Nuclear Surface Energy and the Diffuseness of the Nuclear Surface

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IN the course of an attempt to interpret the empirical magnitude of the nuclear surface energy $S(4\pi R_0^2 S = 15$ Mev, $R_0 = 1.4\times10^{-13}$ cm), we have found reasons to believe that this quantity is intimately related to the diffuseness of the nuclear surface and should in fact provide a measure of the thickness of the surface region. Estimates given below suggest that to account for the observed S it is necessary to assume the thickness of the region in which nucleons experience an appreciable one-sided resultant force and the density is falling to be of the order $6-8\times10^{-13}$ cm. An estimate of some of the most obvious consequences of such a value did not seem to lead to contradictions with known properties of nuclei. Some arguments which appear to support it are discussed in the following Letter. It is suggested that the use of a nuclear potential with sloping sides in which the rise occurs in a distance of $6-8\times10^{-13}$ cm merits attention.

Since the surface energy is intimately connected with the deficiency in binding of particles in the surface, it is clear that any attempt at a quantitative account of the nuclear surface energy will come up against difficulties due to our insufficient understanding of the nature of the effects responsible for nuclear cohesion.[1–3] The idea behind the present attempt is to by-pass this difficulty in the same way that the individual particle model by-passes the question of nuclear interactions, that is, by using the fact that it appears to be possible with some success to replace the effect of interactions by an over-all potential acting on independent particles. This is a prescription which often enables one to make predictions about phenomena which in fact depend on the presence of nuclear interactions without having to specify these explicitly (the nuclear shell model). We have examined the consequence of this prescription for the question of the surface energy. The estimate of S which follows can be regarded as an attempt to apply the ideas of the independent particle model to the calculation of the surface energy.

In practice the problem is essentially that of estimating the surface energy of a Fermi gas of particles moving in some potential well with sloping sides. The total energy (kinetic plus potential) of any one particle in such a well *remains constant* as the particle enters the surface region and the appearance of a surface energy for the Fermi gas as a whole is then due to the circumstance that the faster particles penetrate further into the surface region, which thus becomes enriched in high-total-energy particles. A somewhat more general way of looking at this is to say that when filling a region of space with particles obeying the exclusion principle, the number of particles that can be accommodated in a given energy interval is proportional to the volume of ordinary space available. Since the more energetic (fast) particles are more difficult for the cohesive forces to contain, they will explore a greater volume (by an amount proportional to the surface area) and consequently more of them can be accommodated. We would like to stress that on the basis of the individual-particle model the surface energy is thus due to a simple effect of rather general validity (faster particles explore greater volumes) and to point out the immediate connection between the surface energy and the slope of the potential in the surface region. (The extra energy should increase with decreasing slope.)

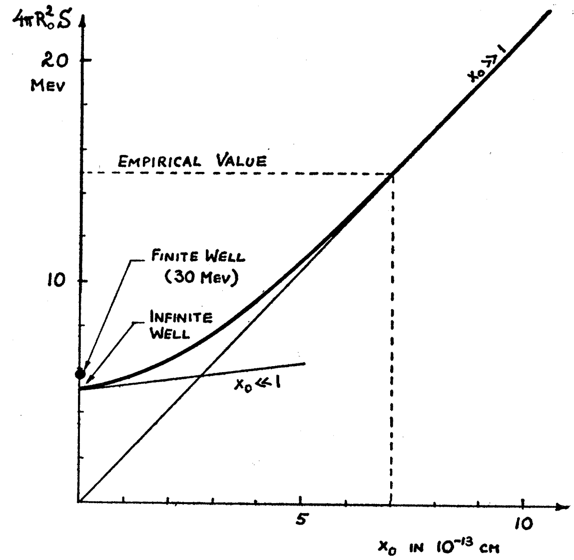

FIG. 1. The surface energy per unit area (defined as the energy associated with a number of particles in a volume touching unit area of surface minus the energy of the same number of particles inside the nucleus) for a Fermi gas with $T_0=22$ Mev bounded by a potential which rises to infinity with a constant slope equal to T_0/x_0. (Compare reference 5.) (The effect of levelling off the potential at about 30 Mev can be neglected due to the small number of particles in the region in question. The effect vanishes for $x\rightarrow\infty$ and is greatest for a square well ($x_0=0$) when it increases S by 12.5 percent as shown in the figure.) The curve plotted is an interpolation between the limits $x_0\rightarrow0$ and $x_0\rightarrow\infty$. The finite value of S at $x_0=0$ is a quantum mechanical effect which can be traced to the more efficient use of the available space by particles with a shorter wavelength.

A quantitative illustration is given in Fig. 1, which shows the surface energy for a semi-infinite Fermi gas (Fermi energy T_0) bounded by a region of linearly increasing potential, plotted as function of a thickness x_0, the distance in which the potential increases by T_0. The only parameter entering the calculation is T_0. No assumptions about nuclear forces have been made. At $x_0=1-2\times10^{-13}$ cm, $4\pi R_0^2 S$ is $5-6$ Mev,[4] to be compared with the empirical 15 Mev. In terms of specific assumptions about nuclear interactions, one could think of effects that would tend to increase or decrease the surface energy (many-body forces, clustering into alpha particles, etc.). The significance of the present estimate lies in the fact that so long as these effect can be represented by an over-all potential (and this is suggested by the validity of the shell model), one would come back to a value $\sim$6 Mev for $x_0=1-2\times10^{-13}$ cm. To obtain the value $4\pi R_0^2 S=15$ Mev with $T_0=22$ Mev we must assume $x_0=6-8\times10^{-13}$ cm.

