

and Mr. J. B. Bulkeley of the Radioactivity Center at the Massachusetts Institute of Technology for his cooperation in arranging the necessary cyclotron bombardments.

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<sup>2</sup> G. Scharff-Goldhaber, Phys. Rev. **90**, 487 (1953); P. Preiswerk and P. Stähelin, Helv. Phys. Acta **24**, 623 (1952).

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<sup>4</sup> We thank Professor R. G. Wilkinson of Indiana University for his advice concerning the plating process.

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### Self-Energy Effects on Meson-Nucleon Scattering According to the Tamm-Dancoff Method

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IN a paper with the foregoing title, Visscher<sup>1</sup> has shown that the self-energy effects in the Tamm-Dancoff method (as modified by Dyson) give rise to an unphysical pole in the meson-nucleon wave function. We show here that a simple and natural modification of the usual way of eliminating the negative frequency components removes this difficulty.

We use Visscher's notation. Define

$$X(\mathbf{p}) = [X^+(\mathbf{p})u + X^-(\mathbf{p})w](M/E_p)^{\frac{1}{2}} \\ = [(\Psi_0^*(0)b_{p\alpha}a_{-p\alpha}\Psi(0))u \\ + (\Psi_0^*(0)b_{p\alpha}w^*a_{-p\alpha}\Psi(0))w](M/E_p)^{\frac{1}{2}},$$

where  $w$  is a negative-energy solution of the Dirac equation; and

$$-i\gamma_4(\mathbf{p}\cdot\boldsymbol{\gamma} - iM)N(p) = \frac{3}{4}iG^2\Omega_R(\mathbf{p}\cdot\boldsymbol{\gamma}),$$

where  $\Omega_R$  is the finite self-energy function defined by Visscher. We suppose the meson negative-frequency components eliminated, since they are not our concern; and we leave out the meson self-energy term for simplicity. Then the second-order equation has the form

$$(\epsilon - H_p - \omega_p)[1 - \gamma_4 N(p)]X(\mathbf{p}) \\ = \int \gamma_4 K(\mathbf{p}, \mathbf{k})X(\mathbf{k})d\mathbf{k}, \quad (1)$$

where  $\mathbf{p} = (\mathbf{p}, \epsilon - \omega_p)$  and  $K$  represents the interaction terms. Visscher drops negative energy components from (1), to give the approximate equation

$$(\epsilon - E_p - \omega_p)[1 - (\bar{u}(\mathbf{p})N(p)u(\mathbf{p}))M/E_p]X^+(\mathbf{p}) \\ = \int (M/E_p)^{\frac{1}{2}}(\bar{u}(\mathbf{p})K(\mathbf{p}, \mathbf{k})v(\mathbf{k}))(M/E_k)^{\frac{1}{2}}X^+(\mathbf{k})d\mathbf{k}, \quad (2)$$

and a zero of the factor in the square bracket causes the unwanted pole in  $X^+(\mathbf{p})$ .

We propose the following alternative prescription. First transfer the square bracket in (1) to the right-hand side, then separate and drop the negative frequency components, to give

$$(\epsilon - E_p - \omega_p)X^+(\mathbf{p}) = \int (M/E_p)^{\frac{1}{2}}(\bar{u}(\mathbf{p})\{[1 - \gamma_4 N(p)]\}^{-1} \\ \times K(\mathbf{p}, \mathbf{k})\}v(\mathbf{k}))(M/E_k)^{\frac{1}{2}}X^+(\mathbf{k})d\mathbf{k}. \quad (3)$$

A natural further approximation gives

$$(\epsilon - E_p - \omega_p)X^+(\mathbf{p}) = (\bar{u}(\mathbf{p})[\gamma_4 - N(p)]^{-1}u(\mathbf{p}))M/E_p \\ \times \int (M/E_p)^{\frac{1}{2}}(\bar{u}(\mathbf{p})K(\mathbf{p}, \mathbf{k})v(\mathbf{k}))(M/E_k)^{\frac{1}{2}}X^+(\mathbf{k})d\mathbf{k} \quad (4)$$

as our alternative to Eq. (2).

The factor in front of the integral in (4) can be written

$$[L(u^2)]^{-1}[1 - (\bar{u}(\mathbf{p})N(-\mathbf{p})u(\mathbf{p}))M/E_p], \quad (5)$$

where  $u = \boldsymbol{\gamma}\cdot\mathbf{p}/(iM)$ , and  $L(u^2)$  is a function defined by Feldman.<sup>2</sup> This is verified by noting that  $N$  is connected with Feldman's function  $f(u)$  by the equation

$$-\frac{1}{3}\gamma_4 N(p) = cf(u).$$

Feldman has found the zeros of  $L(u^2)$  (when  $\mu=0$ ), and none of them is real. Therefore, Eq. (4) does not have the difficulty of Eq. (2).

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### Photoproduction of $\pi^+$ Mesons from Hydrogen Near Threshold\*

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THE total cross section for photoproduction of  $\pi^+$  mesons very close to threshold has been investigated, following the method of a preliminary experiment.<sup>1</sup>  $\pi^+$  mesons and their  $\mu^+$  decay mesons were stopped in a carbon absorber surrounding the target. Since the positrons from the decay of the stopped  $\mu^+$  mesons are emitted with an energy spectrum peaked around 36 Mev,<sup>2</sup> a large fraction will escape with considerable energy and can be counted. Since the decay of the stopped  $\mu^+$  mesons is isotropic, the positrons leaving the absorber will be essentially isotropic, regardless of the angular distribution of the emitted  $\pi^+$  mesons. Except for small corrections due to geometry

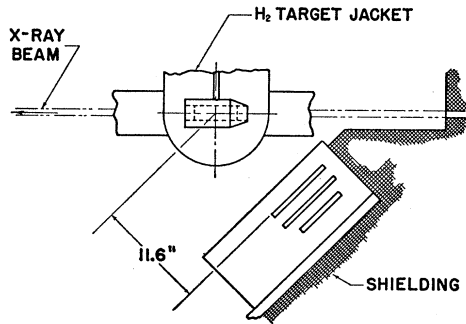


FIG. 1. Experimental arrangement, showing hydrogen target, absorber around target, counters, and shielding.

effects and to positron energy loss in the target and absorber, the spectrum and angular distribution of positrons leaving the absorber are independent of the energy of the photon which initiated the reaction.

