surface energy (many-body forces, clustering into alpha particles, etc.). The significance of the present estimate lies in the fact that so long as these effect can be represented by an over-all potential (and this is suggested by the validity of the shell model), one would come back to a value ~6 Mev for $x_0=1-2\times10^{-13}$ cm. To obtain the value $4\pi R_0^2S=15$ Mev with $T_0=22$ Mev we must assume $x_0=6-8\times10^{-13}$ cm.

[1] H. A. Bethe and R. F. Bacher, Revs. Modern Phys. **8**, 83 (1936), especially p. 164.
[2] E. Feenberg, Phys. Rev. **60**, 204 (1941).
[3] W. J. Swiatecki, Proc. Phys. Soc. (London) **A64**, 226 (1951).
[4] Note that E. Feenberg, reference 2, or D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953), especially p. 1125, would give $4\pi r_0^2S=28$ Mev at $x_0=0$. The discrepancy by a factor 5 is discussed in reference 2.
[5] W. J. Swiatecki, "The effect of a potential gradient on the density of a degenerate Fermi gas," Proc. Phys. Soc. (London) (to be published).

Average Nuclear Potentials and Densities

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According to the preceding Letter,[1] the empirical magnitude of the nuclear surface energy, interpreted on the basis of the individual-particle model, suggests a nuclear potential well with rather gently sloping sides. In the present note we shall discuss some consequences of such a potential. For definiteness, let us consider the nucleus divided into an inner region $r<R_1$ where no average force is present and a surface region where particles experience a constant inward force. The implied potential wells and estimates of the associated densities of a Fermi gas are given in Fig. 1 for $A=27$, 64, 125, 216. The densities are based on the semi-infinite distribution used in (1) and disregard therefore effects of shells, the curvature of the nuclear surface, etc. From other work[2] it then appears that for the small potential gradient assumed here the simple Fermi relation, stating the proportionality of the density ρ and the $\frac{3}{2}$ power of the maximum kinetic energy $T(r)$, is adequate.[3] It will be noted that the wells do not appear to be inconsistent with a shell model potential (intermediate between square well and oscillator potential). For medium and large A, a region of constant density is implied. For small A, all nucleons should be regarded as subject to a field of force.

As a rule the density distributions are more compact than the corresponding potential wells. This is due to (a) the more rapid fall off of density ($\frac{3}{2}$ power of T) which expresses the statistical preference for particles to congregate in a region of deep potential, and (b) the finite nucleon separation energy (~8 Mev) which means that about a quarter of the potential rise occurs

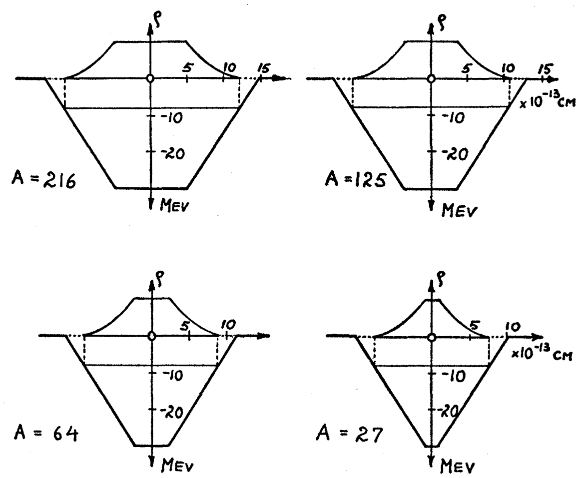

Fig. 1. Potential wells and Fermi densities for $A=27$, 64, 125, 216. The depth of the wells is 30 Mev, the Fermi energy $T_0=22$ Mev, the gradient at the surface T_0/x_0 with $x_0=7\times10^{-13}$ cm. With these density distributions the equivalent radius $\bar{R}$, defined as the radius of the sphere which would contain all the particles at a constant (the central) density, can be approximated by $\bar{R}\doteq R_1+x_0/2$ to better than 3.6 percent. We have taken $\bar{R}=1.4\times10^{-13}A^{\frac{1}{3}}$ cm.

beyond the turning point of the fastest particle where the density is practically zero.