[1] H. A. Bethe and R. F. Bacher, Revs. Modern Phys. **8**, 83 (1936), especially p. 164.
[2] E. Feenberg, Phys. Rev. **60**, 204 (1941).
[3] W. J. Swiatecki, Proc. Phys. Soc. (London) **A64**, 226 (1951).
[4] Note that E. Feenberg, reference 2, or D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953), especially p. 1125, would give $4\pi r_0^2 S=28$ Mev at $x_0=0$. The discrepancy by a factor 5 is discussed in reference 2.
[5] W. J. Swiatecki, "The effect of a potential gradient on the density of a degenerate Fermi gas," Proc. Phys. Soc. (London) (to be published).

Average Nuclear Potentials and Densities

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According to the preceding Letter,[1] the empirical magnitude of the nuclear surface energy, interpreted on the basis of the individual-particle model, suggests a nuclear potential well with rather gently sloping sides. In the present note we shall discuss some consequences of such a potential. For definiteness, let us consider the nucleus divided into an inner region $r<R_1$ where no average force is present and a surface region where particles experience a constant inward force. The implied potential wells and estimates of the associated densities of a Fermi gas are given in Fig. 1 for $A=27$, 64, 125, 216. The densities are based on the semi-infinite distribution used in (1) and disregard therefore effects of shells, the curvature of the nuclear surface, etc. From other work[2] it then appears that for the small potential gradient assumed here the simple Fermi relation, stating the proportionality of the density ρ and the $\frac{3}{2}$ power of the maximum kinetic energy $T(r)$, is adequate.[3] It will be noted that the wells do not appear to be inconsistent with a shell model potential (intermediate between square well and oscillator potential). For medium and large A, a region of constant density is implied. For small A, all nucleons should be regarded as subject to a field of force.

As a rule the density distributions are more compact than the corresponding potential wells. This is due to (a) the more rapid fall off of density ($\frac{3}{2}$ power of T) which expresses the statistical preference for particles to congregate in a region of deep potential, and (b) the finite nucleon separation energy ($\sim$8 Mev) which means that about a quarter of the potential rise occurs beyond the turning point of the fastest particle where the density is practically zero.

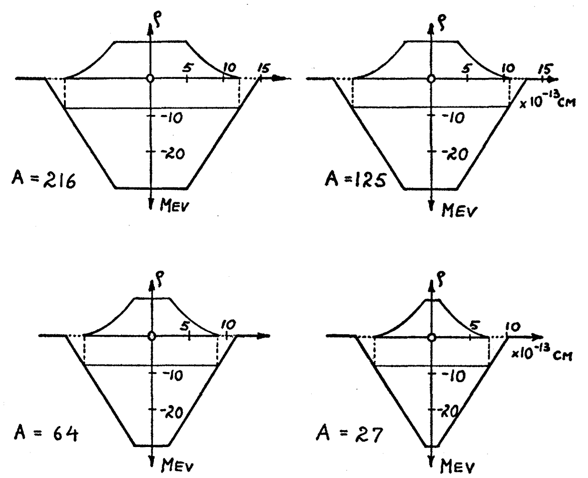

FIG. 1. Potential wells and Fermi densities for $A=27$, 64, 125, 216. The depth of the wells is 30 Mev, the Fermi energy $T_0=22$ Mev, the gradient at the surface T_0/x_0 with $x_0=7\times10^{-13}$ cm. With these density distributions the equivalent radius $\bar{R}$, defined as the radius of the sphere which would contain all the particles at a constant (the central) density, can be approximated by $\bar{R}\doteqdot R_1+x_0/2$ to better than 3.6 percent. We have taken $\bar{R}=1.4\times10^{-13}A^{\frac{1}{3}}$ cm.

The above refers to average nucleon densities. It is interesting to examine the neutron and proton densities (ρ_N,ρ_Z) separately. For a nucleus with $N>Z$, the zero point energies for neutrons are greater than for protons (Fermi energies $T_N>T_Z$) and the faster neutrons will, therefore, penetrate farther into the surface region, this enrichment of the surface in neutrons being inversely proportional to the gradient of the potential across the surface.[4] A simple estimate of the effect can be made by using $T_N/T_Z=(N/Z)^{\frac{2}{3}}$, when the difference in effective neutron and proton radii $\bar{R}_N-\bar{R}_Z$ becomes $\frac{2}{3}[(N-Z)/A]x_0$.[5] (For notation see caption to Fig. 1.)

Figure 2 shows, as a function of A, the values of $\bar{R}_N$ and $\bar{R}_Z$, as well as $R_{\frac{1}{2}}$, the radius at which the average potential has half its central value. If $R_{\frac{1}{2}}$ is taken to represent an average interaction radius for nucleons interacting with a potential well with sloping sides, then the curves for $R_{\frac{1}{2}}$ and $\bar{R}_Z$ illustrate the different radii appropriate for the interpretation of experiments with particles which are (nucleons) and are not (electrons or muons) subject to specifically nuclear interactions. The effect being associated with a sloping potential the difference between $R_{\frac{1}{2}}$ and $\bar{R}_Z$ should provide a measure of the thickness of the surface region. The empirical difference between the two sets of radii (about 20 percent) is consistent in order of magnitude with Fig. 2, based on a value $x_0=7\times10^{-13}$ cm (but not with $x_0=1-2\times10^{-13}$ cm).

A value of x_0 considerably greater than $1-2\times10^{-13}$ cm is suggested by very simple considerations. The thickness of the region in which a particle crossing a