

Lassen also observes an increase of the average charge with increasing gas pressure, which he attributes to a contribution to electron loss of excited states with lifetimes comparable to the time between excitations. He finds an increase in average charge of about 7 percent per 10 mm increase in gas pressure. In our experiment the highest pressures used were about 0.1 mm, so that this effect was not detectable.

Gluckstern has calculated electron capture and loss cross sections using a modification of Bell's theory.⁴ In the following paper¹⁰ he compares his theoretical results

with cross sections deduced from Fig. 4 and with the equilibrium distributions in charge states given in Figs. 5 and 6.

We wish to acknowledge valuable contributions to this experiment by many members of the linear accelerator operating crew and of the groups from Yale University and from the Radiation Laboratory that are designing the heavy-ion accelerators.

¹⁰ R. L. Gluckstern, following paper [Phys. Rev. **98**, 1817 (1955)].

Electron Capture and Loss by Ions in Gases*

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Capture and loss cross sections for ions of intermediate atomic number ($Z_1=8$ to 18) passing through low-pressure gases have been calculated. A modified form of Bell's model was used, leading to lower capture cross sections than the original model. The resulting equilibrium charge distributions obtained compare favorably with the experimental results of Hubbard and Lauer. The individual cross sections were obtained from the observed dependence of the charge distribution on the stripper thickness; these results agree reasonably well with the predictions of the calculations. Capture and loss of two electrons in a single collision may be significant, but the effect on the charge distributions should not be too great.

I. INTRODUCTION

THE heavy-ion linear accelerators at the University of California Radiation Laboratory and at Yale University are being designed to include gas strippers to increase the efficiency of the ion acceleration. Experiments described in the preceding paper have been performed by Hubbard and Lauer¹ to determine the charge distributions for stripping oxygen and neon ions in various gases, using the University of California Radiation Laboratory 4-Mv Van de Graaff injector to the 40-foot proton accelerator. The purpose of this paper is to compare their results with predictions based primarily on the model that Bell² used for investigation of the charge distribution of fission fragments in gases.

The most recent theoretical investigations of electron capture and loss have been undertaken by Bohr,³ Bell,² and Bohr and Lindhard.⁴ Bohr³ and Bohr and Lindhard⁴ use a simplified Fermi-Thomas model for the electron shielding, and base their estimate of capture and loss cross sections on general features of a classical description of the ion-atom collision. Bell² uses a more detailed

Fermi-Thomas model for the position, velocity, and "binding force" of the electrons involved in the collision. According to Bohr and Lindhard,⁴ the agreement between the predictions of the two methods for fission fragments is good, considering the complicated nature of the actual collision. In this paper a slight modification of Bell's method is used to calculate the loss and capture cross sections and the charge distributions for stripping of oxygen through argon ions in a variety of stripping gases for a velocity range $3v_0$ to $7v_0$ ($v_0=c/137$).

II. CALCULATION OF CROSS SECTIONS AND EQUILIBRIUM CHARGE DISTRIBUTIONS

The stripping pressures used by Hubbard and Lauer¹ are sufficiently small that the time between successive loss or capture collisions is much greater than the average lifetime of excited electron states induced in the ion in these collisions.⁵ For this reason the ion can be considered in its ground state before each collision, and Bell's model of a "rarefied" gas stripper applies.

For the capture cross section, Bell's model is applied with the stripper electrons assumed to be located at the "half-charge" radii determined from a Fermi-Thomas model, i.e., the total charge inside the sphere corresponding to the n th electron is $Z_2 - n + \frac{1}{2}$. The capture cross section for each stripper electron by a point charge representing the ion is calculated by kinematic considerations regarding the liberation and

* This work was performed at the Radiation Laboratory, University of California, and was supported by the U. S. Atomic Energy Commission Contract AT (30-1) 1691.

¹ E. L. Hubbard and E. J. Lauer, preceding paper [Phys. Rev. **98**, 1814 (1955)].

² G. I. Bell, Phys. Rev. **90**, 548 (1953).