The above refers to average nucleon densities. It is interesting to examine the neutron and proton densities (ρ_N,ρ_Z) separately. For a nucleus with $N>Z$, the zero point energies for neutrons are greater than for protons (Fermi energies $T_N>T_Z$) and the faster neutrons will, therefore, penetrate farther into the surface region, this enrichment of the surface in neutrons being inversely proportional to the gradient of the potential across the surface.[4] A simple estimate of the effect can be made by using $T_N/T_Z=(N/Z)^{\frac{2}{3}}$, when the difference in effective neutron and proton radii $\bar{R}_N-\bar{R}_Z$ becomes $\frac{2}{3}[(N-Z)/A]x_0$.[5] (For notation see caption to Fig. 1.)

Figure 2 shows, as a function of A, the values of $\bar{R}_N$ and $\bar{R}_Z$, as well as $R_{\frac{1}{2}}$, the radius at which the average potential has half its central value. If $R_{\frac{1}{2}}$ is taken to represent an average interaction radius for nucleons interacting with a potential well with sloping sides, then the curves for $R_{\frac{1}{2}}$ and $\bar{R}_Z$ illustrate the different radii appropriate for the interpretation of experiments with particles which are (nucleons) and are not (electrons or muons) subject to specifically nuclear interactions. The effect being associated with a sloping potential the difference between $R_{\frac{1}{2}}$ and $\bar{R}_Z$ should provide a measure of the thickness of the surface region. The empirical difference between the two sets of radii (about 20 percent) is consistent in order of magnitude with Fig. 2, based on a value $x_0=7\times10^{-13}$ cm (but not with $x_0=1-2\times10^{-13}$ cm).

A value of x_0 considerably greater than $1-2\times10^{-13}$ cm is suggested by very simple considerations. The thickness of the region in which a particle crossing a

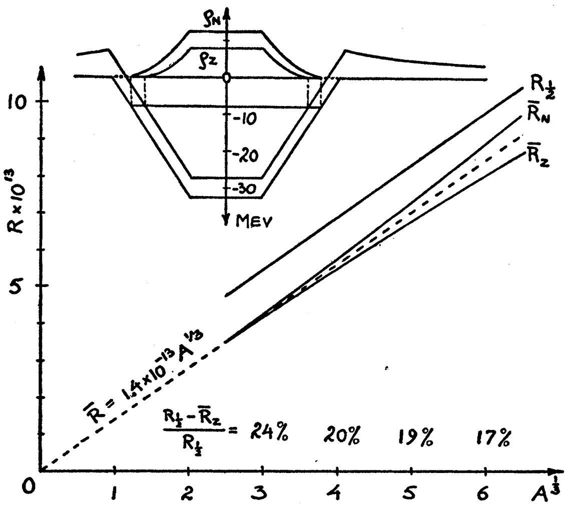

FIG. 2. The neutron and proton wells and densities for $A=216$, the equivalent radii $\bar{R}_N$, $\bar{R}_Z$ and the radius $R_{\frac{1}{2}}$ at half-depth of the average potential well. $\bar{R}$ has been taken as $1.4\times10^{-13}A^{\frac{1}{3}}$ cm, though one would be nearer to the experimental results by choosing $R_{\frac{1}{2}}\sim1.4\times10^{-13}A^{\frac{1}{3}}$ cm (to agree with high-energy neutron cross sections) and have the "electromagnetic" radius $\bar{R}_Z$ come out $\sim$20 percent less. The resulting value of $\bar{R}\sim1.1\times10^{-13}A^{\frac{1}{3}}$ cm corresponds to greater nucleon densities, which in turn imply a well depth of $8+(1.4/1.1)^2\times22=44$ Mev. It may be significant that a well depth $\sim$40 Mev is also suggested by recent evidence from low-energy neutron scattering experiments [R. K. Adair, Phys. Rev. 94, 737 (1954)]. With a greater value of T_0 the arguments of the preceding Letter would lead to a smaller estimate for x_0.

sharp discontinuity in the density would experience an appreciable resultant force would be twice the range of the forces. For a Fermi gas of particles the least diffuseness in the density at the surface is about $(2\pi)^{-1}$ times the wavelength of the fastest particle present (1×10^{-13} cm for a nucleus) realized when the gas is bounded by an infinite square well. This would give $x_0\sim(1+2\times1.4)\times10^{-13}=3.8\times10^{-13}$ cm. Since the density distribution is actually determined by some such resultant field of force and not by an infinite force at a sharp surface (square well), the density would diffuse further and there is no difficulty in understanding a final diffuseness of $x_0=7\times10^{-13}$ cm. On the contrary a value $x_0=1-2\times10^{-13}$ cm would be hard to explain.

