E_1 and E_2 are their total energies in the laboratory system. Upon solving Eqs. (3)–(5) for $\theta=\theta_1+\theta_2$, one finds

$$\cos\theta=\frac{E_1E_2-A}{(E_1^2-m_1^2)^{\frac{1}{2}}(E_2^2-m_2^2)^{\frac{1}{2}}},\tag{6}$$

where

$$A\equiv(m_0^2-m_1^2-m_2^2)/2.\tag{7}$$

Upon differentiating Eq. (6), one obtains (using $\partial E_2/\partial E_1=-1$),

$$-\sin\theta d\theta=\cos\theta\left[\frac{E_2-E_1}{E_1E_2-A}-\frac{E_1}{(E_1^2-m_1^2)^{\frac{1}{2}}}+\frac{E_2}{(E_2^2-m_2^2)^{\frac{1}{2}}}\right]dE_1.\tag{8}$$

From (2) and (8), one finds

$$\frac{dn}{d\theta}=\frac{m_0}{2p_0p_1^*}\left|\tan\theta\Big/\left[\frac{E_1-E_2}{E_1E_2-A}+\frac{E_1}{(E_1^2-m_1^2)^{\frac{1}{2}}}-\frac{E_2}{(E_2^2-m_2^2)^{\frac{1}{2}}}\right]\right|.\tag{9}$$

$dn/d\theta$ is found by first calculating θ for several assumed values of E_1 from (6), and then $dn/d\theta$ from (9). In these expressions, $E_2=E_0-E_1$, where E_0= total energy of the incident particle. The extreme values of E_1 are obtained when $\theta=0°$ or 180°. Figure 1 shows $dn/d\theta$ for θ^0 particles of various kinetic energies T_0. There are two regions of T_0: (1) For $T_0<378$ Mev, the velocity v_0 of θ^0 is less than the velocity v_1^* of the π's in the θ^0 rest system. There is a minimum angle θ_s corresponding to symmetric decay about the incident direction ($\theta_1=\theta_2$, $E_1=E_2=E_0/2$). θ_s is given by

$$\cos\theta_s=(\tfrac{1}{4}E_0^2-A)/(\tfrac{1}{4}E_0^2-m_1^2).\tag{10}$$

$dn/d\theta$ becomes infinite at $\theta=\theta_s$, since the denominator of (9) vanishes for $E_1=E_2$. This case is similar to π^0 decay. (2) For $T_0>378$ Mev, $v_0>v_1^*$, the possible values of θ extend from 0° to a maximum θ_m. By a method given previously,[3] it can be shown that for $m_1=m_2$, θ_m is given by

$$\tan\theta_m=(1-v_0^2)^{\frac{1}{2}}/(v_0^2-v_1^{*2})^{\frac{1}{2}}.\tag{11}$$

At $\theta=\theta_m$, $dn/d\theta$ becomes infinite since $\partial\theta/\partial E_1=0$ [Eq. (2)]. The curves of $dn/d\theta$ have two branches between θ_s and θ_m, because for a given θ in this region, there are two possible values of E_1, which lead to the two values of $dn/d\theta$. The total $dn/d\theta$ is the sum of the terms due to the two branches.

We note that for case (1), θ is close to θ_s for a large fraction of the decays. Thus for $T_0=100$ Mev, θ lies between θ_s (=101.8°) and 110° for ~65 percent of the decays. Similarly, for $T_0=500$ Mev [case (2)], θ lies between θ_s (=50.3°) and θ_m (=61.1°) for ~95 percent of the decays. This correlation between T_0 and the most likely value of θ could be used to estimate the energy of the incident θ^0 particles in a suitable counter experiment.[4] Moreover, the existence of a minimum angle θ_s for case (1) and of a maximum angle θ_m for case (2) may be used to put an upper or a lower limit on T_0.