³ N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **18**, No. 8 (1948).

⁴ N. Bohr and J. Lindhard, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 7 (1954).

⁵ As estimated by Bohr and Lindhard [reference 4, Eq. (6.4)].

probability of escape of that particular electron. The total capture cross section is then the sum of the individual capture cross sections for each electron.

For the loss cross section, positive-ion Fermi-Thomas functions are obtained by a perturbation technique from the neutral atom functions.⁶ The loss cross section for each ion electron (located at the appropriate "half-charge" radius) is obtained by considering the electron to be scattered by a Coulomb-like stripper core according to the Rutherford cross section. The core charge is determined from a Fermi-Thomas charge distribution

TABLE I. Loss and capture cross sections for oxygen ions in various strippers calculated using Bell's model directly.

$p \rightarrow q$	v/v_0	$\sigma_{pq}/\pi a_0^2$			
		$Z_2=1^a$	$Z_2=7$	$Z_2=18$	$Z_2=80$
2→3	3	0.47	3.6	6.6	12.2
	5	0.20	2.4	4.9	9.7
	7	0.099	1.57	3.5	7.5
3→4	3	0.170	1.80	3.6	6.8
	5	0.089	1.42	3.2	6.8
	7	0.049	0.97	2.4	5.5
4→5	3	0.056	0.77	1.69	3.3
	5	0.042	0.80	1.85	4.2
	7	0.026	0.56	1.59	3.9
5→6	3	0.010	0.24	0.64	1.57
	5	0.016	0.35	0.88	2.2
	7	0.013	0.32	0.92	2.4
3→2	3	0.56	2.0	3.2	7.4
	5	0.063	0.37	0.72	1.66
	7	0.016	0.11	0.22	0.65
4→3	3	1.00	3.4	5.2	11.8
	5	0.111	0.62	1.21	2.7
	7	0.028	0.19	0.40	1.07
5→4	3	1.54	5.0	8.2	16.6
	5	0.173	0.95	1.77	4.0
	7	0.043	0.29	0.61	1.60
6→5	3	2.2	7.0	10.7	22.0
	5	0.25	1.3	2.4	5.4
	7	0.063	0.40	0.84	2.2

^a See reference 7.

on the basis of a radius determined by classical considerations involving the impact parameter. Only those electrons deflected sufficiently to escape from the ion are considered lost, and the total loss cross section is then taken as the sum of the individual loss cross sections.

The results of calculations of loss and capture of electrons for oxygen ions stripped by hydrogen ($Z_2=1$),⁷ nitrogen ($Z_2=7$), argon ($Z_2=18$), and mercury ($Z_2=80$), using Bell's model, are given in Table I, where σ_{pq} is the

⁶ E. B. Baker, Phys. Rev. **36**, 630 (1930); P. Gombas, *Die statistische Theorie des Atoms und ihre Anwendungen* (Springer-Verlag, Berlin, 1949).

⁷ The stripper atom model for $Z_2=1$ was taken as the usual Bohr hydrogen atom rather than the Fermi-Thomas model used for the other strippers.

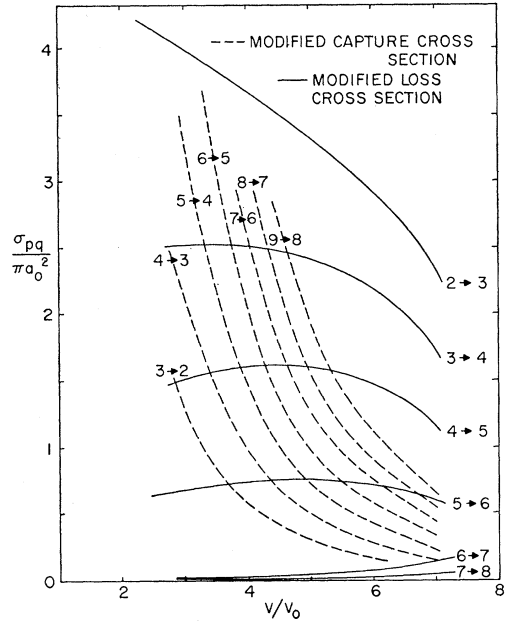


FIG. 1. Modified capture and loss cross sections for oxygen ions in argon.

cross section for going from an ionic charge p to an ionic charge q , $a_0 = \hbar^2/mc^2$ is the Bohr radius, and $v_0 = c/137$.

