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Slope of the λ Curve of Liquid Helium*

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Landau's theory of liquid helium is developed to give an approximate value of the slope of the λ curve in terms of the quantities $(\rho/\Delta)\partial\Delta/\partial\rho$, $(\rho/p_0)\partial p_0/\partial\rho$, and $(\rho/\mu)\partial\mu/\partial\rho$. From the experimental data on the coefficient of expansion and the pressure variation of the velocities of first and second sound, the following numerical values are derived for these quantities:

$$\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}=-0.57,\quad \frac{\rho}{p_0}\frac{\partial p_0}{\partial\rho}=+0.26,\quad \frac{\rho}{\mu}\frac{\partial\mu}{\partial\rho}=-1.8.$$

The negative slope of the λ curve can then be explained and its magnitude predicted with an accuracy of about 15 percent.

LANDAU'S latest theory of liquid helium[1] contains three arbitrary parameters, Δ, p_0, and μ, which occur in the expression relating the energy ϵ of a roton to its momentum p:

$$\epsilon=\Delta+(p-p_0)^2/2\mu. \tag{1}$$

The contribution of the rotons to the entropy can be shown to be

$$S_r=\frac{2k^{\frac{1}{2}}\mu^{\frac{1}{2}}p_0{}^2\Delta}{(2\pi)^{\frac{3}{2}}\rho T^{\frac{1}{2}}\hbar^3}\left(1+\frac{3kT}{2\Delta}\right)e^{-\Delta/kT}. \tag{2}$$

The contribution of the rotons to the fraction of normal component is

$$x_r=\rho_{nr}/\rho$$
$$=\frac{2\mu^{\frac{1}{2}}p_0{}^4}{3(2\pi)^{\frac{3}{2}}\rho(kT)^{\frac{1}{2}}\hbar^3}e^{-\Delta/kT}. \tag{3}$$

Landau pointed out that the temperature of the λ point, T_λ, may be obtained approximately by putting $x_r=1$, or

$$\frac{2\mu^{\frac{1}{2}}p_0{}^4}{3(2\pi)^{\frac{3}{2}}\rho(kT_\lambda)^{\frac{1}{2}}\hbar^3}e^{-\Delta/kT_\lambda}=1. \tag{4}$$

Using the experimental data on the entropy, S, and the velocity of second sound, u_2, along the vapor pressure curve, Khalatnikov[2] deduced that $\Delta/k=8.9\pm0.2$°K, $p_0=(2.1\pm0.05)\times10^{-19}$ deg cm sec^{-1}, $\mu=(1.72\pm0.68)\times10^{-24}$ g. Inserting these values in Eq. (4) one obtains $T_\lambda=2.55$°K, as compared with the observed value of 2.19°K. Exact agreement is not to be expected, because the simple theory underlying Eq. (3) is invalid above 1.6°K where the mutual interactions of the rotons become important. However, it is clear that Eq. (4) is capable of giving the right order of magnitude for T_λ and it might be added that the rigorous mathematical treatment of λ transitions is now known to be so complicated that no simple theory could expect to make exact numerical predictions.

We might therefore hope to obtain an approximate value for the slope of the λ curve by differentiating Eq. (4) with respect to the density ρ. The result is

$$\frac{\rho}{T_\lambda}\frac{\partial T_\lambda}{\partial\rho}\left(\frac{\Delta}{kT_\lambda}-\frac{1}{2}\right)=\frac{\Delta}{kT_\lambda}\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}-\frac{1}{2}\frac{\rho}{\mu}\frac{\partial\mu}{\partial\rho}-\frac{4\rho}{p_0}\frac{\partial p_0}{\partial\rho}+1. \tag{5}$$

From their measurements of the coefficient of expansion, Atkins and Edwards[3] deduced that

$$\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}=-0.57, \tag{6}$$

$$\frac{1}{2}\frac{\rho}{\mu}\frac{\partial\mu}{\partial\rho}+\frac{2\rho}{p_0}\frac{\partial p_0}{\partial\rho}=-0.38. \tag{7}$$

In addition, it is necessary to know the value of $(\rho/p_0)\times\partial p_0/\partial\rho$. Khalatnikov[2] has quoted $(\rho/p_0)\partial p_0/\partial\rho\sim+\frac{1}{3}$, but gives no details of his calculations. He probably used a method similar to that given below, but his estimate must have been very rough since it was made before the recent measurements of entropy[4] and coefficient of expansion[3] and it is also not clear whether he made an adequate allowance for phonon effects.