The geometry used is shown in Fig. 1. The liquid hydrogen target is a cylinder  $1\frac{1}{2}$  in. in diameter and 4 in. long with 0.001-in. brass windows for the x-ray beam. The beam diameter at the target was 0.681 in. The hydrogen target was surrounded by a hollow carbon cylinder 6 in. long with 0.500-in. wall, to stop most mesons produced by photons of less than 185 Mev. Triple coincidences in the three  $\frac{3}{8}$ -in. plastic scintillators were counted. Background from hydrogen, due to pair production, is small and was subtracted on the basis of counting rates below meson threshold. Activation curves were obtained in 2-Mev steps from 140 to 200 Mev.

The cross sections determined from the activation curves<sup>3,4</sup> must be corrected for the counting efficiency, composed of three factors: (1) the fraction of mesons which stop in the hydrogen or carbon ( $\sim 90$  percent); (2) the solid angle of the counters ( $\sim 0.13$  steradian); (3) the fraction of positrons heading for the detector which are counted ( $\sim 50$  percent). The efficiency has been calculated using Monte Carlo methods and the Illiac and is believed accurate to  $< 10$  percent.

Using arguments given by Bernardini and Goldwasser,<sup>5</sup> one can express the total cross section by

$$\sigma_T/4\pi\chi = A_0 + A_2\eta^2,$$

where

$$\chi = (k/\nu)(1+\nu/M)^{-1}(1+\omega/M)^{-1}, \quad \eta = k/\mu c,$$

$\nu$  = photon momentum or energy, and  $k$  and  $\omega$  = meson momentum and energy, respectively. The best straight-line fit to our data in a plot of  $\sigma_T/4\pi\chi$  vs  $\eta^2$  gives  $A_0 = (1.60 \pm 0.10) \times 10^{-29}$ ,  $A_2 = (1.20 \pm 0.56) \times 10^{-29}$ . See Fig. 2. The errors shown for the points are statistical only. In a similar plot for the  $90^\circ$  cross section, using data of this experiment together with that of other work at higher energies<sup>6</sup> (near threshold  $\sigma_T/4\pi \cong \sigma_{90^\circ}$ ), one obtains  $A_0 = (1.56 \pm 0.06) \times 10^{-29}$ ,  $A_2 = (1.46 \pm 0.19)$

$\times 10^{-29}$ . This shows that  $A_0$  is insensitive to the energy range assumed, and is therefore reasonably well determined.

Using a weighted mean value of  $A_0 = (1.57 \pm 0.05) \times 10^{-29}$  and the value  $1.4 \pm 0.1$  for the ratio of  $\pi^-$  to  $\pi^+$  photoproduction in deuterium at threshold,<sup>7</sup> we have determined the renormalized symmetric coupling constant  $g$ , and the coupling constant  $f$  of the Chew cutoff theory<sup>7</sup> to be  $g^2 = 11.7 \pm 0.7$ ,  $f^2 = 0.069 \pm 0.004$ .

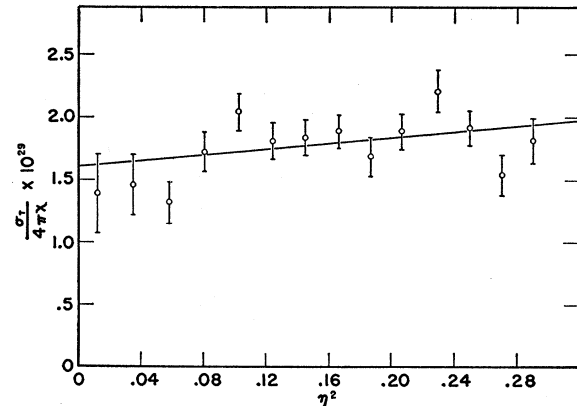


FIG. 2. Total cross section,  $\sigma_T/4\pi\chi$ , plotted versus  $\eta^2$ . Solid curve is  $\sigma_T/4\pi\chi = (1.60 + 1.20\eta^2) \times 10^{-29}$ . The range of photon energy for the points is from 152.8 to 178.7 Mev.

Using the foregoing data, the  $\pi^-/\pi^+$  ratio in deuterium,<sup>8</sup> and the branching ratio in the Panofsky experiment,<sup>9</sup> one may calculate<sup>10,11</sup> a threshold value for the difference of the  $S$ -wave scattering phase shifts,  $(\alpha_1 - \alpha_3)/\eta = 10.5^\circ \pm 1.3^\circ$ . (Bethe and de Hoffmann<sup>11</sup> obtain  $10.7^\circ \pm 2.5^\circ$  from the data of Bernardini and Goldwasser.) This result, when compared with the value  $\sim 17^\circ$  from scattering experiments at higher energies, is further evidence that  $\alpha_1 - \alpha_3$  does not depend linearly on  $\eta$ . It should be noted, however, that this interpretation of the Panofsky experiment, based on charge independence, is in some doubt because of the  $\pi^- - \pi^0$  mass difference.

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