The rather diffuse potential wells discussed above are not necessarily inconsistent with the recent analyses of high-energy electron scattering experiments which suggest that the transition region ΔR in which the *proton density* falls from 90 percent to 10 percent of its central value is around 2.4×10^{-13} cm,[6] or between 2.5 and 4.5×10^{-13} cm.[7] The considerations of the present note give $\Delta R=0.717(2Z/A)^{\frac{1}{3}}x_0$. Thus, for Au, $\Delta R=4.3\times10^{-13}$ cm if $x_0=7\times10^{-13}$ cm, or $\Delta R=3.1\times10^{-13}$ cm if $x_0=5\times10^{-13}$ cm.

Quantitative estimates apart, we would like to stress the qualitative considerations underlying this and the preceding Letter, namely the observation that faster particles are expected to explore greater volumes and that this points to a connection between the magnitude of the surface energy, the diffuseness of the nuclear surface, and the enrichment of the surface in neutrons.

[1] W. J. Swiatecki, preceding Letter [Phys. Rev. 98, 203 (1955)].
[2] W. J. Swiatecki, "The effect of a potential gradient on the density of a degenerate Fermi gas," Proc. Phys. Soc. (London) (to be published).
[3] For example, this relation gives $\rho=0$ at $T=0$; the more correct density calculated according to reference 2 would give $\rho=2.1$ percent of the central density.
[4] Compare reference 1. This is a way of stating the first of the two arguments suggested by M. H. Johnson and E. Teller [Phys. Rev. 93, 357 (1954)] to account for a greater neutron radius than proton radius. The second effect, associated with the region of exponential decay of wave functions, should be negligible in our case (see reference 3).
[5] A more general expression for T_N/T_Z, which corrects for the smaller proton radius implied by the effect under discussion, leads to $\bar{R}_N-\bar{R}_Z=\frac{2}{3}[(N-Z)/A](1/x_0+1/3\bar{R})^{-1}$.
[6] G. Ravenhall, Stanford University (private communication).
[7] G. Brown, University of Birmingham (private communication).

Angular Distribution of Pions from θ^0 Decay*

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IN view of the possibility of counter experiments to detect the θ^0 particles, it seems of interest to calculate the expected distribution in angle between the π^+ and π^- mesons as a function of the kinetic energy of the θ^0. The derivation follows closely that given by Rossi[1] for the π^0 decay. For the sake of generality, the masses of the decay particles will be taken as different, m_1 and m_2.

As shown by Rossi,[1] the probability that in a two-body decay, one of the outgoing particles has total energy E_1 in the laboratory system is given by

$$dn/dE_1=m_0/(2p_0p_1^*), \tag{1}$$

where dn = fractional number of decays in dE_1, m_0, and p_0 are the mass and momentum of the decaying particle, p_1^* = momentum of the emitted particles in the rest system. Hence, if θ is the angle between the emitted particles in the laboratory,

$$\frac{dn}{d\theta}=\frac{m_0}{2p_0p_1^*}\left|\frac{dE_1}{d\theta}\right|. \tag{2}$$

The conservation equations are[2]

$$(E_1^2-m_1^2)^{\frac{1}{2}}\sin\theta_1-(E_2^2-m_2^2)^{\frac{1}{2}}\sin\theta_2=0, \tag{3}$$

$$(E_1^2-m_1^2)^{\frac{1}{2}}\cos\theta_1+(E_2^2-m_2^2)^{\frac{1}{2}}\cos\theta_2=p_0, \tag{4}$$

$$p_0^2+m_0^2=(E_1+E_2)^2, \tag{5}$$

where θ_1 and θ_2 are the laboratory angles of the emitted particles with the direction of the incoming particle,