If the ions pass through a sufficient thickness of gas, an equilibrium distribution of ion charge states is reached long before any significant energy degradation takes place. The fractions of the various ions in this equilibrium distribution are given by

$$N_p/N_q = \sigma_{qp}/\sigma_{pq}; \quad \sum N_p = 1.$$

The resulting average charge for oxygen ions, using the information in Table I, is given in Table II.

The average charge, as well as the entire charge distribution, seems to increase slightly with increased Z of the stripper, not inconsistent with the experimental determination of no significant variation with Z_2 . The average charge measured, however, is about 1 unit greater than that predicted. As this would correspond to a fairly large discrepancy in the pertinent cross sections, Bell's model was re-examined.

In Bell's calculation of capture, the cross section is obtained as a sum of the individual capture cross sections for each stripper electron. The model for capture that Bell uses, however, does not take into account the fact that in collisions with small impact parameters,

TABLE II. Average charge for equilibrium distribution of oxygen ions in various strippers.

v/v_0	$Z_2=1^a$	$Z_2=7$	$Z_2=18$	$Z_2=80$
4	3.0	3.8	3.8	3.8
5	3.4	4.3	4.5	4.5
6	3.8	4.7	4.9	5.0

^a See reference 7.

for which the capture cross sections are greatest, one should *not* sum the individual capture cross sections. One should instead consider, at an impact parameter r_0 , the probability $P(r_0)$ of capturing *any* electron. The total capture cross section, according to Bell's prescription, would then be

$$\sigma_c = 2\pi \int P(r_0) r_0 dr_0.$$

Application of this modification leads to the conclusion that the results given in Table I for capture are overestimates. A survey of different ions and different strippers indicated that a simple but adequate modification is to take 40 percent of the previously calculated capture cross sections in the range of ion Z_1 , stripper Z_2 ,

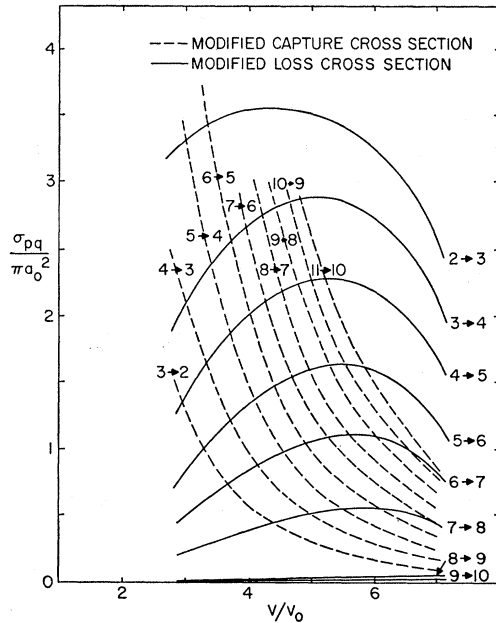


FIG. 2. Modified capture and loss cross sections for neon ions in argon.

and ion velocity of interest. Since this raises the average equilibrium charge, it can be expected to improve the agreement with experiment.

