PHYSICAL REVIEW VOLUME 98, NUMBER 6 JUNE 15, 1955

Lattice Thermal Conductivity

J. S. Dugdale and D. K. C. MacDonald
Division of Physics, National Research Council, Ottawa, Canada

(Received February 8, 1955)

Consideration of the anharmonic properties of a linear chain suggests that the dimensionless quantity $\alpha\gamma T$ is in general a measure of the anharmonicity of a lattice (α is the thermal expansion coefficient and γ the Grüneisen parameter). It is therefore proposed that the mean free path for lattice vibrations in an insulator should, in the classical temperature region, be approximately $A_0/\alpha\gamma T$, where A_0 is the lattice spacing. Reasonably good agreement with experiment is found.

DESPITE the pioneer work of Debye[1] and Peierls[2,3] in the theory of the thermal conductivity of insulating solids, we have still no quantitative expressions for the thermal conductivity in terms of the equilibrium properties of the solid. It is now generally agreed that a finite thermal conductivity depends essentially on the anharmonicity of the interatomic potential and consequently one should expect that there would be a close relationship between thermal conductivity and the thermal expansion which also depends intrinsically on the anharmonicity.

A particular problem of considerable interest is the interchange of momentum between the thermal flow and the lattice structure. Although Peierls has emphasized that the so-called "Umklappprozess" involving a Bragg reflection of the "phonons" with the quasi-periodic lattice will provide momentum transfer, we would like to point out that the essential differential expansion of the lattice under a thermal gradient itself already calls for such a momentum transfer. If, as is normal, the expansion coefficient is positive, the momentum will be absorbed by the differentially displaced atoms from the thermal flow in going from the hot to the cold end. On the other hand, were the thermal expansion to be negative (which in itself involves no contradiction with thermodynamics), this would imply a progressive absorption of momentum from the lattice by the thermal flow. The magnitude of the force per unit volume, say F, corresponding to this momentum transfer may be inferred from Brillouin's[4] expression for the radiation pressure due to diffuse thermal radiation in a solid, yielding $F \simeq \gamma C dT/dx$ (where γ is the Grüneisen parameter and C is the specific heat per unit volume).

Although we have not succeeded in deriving on a sound theoretical basis quantitative expressions for the thermal conductivity, it appears that in the analysis of lattice properties the dimensionless parameter $\alpha\gamma T$ (where α is the thermal expansion coefficient) always appears as a measure of the departure of the lattice from harmonic behavior (cf. Dugdale and MacDonald[5] and MacDonald and Roy[6]). If it is permissible, as Debye first did, to introduce a mean free path l for the lattice vibrations (or "phonons"), then in the classical temperature region it should vary inversely with temperature (as first predicted by Debye) and would be expected from Peierls' analysis (cf. Blackman[7]) to vary inversely with the square of the anharmonic coefficient. It, therefore, seems reasonable to assume that it has the form

$$l \simeq A_0/\alpha\gamma T, \tag{1}$$

where A_0 is the lattice constant. There may of course be some further dimensionless factor involving the frequency spectrum of the lattice, but we shall just assume the above simple form for the mean free path and estimate the thermal conductivity of some insulators on this basis. The thermal conductivity, K, is related to the mean free path by

$$K \simeq \tfrac{1}{3} C_v l v, \tag{2}$$

where C_v is the specific heat per unit volume and v is the velocity of sound in the solid, and Table I shows the results for those alkali halides for which experimental data are available. This table suggests that Eq. (1) offers a reasonable estimate of the mean free path.

Table I. Calculated and observed thermal conductivities for some alkali halides.[a]

Substance	$10^4\alpha$ (°C^{-1})	γ	C_v (cal/cc °C)	$10^{-5}v$ (cm/sec)	$10^8 A_0$ (cm)	$100K_{\text{calc}}$ at 0°C (cal/cm °C sec)	$100K_{\text{obs}}$ at 0°C (cal/cm °C sec)
NaF	0.98	1.55	0.72	4.1_6	2.31	5.5	2.5
NaCl	1.10	1.63	0.43_2	3.3_2	2.81	2.8	2.4
NaBr	1.19	(1.56)	0.38	2.4_6	2.98	1.8	0.6
KF	1.00	1.45	0.29	3.2_8	2.67	2.1	1.7
KCl	1.01	1.60	0.32	3.0_0	3.14	2.3	2.5
KBr	1.10	1.68	0.28	2.4_8	3.29	1.5	0.9
KI	1.25	1.63	0.22_3	1.9_4	3.53	0.9	0.7
RbBr	1.04	(1.37)	0.24_1	1.9_5	3.43	1.4	0.9
RbI	1.19	(1.41)	0.20_2	1.7_2	3.66	0.9	0.8

[a] Experimental data on thermal conductivity from A. Eucken and G. Kuhn, Z. physik. Chem. **134**, 193 (1928). Remaining data from Landolt-Börnstein Tables (Verlag Julius Springer, Berlin, 1923); J. C. Slater, *Introduction to Chemical Physics* (McGraw-Hill Book Company, Inc., New York, 1939); and P. W. Bridgman, *The Physics of High Pressure* (G. Bell and Sons, Ltd., London, 1949).

---

[1] P. Debye, *Vorträge über die kinetische Theorie* (B. G. Teubner, etc., Leipzig, 1914).

[2] R. Peierls, Ann. Physik **3**, 1055 (1929).