To obtain $(\rho/p_0)\partial p_0/\partial\rho$, we start with the equation for the velocity of second sound:

$$u_2{}^2=\frac{\rho_s}{\rho_n}\frac{TS^2}{C}. \tag{8}$$

Differentiating with respect to the pressure,

$$\frac{1}{x(1-x)}\frac{\partial x}{\partial p}=-\frac{2}{u_2}\frac{\partial u_2}{\partial p}-\frac{2V\alpha_p}{S}+\frac{TV}{C}\left(\frac{\partial\alpha_p}{\partial T}+\alpha_p{}^2\right). \tag{9}$$

Hence, knowing the coefficient of expansion α_p as a function of temperature[3] and u_2 as a function of pressure,[5,6] $\partial x/\partial p$ can be derived. Here x is the total

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fraction of normal component including both roton and phonon contributions. But the phonon contribution is

$$x_{ph}=TS_{ph}/c^2, \tag{10}$$

where c is the velocity of first sound. Hence

$$\frac{\partial x_{ph}}{\partial p}=-\frac{T}{c^2}\left(V\alpha_{ph}+2S_{ph}\frac{1}{c}\frac{\partial c}{\partial p}\right), \tag{11}$$

and as S_{ph} and $(1/c)\partial c/\partial p$ are known,[7,8] $\partial x_{ph}/\partial p$ can be obtained and subtracted from $\partial x/\partial p$ to give $\partial x_r/\partial p$.

Differentiating (2) and (3) with respect to ρ, $(\rho/p_0)\times\partial p_0/\partial\rho$ may finally be determined from

$$2\frac{\rho}{p_0}\frac{\partial p_0}{\partial\rho}=\frac{1}{x_rK_T}\frac{\partial x_r}{\partial p}+\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}\frac{1}{(1+3kT/2\Delta)}+\frac{V\alpha_r}{S_rK_T}, \tag{12}$$

where K_T, the isothermal compressibility, and α_r, the contribution of the rotons to the coefficient of expansion, are known.[3] Taking $(1/u_2)\partial u_2/\partial p$ from the measurements of Peshkov and Zinoveva,[5] $(\rho/p_0)\partial p_0/\partial\rho$ is found to be $+0.26$. However, this result is very sensitive to $(1/u_2)\partial u_2/\partial p$ and, if the measurements of Maurer and Herlin[6] are used, $(\rho/p_0)\partial p_0/\partial\rho$ varies from -1.12 at 1.0°K to -0.23 at 1.6°K. As it should, in principle, be independent of temperature, we have preferred to use the Russian measurements, but it would be desirable to make a further investigation of the variation of u_2 with pressure, placing particular emphasis on the lower pressures to obtain an accurate value for the initial slope. It is also important to note that all the parameters discussed here are sensitive to the values adopted for S and C. We have used the recent values of Hercus and Wilks[4] which are about 10 percent higher than previous values.

If we accept $(\rho/p_0)\partial p_0/\partial\rho=+0.26$, Eqs. (6) and (7) lead to $(\rho/\mu)\partial\mu/\partial\rho=-1.8$. It appears that the effective mass μ varies much more rapidly with ρ than either Δ or p_0.

Equation (5) may now be evaluated to yield $(\rho/T_\lambda)\times\partial T_\lambda/\partial\rho=-0.42$. The experimental value is -0.37. In view of the approximation involved in neglecting roton interactions, the agreement is very satisfactory. It seems that both the negative coefficient of expansion and the negative slope of the λ curve are caused primarily by the negative value of $(\rho/\Delta)\partial\Delta/\partial\rho$.

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Phenomenological Theory of Townsend Breakdown in Dielectrics

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The Townsend model of dielectric breakdown is discussed. The transient equations describing the behavior of the electron and positive ion current densities are solved rigorously with boundary conditions appropriate to a simplified Townsend model employing photoelectron production at the cathode as the only secondary mechanism. The correct boundary condition is found to lead to a set of integro-difference equations. Solution of the integro-difference equations allows rigorous specification of the criterion for breakdown. This criterion is found to become equivalent to the familiar Townsend criterion in the asymptotic limit of large time intervals. The results of our development may be applied to the calculation of formative time lags and to the estimation of space-charge distortion effects.

1. INTRODUCTION

When a dielectric material is subjected to an electrical stress of sufficient magnitude it is converted into a conductor. Breakdown occurs when the conductivity of the material becomes sufficient to maintain a current limited only by the external circuit. In his pioneering work on electrical discharges Townsend found that in most instances the conversion of an insulator to conductor is caused by the ionization of the original dielectric material.

The ionization of the dielectric can be initiated by any of several well-known external means, or by the presence of stray electrons in the system. The electrons created by the initial ionization burst become accelerated in the applied electric field and in turn produce additional ionization. Such an avalanche of ionization can, however, produce no breakdown as long as the field remains static and essentially homogeneous since the electrons created in the ionization avalanche are removed from the system by the applied field. In order to achieve a self-sustaining discharge, it becomes necessary to introduce a secondary, regenerative source of electron production; although it is conceivable that in some rare instances breakdown can be achieved purely by some highly effective field distortion mechanism instead of a secondary electron production process.

Various schemes have been proposed to explain the secondary electron production process active in the breakdown of dielectrics by static fields. We shall confine our attentions to what we shall term Townsend