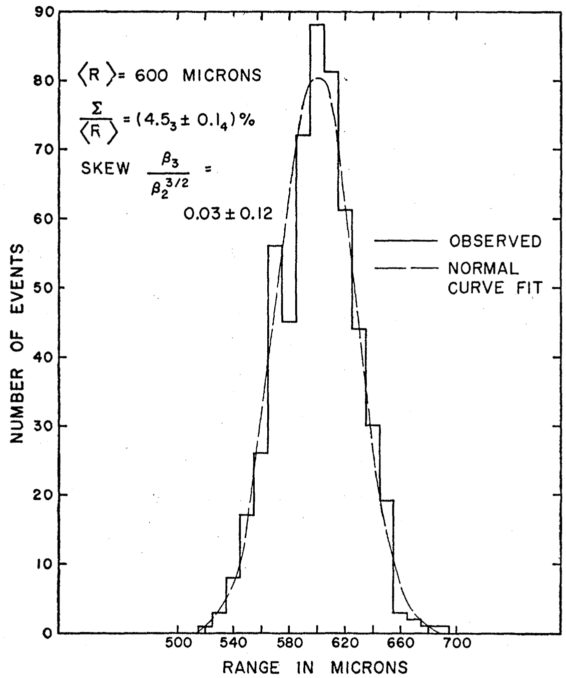

FIG. 1. Range distribution observed for positive mu mesons arising from the decay of stopped pi mesons in C.2 emulsion. The straggling contains contributions from a number of the effects that are analyzed in the text. Data from several plates are shown. The histogram is fitted well by a normal curve of the same standard deviation.

the range variance of the combined data by only a negligible amount.

The histogram has also been corrected for the "staying-in" probability. The probability is $\approx t/2R$ that a track of range R emitted in a random direction in an emulsion of thickness t ($t<R$) will remain in the emulsion. The mean range, including a correction of one micron for this effect, was close to 600 microns. Each plate was therefore normalized separately to this figure. The resultant range straggling is 4.53 percent. This is a typical figure for μ mesons, but it is clear that it will change for different plates and observers because the additional more or less controllable sources of straggling in emulsion always raise the measured values above the irreducible amount expected in homogeneous, rigid materials. The various straggling effects in three plates have been itemized in Table III. It is apparent that the known contributions to the straggling account reasonably well for the observed range variance of muons in emulsion.

C. Straggling of Protons

Measurements were made on six groups of protons as shown in Table III. The proton range was so great (about 4600 microns) that the distortion effects are negligible, and the target was so small that the momentum variance also was very small. Under these conditions, the range straggling should be approximately that derived from the theory of Bohr. Actually the straggling of 1.2 ± 0.1 percent observed, while reconcilable with the theoretical figure of 1.4 percent, is so low that a further investigation is justified. It is possible that requiring these long tracks to remain in the emulsion for their full length imposes some kind of restriction on the range variance. Other proton data of Han and Endt[13] at lower velocities are also listed in Table III. The agreement here is excellent. Incidental to this study, the range of 33.64-Mev protons in Ilford C.2 emulsion was found to be 4580 ± 18 microns. Although the emulsion humidity was not measured, the figure given is derived from six plates exposed at different times. In each case the plate was kept near the humidity at which it was packed, except for about one-half hour during which it was in vacuum. Variations in stopping power of about 1 percent were observed for various samples of emulsion.

D. Straggling of Pions

For a particle of a group distributed in a finite velocity interval, the product Rp^{-q} is called the "normalized range." The value 3.44 has been adopted[7] as the value of q in the vicinity of $\beta=0.27$. The value of q is determined by the condition that the expectation value of Rp^{-q} not depend on p. The error in q is known to be too small to affect the present measurements. In Table III are listed negative pion groups, which were measured on three plates exposed to mesons coming from a small target in the circulating beam of the 184-inch cyclotron. Also listed are six similar groups of positive pi mesons. In these cases the momentum uncertainty, although small, must be included in the calculated variance of the normalized range. It will be seen, on comparing the calculated and measured straggling, that here again the known effects probably are adequate to account for the observed straggling, although the average measured straggling somewhat exceeds the theoretical value.

IV. DISCUSSION

A rather complete analysis of the contributions to the range straggling in nuclear track emulsion is possible, so that from data describing the physical condition of the emulsion, the range straggling of particles of various masses becomes predictable. The unusually large straggling of μ^+-meson ranges found by Fry and White,[14] for example, although incomprehensible as straggling in a rigid homogeneous medium, can be reconciled with the theory if one assumes that considerable distortion was present in their emulsions. Observations that the range straggling is generally larger in thicker emulsions are also readily understood, since both the effect of distortion and errors in the shrinkage factor become more important as the thickness of the emulsion is increased.

[13] K. K. Han and P. M. Endt, Physica **20**, 311 (1954).
[14] W. F. Fry and G. R. White, Phys. Rev. **90**, 207 (1953).