Ambipolar Diffusion in a Magnetic Field

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Diffusion of ions in a plasma across a magnetic field is shown to be *not* ambipolar in character in most arc experiments. Owing to the highly anisotropic conductivity of the medium, the ions diffuse across the field at their own intrinsic rate. Space-charge neutralization is maintained by slight adjustments of the currents in the direction of the magnetic field lines. The discrepancy between theory and experiment noted by Bohm is thus resolved and no additional mechanisms, such as plasma oscillations, need be postulated.

In the absence of a magnetic field, electrons would tend to diffuse out of a plasma much more readily than the ions. These unequal diffusion rates immediately produce large space charges and hence electric fields in the plasma so as to reduce the electron current, increase the ion current and maintain space charge neutralization. The resultant ambipolar diffusion coefficient D, which is the same for both electrons and ions, is of the order of magnitude[1]

$$D \cong 2D_+D_-/(D_+ + D_-), \tag{1}$$

where D_+ and D_- are the intrinsic ion and electron diffusion coefficients respectively. Since $D_- \gg D_+$, we have in the absence of a magnetic field

$$D \cong 2D_+, \tag{2}$$

and the diffusion proceeds approximately at the rate of the *slower* component.

The effect of a strong magnetic field is to reduce the coefficients of diffusion, $D_\perp$, across the magnetic field to the values

$$D_{+\perp} = \frac{D_+^0}{1+(\omega_+\tau_+)^2}, \qquad D_{-\perp} = \frac{D_-^0}{1+(\omega_-\tau_-)^2}, \tag{3}$$

where the superscript zero denotes the coefficient for $H=0$, $\omega_\pm = eH/m_\pm c$, and $\tau_\pm$ = mean free time between collisions for ions or electrons respectively. Diffusion in the direction of the magnetic field, $D_{\parallel}$, is unchanged from its field-free value. In most arc experiments $(\omega_-\tau_-) \gg (\omega_+\tau_+) \gg 1$ and as a result

$$D_{-\perp} \ll D_{+\perp} \ll D_\pm^0 = D_{\pm\parallel}. \tag{4}$$

It might now be supposed, since the ions diffuse more rapidly across the magnetic field, that electric fields would arise which would reduce the ion current and which would result in a new ambipolar diffusion coefficient, as before, of the magnitude

$$D_\perp \cong 2D_{-\perp}. \tag{5}$$

Just such an analysis has been applied by Bohm[2] to experiments on the diffusion of ions in an arc plasma across a magnetic field. For the conditions of these arcs, such as occur in Calutron ion sources, $(\omega_-\tau_- \cong 10^5)$, Eq. (5) yielded an expected value of $D_\perp = 20$ cm²/sec. The observed values[3] were more like $D_\perp = 3\times 10^3$ cm²/sec. As a result of this large discrepancy, Bohm postulated that the principal mechanism of diffusion was by means of plasma oscillations. However, subsequent experimental attempts[4] to produce these oscillations have proven to be fruitless.

It is the purpose of this note to point out that diffusion across the magnetic field in all experiments of this type is *not* ambipolar in character. The discrepancy between theory and experiment noted by Bohm therefore does not exist and no additional mechanism, such as plasma oscillations, need be postulated.

The absence of ambipolar diffusion is due to the highly anisotropic conductivity in the medium. A section of an arc chamber is sketched in Fig. 1. Consider a fluctuation in which some positive ions begin streaming to the right, producing a region, 2, of net positive charge, and leaving a region, 1, of net negative charge to the left. An electric field immediately builds up to counteract this separation. However, this electric field has an enormously greater effect on the currents in the direction of the magnetic field lines because of the enormously greater $[(\omega\tau)^2 \gg 1]$ conductivity of the medium in this direction compared to that across the field lines. As a result, space charge neutralization is maintained by slightly reducing the electron current from region 2 to the end wall and slightly increasing it from region 1 to the end wall. Of course, an electric field of the same order of magnitude is produced in the horizontal direction. This field is entirely too small to have any effect on the perpendicular diffusion because of the greatly reduced conductivity in this direction. One way of picturing the effect is to observe that the combined effect of the increased electron current to the wall from region 1, the presence of the end wall, and

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[1] See M. A. Biondi and S. C. Brown, Phys. Rev. **75**, 1700 (1949).

[2] A. Guthrie and R. K. Wakerling, *The Characteristics of Electrical Discharges in Magnetic Fields* (McGraw-Hill Book Company, Inc., New York, 1949), National Nuclear Energy Series, Plutonium Project Record, Vol. 5, Div. 1, p. 197.

[3] See reference 2, p. 201, Eq. (12).

[4] D. H. Looney and S. C. Brown, Phys. Rev. **93**, 965 (1954).