Another difficulty in applying Bell's model to capture and loss of electrons from relatively light ions is that the Fermi-Thomas description of the ion electrons cannot be expected to be accurate for high-charge states. For this reason, and also to simplify the calculation of the loss cross sections for different ions and different strippers, a modification was made in which the ion electrons were located in concentric shells with radii chosen to match the known ionization potentials. Although no account of the readjustment of the orbits is taken as electrons are added, the model should be more realistic for the K electrons and even possibly for the first few L electrons. For ions with many electrons

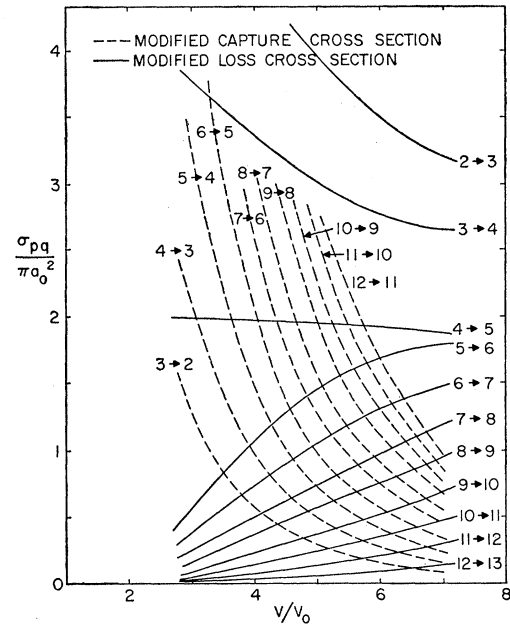


FIG. 3. Modified capture and loss cross sections for phosphorous ions in argon.

one should undoubtedly return to a Fermi-Thomas distribution.

Using the modifications described above (i.e., capture cross sections reduced to 40 percent of the original values and loss cross sections obtained from an ion whose electron orbits are determined directly from the ionization potentials) one obtains the loss and capture cross sections shown in Figs. 1 through 4 for oxygen

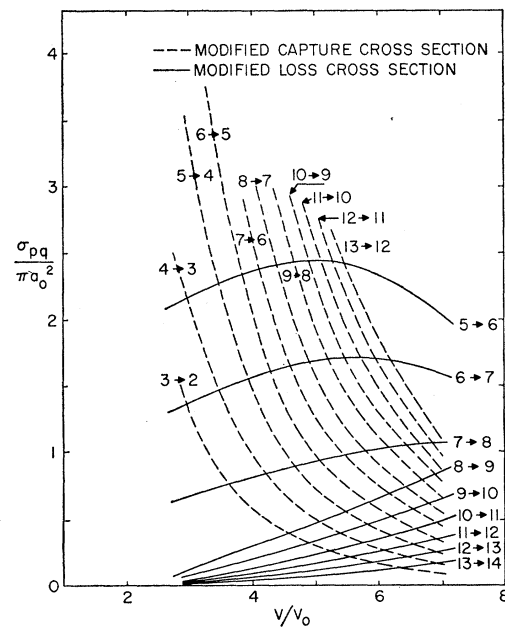


FIG. 4. Modified capture and loss cross sections for argon ions in argon.

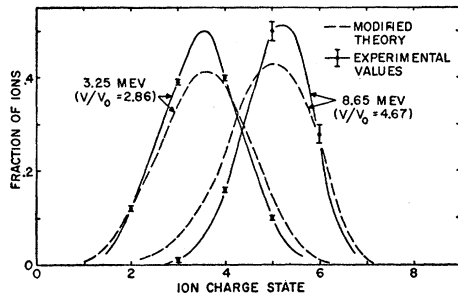


Fig. 5. Charge distribution at equilibrium for oxygen ions in argon.

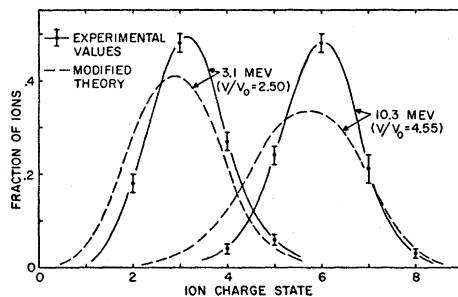


Fig. 6. Charge distribution at equilibrium for neon ions in argon.

($Z_1=8$), neon ($Z_1=10$), phosphorus ($Z_1=15$), and argon ($Z_1=18$) ions being stripped in argon. Figures 5 and 6 contain corresponding charge distributions calculated for subsequent comparison with experiment, and Fig. 7 contains the predictions for argon ions with $v/v_0=4$ and 6.