[3] R. Peierls, Ann. inst. Henri Poincaré **5**, 177 (1935).

[4] L. Brillouin, *Wave Propagation in Periodic Structures* (McGraw-Hill Book Company, Inc., New York, 1946).

[5] J. S. Dugdale and D. K. C. MacDonald, Phys. Rev. **96**, 57 (1954).

[6] D. K. C. MacDonald and S. K. Roy, Phys. Rev. **97**, 673 (1955).

[7] M. Blackman, Phil. Mag. **19**, 989 (1935).

If, further, we apply Eq. (2) to an electrical conductor such as sodium, at room temperature we find $K_{\text{latt}}\simeq 8\times 10^{-3}$ cals/cm °C sec, which is about 2 percent of the observed thermal conductivity of the metal. This is consistent with the usual assumption that the lattice conductivity in such metals is practically negligible in comparison with that due to the free electrons.

In the case of beryllium, the effective number of conduction electrons is sufficiently low that the intrinsic lattice conductivity can be observed experimentally. Powell[8] finds that the data may be expressed by

$$K_{\text{latt}}=[(64/T)-0.036] \text{ cal/cm °C sec}, \qquad (3)$$

valid between 320°K and 700°K. Because of the high characteristic temperature of beryllium (estimates vary between ~650°K and ~1040°K), this temperature-region cannot be considered as fully classical and this may account for the particular form of Eq. (3). However, if we take a value for α appropriate to classical behavior, Eq. (2) gives

$$K_{\text{latt}}\simeq(68/T) \text{ cal/cm °C sec}. \qquad (4)$$

The close agreement with the dominant term of Eq. (3) must be regarded as fortuitous.

The ideal subjects for experimental investigation would seem to be the heavier inert gas solids. It is hoped to carry out experiments before long on such solids in our laboratories.

Note added in proof.—Two other recent articles of interest are: G. Leibfried and E. Schlömann, Gött. Nachr. IIa, 71 (1954); A. W. Lawson, "On the relation between thermal expansion and thermal conductivity in one-dimensional lattices" (unpublished).

---

[8] R. Powell, Phil. Mag. 44, 645 (1953).

PHYSICAL REVIEW VOLUME 98, NUMBER 6 JUNE 15, 1955

Magnetization Reversal in Thin Films*

R. L. CONGER
U. S. Naval Ordnance Laboratory, Corona, California

(Received February 9, 1955)

An experiment has been performed which indicates that magnetization reversal in evaporated films of 80 percent Ni, 20 percent Fe, 2×10^{-5} cm thick takes place by domain rotation rather than by the motion of 180° domain walls.

THE magnetization reversal in single crystals with dimensions of the order of a centimeter has been investigated by Williams[1] and others. They have shown that the reversal takes place by the motion of 180° domain walls and that the wall velocity is controlled by eddy current effects. Magnetization reversal in most polycrystalline materials probably takes place by this process. However, if a ferromagnetic film is too thin to support domain walls, the magnetization reversal would be expected to take place by a different mechanism.

The thickness at which a thin ferromagnetic film ceases to support domain walls has been calculated by Kittel.[2] For a material with an anisotropy energy of 5×10^5 ergs/cm^3 and a wall energy of 3 ergs/cm, this thickness is 3×10^{-5} cm. Films thinner than this will tend to be single domain structures. Materials with anisotropy energy less than 5×10^5 ergs/cm^3 will have a poly-domain to single-domain transition thickness greater than 3×10^{-5} cm.

We have prepared ferromagnetic films 2×10^{-5} cm thick which should be single domain structures and have observed the magnetization reversal of these films. During the reversal process there is a large component of the magnetization perpendicular to the direction of the applied field in the plane of the film. This perpendicular component was not observed with thicker sheets of ferromagnetic material.

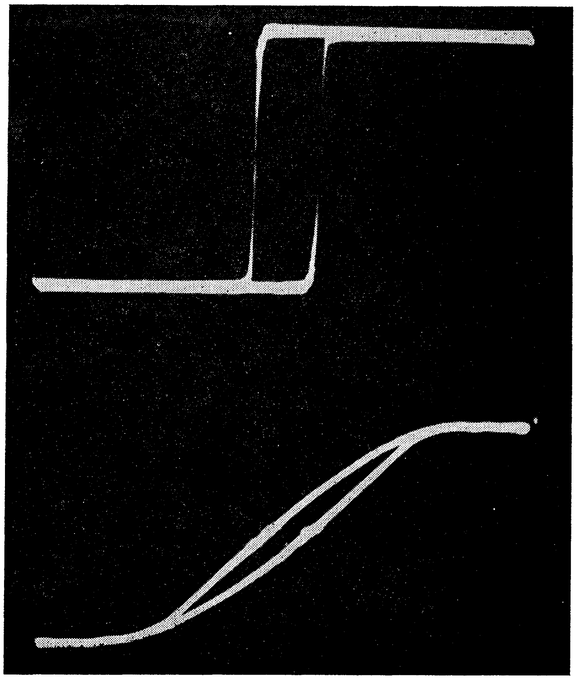

FIG. 1. Hysteresis loops of evaporated film.

---

* This work was supported by the Office of Naval Research.

[1] Williams, Shockley, and Kittel, Phys. Rev. 80, 1090 (1950).

[2] C. Kittel, Phys. Rev. 70, 965 (1946).