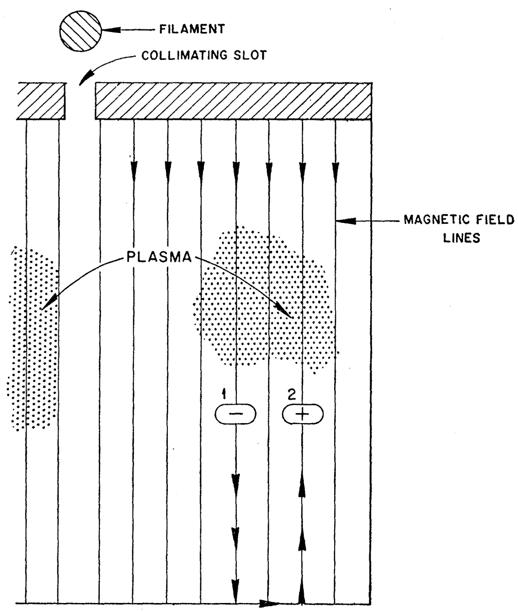

FIG. 1. Illustration of the "short-circuit" effect.

the decreased electron current from region 2 is equivalent to an electron "short circuit" which obviates the need for an ambipolar electric field in the perpendicular direction. As a result, electrons and ions do not diffuse at the same rate across the magnetic field, but rather with the coefficients given by Eq. (3). Space-charge neutralization is maintained by slight modification of the currents to the wall in the direction of the magnetic field lines.

Another way of looking at this effect is to note that in a medium with anisotropic conductivity it is no longer necessary that the currents flowing to the wall balance at the same equilibrium value in each direction. A positive excess to the side wall will be balanced by a negative excess to the bottom and top walls. As a consequence, ambipolar diffusion may be restored if the end walls are separately insulated and allowed to float.

A formal derivation of this effect may be obtained by revising the results of reference 2 so as to include the effect of the potential V, due to the deviation of the plasma from neutrality, upon the diffusion in the direction of the magnetic field lines. Equations (6) and (7) of reference 2 now become (in our notation)

$$D_{+\perp}\frac{d^2n_+}{dx^2}+\frac{D_{+\perp}e}{kT_+}\frac{d}{dx}\left(n_+\frac{dV}{dx}\right)+\frac{D_{+\parallel}e}{kT_+}\frac{d}{dz}\left(n_+\frac{dV}{dz}\right)=\frac{\beta n_+}{l}, \quad (6)$$

$$D_{-\perp}\frac{d^2n_-}{dx^2}-\frac{D_{-\perp}e}{kT_-}\frac{d}{dx}\left(n_-\frac{dV}{dx}\right)-\frac{D_{-\parallel}e}{kT_-}\frac{d}{dz}\left(n_-\frac{dV}{dz}\right)=\frac{\gamma n_-}{l}. \quad (7)$$

These equations are a statement of the conservation of charge for ions (n_+) and electrons (n_-) respectively. The x-direction is perpendicular to the magnetic field direction and the z-axis is parallel to this field. The quantities β and γ represent the mean velocity of diffusion of ions and electrons respectively along the field lines to the end walls of the arc chamber in the absence of the small perturbation due to the potential V.

The terms involving dV/dz were omitted in reference 2. However, these terms are many orders of magnitude larger than the terms involving dV/dx. To see this it is sufficient to recognize that $d[n_+dV/dx]/dx$ will be of the same order of magnitude as $d[n_+dV/dz]/dz$ while $(D_{+\parallel}/D_{+\perp})\cong(\omega_+\tau_+)^2\cong10^5$ for the usual arc conditions. Hence the second term on the left side of Eqs. (6) and (7) may be neglected. Assuming space-charge neutralization, $n_+\cong n_-$, dV/dz can be eliminated between these equations, giving

$$(D_{+\perp}D_{-\parallel}T_++D_{-\perp}D_{+\parallel}T_-)\frac{d^2n_+}{dx^2}=\frac{n_+}{l}(\beta D_{-\parallel}T_++\gamma D_{+\parallel}T_-). \quad (8)$$

The solution of this differential equation which vanishes for large values of x is

$$n_+=A\exp\{-x/x_0\},$$

where

$$x_0=\left[\frac{l(D_{+\perp}D_{-\parallel}T_++D_{-\perp}D_{+\parallel}T_-)}{(\beta D_{-\parallel}T_++\gamma D_{+\parallel}T_-)}\right]^{\frac{1}{2}}. \quad (9)$$

Now $D_{+\perp}\gg D_{-\perp}$, $D_{-\parallel}\gg D_{+\parallel}$ and $\beta\cong\gamma$. Hence Eq. (9) may be rewritten as

$$x_0\cong[lD_{+\perp}/\beta]^{\frac{1}{2}}, \quad (10)$$

which differs from the result given in Eq. (11) of reference 2 in that the quantity $D_{+\perp}$ has replaced $(1+T_+/T_-)D_{-\perp}$.

The theoretically expected value of $D_{+\perp}$ as calculated from Eq. (3) for the conditions of the Bohm experiment is

$$D_{+\perp}\cong3\times10^3\ \mathrm{cm^2/sec},$$

in agreement with the observed value.