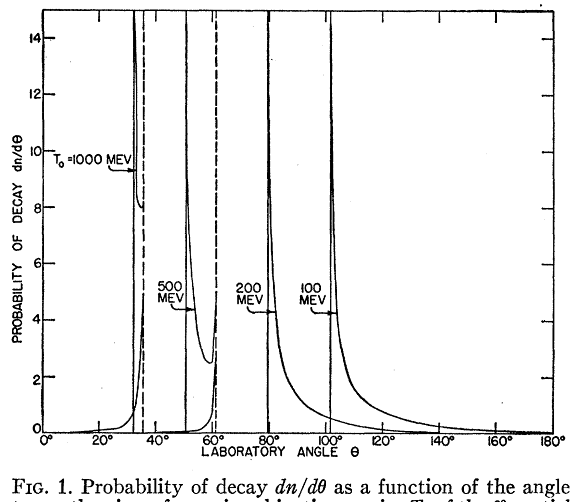

FIG. 1. Probability of decay $dn/d\theta$ as a function of the angle θ between the pions, for various kinetic energies T_0 of the θ^0 particle. The full vertical lines correspond to the angle θ_s for symmetric decay; the broken vertical lines correspond to the maximum angle θ_m.

I would like to thank Dr. R. Madey for suggesting this problem and for helpful discussions.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

1 B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., New York, 1952), pp. 192, 198.

2 It is assumed that the units are such that $c=1$.

3 R. M. Sternheimer, Phys. Rev. **93**, 642 (1954).

4 Equation (9) can also be used to obtain the distribution in θ for the Λ^0 decay. In this case, for $T_0<252$ Mev, when the velocity v_0 of Λ^0 is less than the velocity v_π^* of the pion in the Λ^0 rest system, all angles θ are possible. For $T_0>252$ Mev, $v_0>v_\pi^*$, there is a maximum angle θ_m, and the spectrum is double-valued between $\theta=0°$ and θ_m, where $dn/d\theta$ becomes infinite.

Angular Correlation in Two-Photon Annihilation in Quartz*

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(Received January 7, 1955)

A NARROW component has been found in the angular correlation of two-quantum annihilation radiation in fused quartz at room temperature, while crystalline quartz exhibits no such anomaly. A brief description of the experimental evidence follows.

Measurements are being made of the angular correlation, *circa* 180°, between two photons in coincidence, arising when a positron has entered a thick absorbing

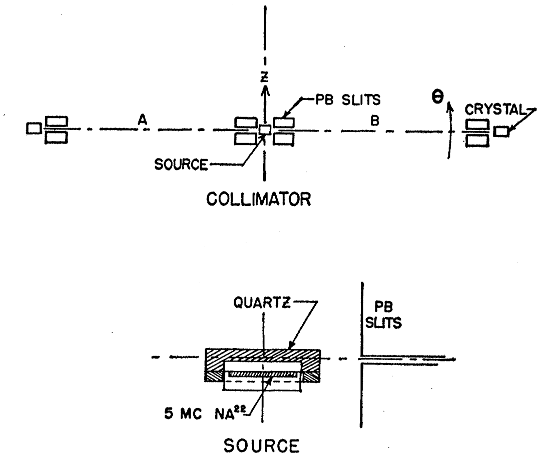

FIG. 1. Apparatus. Each detector-crystal is 120 cm from source. The x-axis is perpendicular to the plane of the paper.

sample. The apparatus measures essentially the z-component of total momentum of the two photons detected (see Fig. 1). Collimator-detector system A remains fixed, while identical system B may be rotated in the plane normal to an axis, x, centered on the region in the sample where the annihilation events take place. The geometry is thus similar to that first used by DeBenedetti *et al.*,[1] except that here it is arranged that the detectors see, for practical purposes, only radiation from the sample itself.

Some experimental work on a "crystal" of native graphite to measure the relative width-at-half-maximum for different basal plane orientations showed that the crystal structure's effect should be less than 1 percent[2] and the apparatus and circuitry is reliable to approximately 0.2 percent (based on some 10^7 coincidences, with the slit width $\Delta\theta=5\times10^{-3}$ radian). We do not know at this time, nor are we concerned about, the orientation of the crystal axes of the particular sample in use.

Geometrically identical fused and crystal quartz samples, denoted by F and C, respectively, were compared first as regards relative half-width, and with moderate resolution, $\Delta\theta=5\times10^{-3}$ radian. The positron source,[3] five mC of Na^{22}, provides some twenty coincidences/second of which less than 0.5 percent are accidentals. The result was (width C)/(width F)$\sim$1.2 uncorrected for finite resolution of apparatus. The nominal half-width is 12×10^{-3} radian.