III. COMPARISON WITH EXPERIMENT

A. Charge Distributions

As can be seen from Figs. 5 and 6, the agreement between the measured and calculated charge distributions is quite good. The peaks coincide fairly well although the measured widths are somewhat smaller than those calculated. The predicted and measured average charges as a function of energy are shown in Fig. 8. Experimental results of Reynolds, Scott, Wyly, and Zucker⁸ and Stephens and Walker⁹ for nitrogen ions in

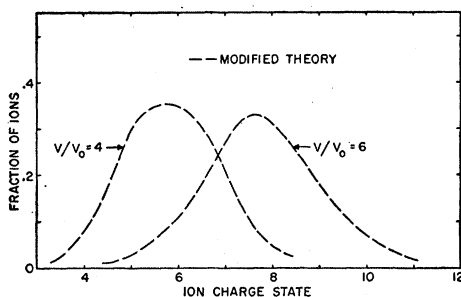


Fig. 7. Charge distribution at equilibrium for argon ions in argon.

⁸ Reynolds, Scott, and Zucker, Phys. Rev. **95**, 671 (1954); H. L. Reynolds and A. Zucker, Phys. Rev. **95**, 1353 (1954); Reynolds, Wyly, and Zucker, Phys. Rev. **98**, 1825 (1955).

⁹ K. G. Stephens and D. Walker, Phil. Mag. **45**, 543 (1954).

foils are also included. The agreement with the results of Hubbard and Lauer¹ is good, the largest discrepancy being 0.3 of a charge state. The same is true of the data using foils,^{8,9} where the experimental results are 0.2 to 0.4 of a charge state higher than the predicted values. Bohr and Lindhard⁴ point out that foils are expected to give higher average charges than gases because the loss cross sections should be greater for ions that do not have sufficient time to return to their ground states after a collision. It would, however, be dangerous to consider this comparison between experiment and theory in Fig. 8(c) as a verification of the "density" effect, because of the crudeness of the basic model and inaccuracies in the experiment.

B. Cross Sections

The individual capture and loss cross sections can in principle be obtained from the dependence of the charge distribution on stripper thickness shown in Fig. 4 of

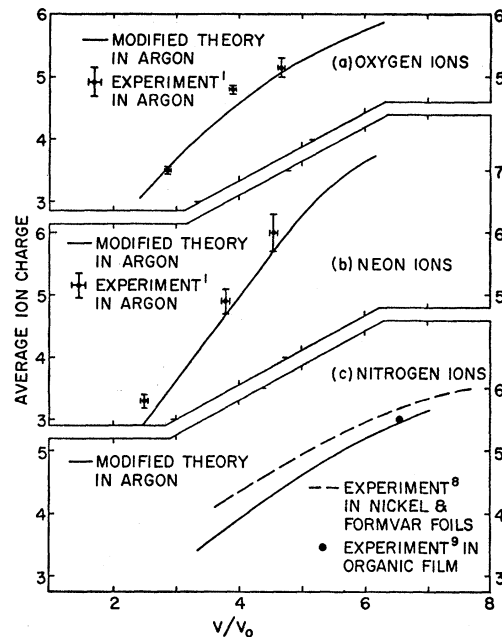


Fig. 8. Average ion charge vs ion velocity.

the preceding paper.¹ One has, for example,

$$dN_4/dx = -\sigma_{43}N_4 - \sigma_{45}N_4 + \sigma_{34}N_3 + \sigma_{54}N_5,$$

where x is the stripper thickness measured in atoms/cm² and N_p is the fraction of ions with charge p . Knowledge of the x dependence of the N_p and dN_p/dx therefore overdetermine the σ_{pq} . The experimental uncertainties, however, particularly in the dN_p/dx , prevent obtaining a unique set of values for the cross sections. A least-squares fit was obtained leading to the values for 8.7-Mev oxygen ions in argon in Table III. The predicted cross sections, obtained from Fig. 1, are given for comparison.