There is a difference in shape between the F-curve and the C-curve occurring mainly for angles between the half-value points. Figure 2(a) shows the F- and C-curves superposed, for $\Delta\theta=5\times10^{-3}$ radian. The "difference," $(F'-C)/C_{\max}$ denoted by γ, is shown as Fig. 2(b). F' is the F-rate arbitrarily normalized. The geometry was then narrowed a factor of two, with $\Delta\theta=2.5\times10^{-3}$ radian, and the resulting difference is Fig. 2(c). Finally, with $\Delta\theta=0.8\times10^{-3}$ radian, the F'- and

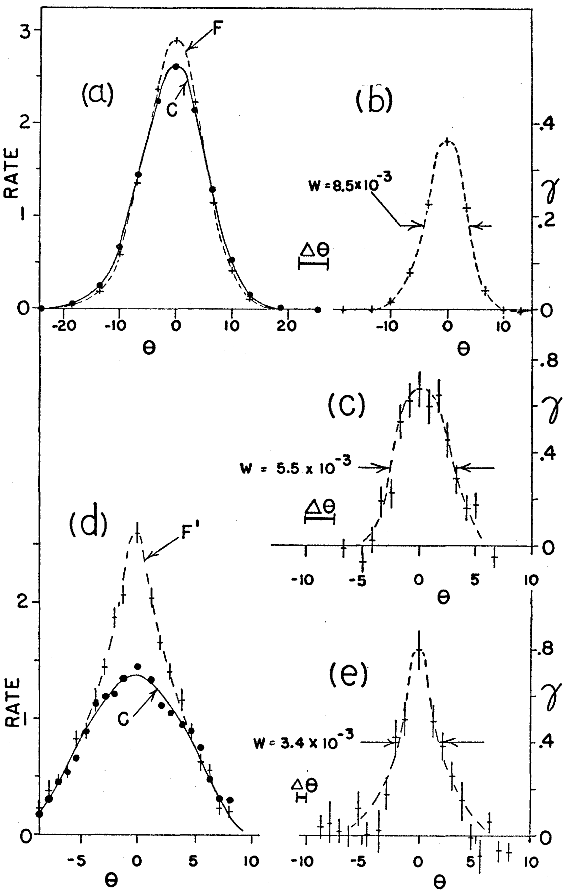

FIG. 2. Angular correlation curves for fused and crystal quartz. Angle θ is in units of 10^{-3} radian. Half-width, w, is in radians.

C-curves of Fig. 2(d) were run, yielding the difference curve Fig. 2(e). The half-width of the difference curve has thus narrowed, without affecting its area significantly.

The best estimate of the fraction of two-quantum annihilation belonging to this narrow component is to be had from the data of curves (a) and (b) which have good statistics. Actually an empirical integration has been made by moving scintillator A by half-inch steps parallel to the x-axis, running complete θ dependence at each step; this procedure eliminates from consideration the finite extension of the scintillators (only $1\frac{1}{4}$ inches) in the x-direction. It is this method which gives difference curve (b) whose area relative to the total two-quantum area for the F-sample is found to be 0.176 ± 0.015. The remaining curves of the figure are based on one x-position only.

The ultimate half width of the narrow component in fused quartz seems to be $\sim3\times10^{-3}$ radian, while the broader component, taken to be that for crystalline quartz, appears to be $\sim11\times10^{-3}$ radian.

That the narrow component should appear in fused quartz and not in crystalline quartz is reminiscent of the behavior of the long-life component found some years ago by Bell and Graham for annihilation in quartz and various other substances.[4] Perhaps the narrow component of the two photons may stem from a simple positron-electron system with v/c sensibly less than 1/137. If the narrow component should be formed only "in parallel" with the long-half-life component, one might expect the narrow fraction to be one-third of the long-half-life fraction of Bell and Graham, which was 0.29.[4] Perhaps a portion of the two-quantum events supposed to arise from the triplet state appear in the narrow component.

The present experiment is being extended to include other materials under various conditions. Preliminary work on teflon, at room temperature, indicates an angular correlation similar to that for fused quartz.

* Work done in the Sarah Mellon Scaife Radiation Laboratory and supported by the Office of Ordnance Research.