The cross sections agree remarkably well with those predicted in Fig. 1. The only consistent discrepancy seems to be that the capture cross sections for low-charge states are smaller than expected, causing the experimental equilibrium charge distributions to be slightly narrower than those predicted. The variation of the charge distributions with stripper thickness corresponding to the cross sections in Table III for the least-squares fit are shown in Fig. 9, together with the experimental points. As can be seen, the curves in Fig. 9 fall primarily within the experimental error. Two possible weak points in the comparison, however, are

- (1) The peak in the fraction of 4+ ions seems to occur somewhat earlier than indicated by the data;
- (2) The fractions of 5+ and 6+ ions rise a little more slowly than indicated by the data.

TABLE III. Comparison of experimental and theoretical cross sections for 8.7-Mev oxygen ions ($v/v_0=4.7$) in argon. The values listed are $\sigma_{pq}/\pi a_0^2$.

$p \rightarrow q$	Experimental ^{a,b}	Theoretical ^c
3→4	2.27 ± 0.1	2.65
4→5	1.77 ± 0.15	1.75
5→6	0.81 ± 0.2	0.80
4→3	0.14 ± 0.1	0.65
5→4	0.57 ± 0.15	0.95
6→5	1.22 ± 0.2	1.30

^a Determined from a least-squares fit to the data in Fig. 4 of the preceding paper.

^b Errors listed are estimated from the uncertainty in the data.

^c Obtained from Fig. 1.

IV. DISCUSSION OF RESULTS

The following general features of the results appear evident:

(1) The capture cross section is independent of the particular ion, and varies approximately as the square of the ionic charge and inversely as the 3.5 power of the velocity. The numerical values agree quite well with those obtained from Eq. (4.5) of Bohr and Lindhard.⁴

(2) The loss cross sections reach a maximum in the region of interest. The variation of these loss cross sections depends quite sensitively on the model used for the ion electrons. There appears to be some question whether Eq. (4.2) of Bohr and Lindhard is applicable in the region of ion Z_1 of interest here.

(3) The charge distributions do not vary much in width as the ion velocity is changed. The average charge is independent of the stripper material over a

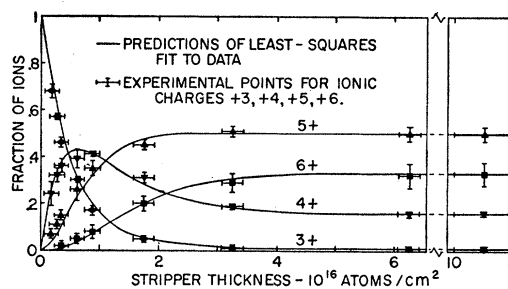


FIG. 9. Charge distribution vs stripper thickness for 8.7-Mev oxygen ions.

wide range of Z_2 , and increases with velocity until one starts to ionize the K shell, at which time the average charge increases less rapidly.

As can be seen from Fig. 9, the least-squares fit to the data does not go through the experimental points as closely as one might hope. One possible reason is that the assumption of transitions only between adjacent charge states may not be valid. The possible presence of double and triple jumps is suggested by the rapid rise of the 5+ and 6+ ions in Fig. 9. Attempts were made to fit the data using double and triple jumps; although the new parameters lead to a closer fit to the data, the experimental uncertainties do not permit one to show the definite existence of these two- and three-charge state transitions. On the other hand, an attempt was made to use Bell's model to calculate these multiple-jump cross sections. This proved difficult; however, crude estimates lead readily to the conclusion that the cross sections for both loss and capture transitions of two-charge states may be as high as 50 percent of the corresponding single-charge transition cross sections. Under these circumstances the single-jump cross sections in Figs. 1 through 4 would have to be modified. The resolution of the question whether or not multiple-charge transitions are important evidently requires either a more detailed model of the collision, or sufficiently accurate data as a function of stripper thickness, particularly near the zero thickness limit.

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