1 DeBenedetti, Cowan, Konneker, and Primakoff, Phys. Rev. **77**, 205 (1950).

2 University of Pittsburgh (unpublished).

3 We are indebted to Dr. A. J. Allen for the preparation of the source.

4 R. E. Bell and R. L. Graham, Phys. Rev. **87**, 236 (1952); **90**, 644 (1953).

Angular Distribution of Pickup Deuterons for 95-Mev Protons on Carbon, and Implications as to Internal Interactions in Carbon*

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(Received January 24, 1955)

The angular distribution of the sharp energy group of deuterons observed in this reaction has been analyzed by Born-approximation pickup theory, to give the internal momentum distribution of the picked-up neutrons. The momentum distribution shows relatively strong high-momentum components, and these in turn indicate the presence of a strong short-range interaction in carbon.

A previous brief report[1] described the sharp energy distribution of deuterons observed in this reaction. The sharply defined group has been studied for laboratory angles between 6° and 60°. At the largest angles, the group still is clearly recognizable, although because of the increasing relative intensity of a continuum distribution of deuterons, the absolute cross section to be attributed to the "line" is uncertain to a factor of perhaps two. An energy distribution at 42° is shown in Fig. 1, and Fig. 2 shows the angular distribution of the sharp group.

Following Chew and Goldberger,[2] a Born-approximation calculation of the reaction yields from the angular distribution the internal momentum distribu-

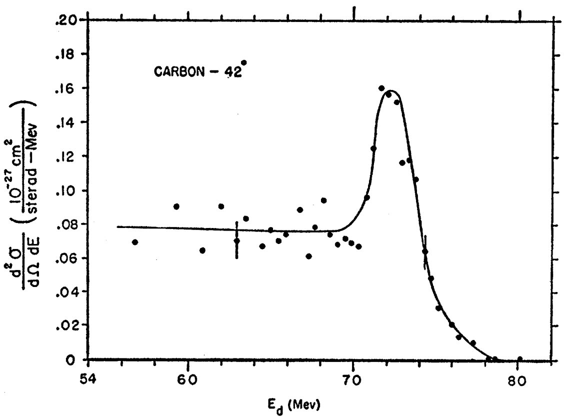

Fig. 1. Energy distribution of deuterons from the $C^{12}(p,d)$ reaction at 95 Mev. Statistical uncertainty is shown for two representative points.

tion of the picked-up neutron. CG show that the differential cross section $\sigma(\theta)$ is proportional to $N(\mathbf{n})F(q)$, where $N(\mathbf{n})$ is the momentum distribution density of neutrons of momentum $\mathbf{n}$ and $F(q)$ is a factor which is a function of the internal momentum of the formed deuteron; if one uses a Hulthén wave function for the deuteron then $F(q)$ has a relatively weak angular dependence. In the present case it falls by a factor of about $2\frac{1}{2}$ while the center-of-mass differential cross section is falling by 40, over the angular range covered.

Some discussion is in order with regard to the interpretation of the deuteron pickup results in terms of internal momentum distributions, in view of the fact that in the same type of theory applied by Butler[3] at low energies with very good success, the results do not seem to depend on the internal wave function $u(r)$ of the picked-up nucleon. In fact the effect of Butler's procedure is that the internal contribution to the

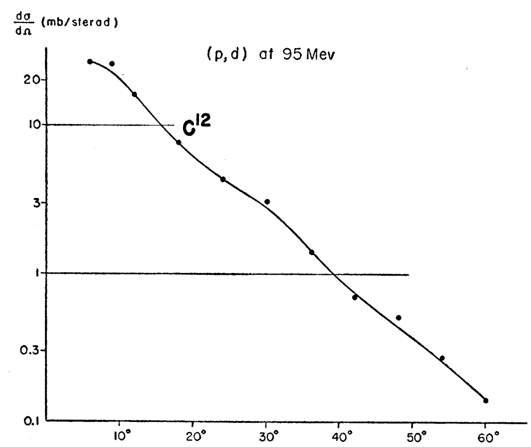

Fig. 2. Angular distribution (plotted *vs* lab angle) of the sharp energy group in the $C^{12}(p,d)$ reaction. The statistical uncertainty for the individual points is about 10 percent. There is additional systematic uncertainty in the separation of the "line" from the continuum—this uncertainty is negligible at small angles, but becomes a factor of about 2 at the largest angles.