gas and not by impurity ions. A great deal more work, however, would be necessary to establish this point definitely.

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Interpretation of Field Measurements in the Cathode Region of Glow Discharges*

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A theory for the cathode region of glow discharges is developed in which the major assumptions are: (a) electron emission from the cathode due to positive ion impact is the dominant secondary mechanism; (b) the probability of ionization by electrons occurring in the high-field cathode region is small, so that electrons carry a negligible portion of the total tube current there; and (c) in this region the positive ions drift towards the cathode with a velocity governed by a mobility law. This theory is found to agree well with measurements in abnormal discharges in various gases at pressures above about 0.1 mm Hg. For lower pressures, measurements indicate that a great number of electrons are present in the cathode-negative glow space, probably due to ionization by high-energy positive ions and molecules; and, that at high enough values of E/p, the application of a mobility theory becomes completely invalid.

GENERAL THEORY

General Mechanism of a Glow Discharge

It is usually assumed that the dominant mechanisms operating in a glow discharge are the primary one of ionization of gas molecules by high-speed electrons and a secondary one of electron emission from the cathode due to the impact of positive ions. γ_i, the ratio of the electron current emitted from the cathode to the positive ion current striking it, ranges between 0.01 and 0.1. Other secondary mechanisms additional to γ_i but akin to it in action are known to occur in gaseous discharges. Some of these are: (a) photoemission from the cathode due to light coming mainly from the negative glow (Little and von Engel[1] have recently proposed a theory based upon this as the dominant cathode mechanism), (b) emission from the cathode due to the impact of metastable atoms or molecules (especially important in the rare gases), (c) ionization of the gas molecules by photons, excited molecules, and/or positive ions. However, because of the relatively high cathode fields and low pressures which favor large values of γ_i compared to other processes, the γ_i mechanism appears to dominate other secondary actions in glow discharges.

The Economy Condition

It will be assumed that all secondary mechanisms other than γ_i can be ignored. Since a glow discharge is self-sustaining and constant in all of its characteristics assuming that no oscillations exist, a certain economy relationship must hold. Namely, each electron emitted from the cathode must on the average cause just enough ionization so that those positive ions formed which return to the cathode cause just one new electron to be emitted. The economy condition may be written $\gamma_i GF=1$, where F is the total number of new electrons and positive ions formed per original electron and G is that fraction of the positive ions formed which arrive at the cathode. G includes losses of electrons and ions to wall and volume recombination. It may have a wide range of possible values.

The Current Continuity Condition

The sum of the electron and positive ion currents at any point in the tube must be a constant equal to the total tube current. Now, $1/(1+\gamma_i)$ of the tube current *at the cathode* must be carried by positive ions. Since positive ions move much more slowly than electrons of the same energy, the space charge of these positive current carriers will be much greater than that of the electrons and thus the net positive charge will be quite large near the cathode. The electric field necessary to achieve high electron velocities, however, is quite low. Therefore, the field in the discharge tube will fall almost to zero as soon as each emitted electron has formed more than $1/\gamma_i$ new ones by ionization. For when this relationship is satisfied, the electron density is high enough so that the tube current of electrons alone will be too high to satisfy the current continuity condition if some minimum field is exceeded. Therefore, the field

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[1] P. F. Little and A. von Engel, Proc. Roy. Soc. (London) **A224**, 209 (1954).

must be quite low with the consequence that some of the electrons are lost to the walls. This condition of high electron and positive-ion densities and low field occurs throughout the major part of the negative glow and to a lesser extent in the Faraday dark space.[2] At the end of the Faraday dark space and the beginning of the positive column, or first striation, the electron density has, mainly through wall losses, become sufficiently low so that the field needed to support the necessary tube current has risen. Ultimately the field rises sufficiently to give to the electrons an energy suitable for excitation and further ionization. In this way, the dwindling electron density is replenished, either constantly, as in a uniform positive column, or periodically as in striations.[2] The new electrons are themselves ultimately lost to the walls.[2] This outline of the variation with position to be expected in electron and positive ion densities and field values is in essential agreement with the explanations of other authors and most measurements. A few of the more striking agreements between observations and this outline will be given.

The current of positive ions and electrons lost to the walls of a tube for every centimeter of its length is proportional to the density of the positive ions and electrons and to the circumference of the tube. The tube current, however, is proportional to the positive ion and electron density and to the area of the tube. Therefore, for the lowest wall losses per cm of tube length, a large diameter tube is desirable. Now, it is observed that glow discharges in spherical vessels, i.e., very distant walls, are characterized by the absence of a positive column. The Faraday dark space appears to fill almost the whole tube. This is, thus, a rather concrete example of low wall losses in the Faraday dark space making unnecessary a positive column with its action of regeneration of positive ions and electrons.

Other observations show that the field in uniform positive columns, or the number of striations per cm of tube length in striated columns, increases as the tube diameter decreases other things remaining fixed. This can be explained by the higher rate of positive ion and electron production needed to offset the higher wall losses resulting from the decreased tube diameter. This higher production can only occur when the field or the striation density are higher, as observed.

A third example deals with the length of the Faraday dark space in a discharge run with a constant gas pressure but varying tube potentials and currents. Under any conditions, the electron current leaving the cathode is $\gamma_i/(1+\gamma_i)$ of the total tube current I. As will appear later in this report, for high and medium pressures and for abnormal discharges, no appreciable ionization occurs in the cathode region. Thus the electron current of $\gamma_i I/(1+\gamma_i)$ ma flows into the negative glow creating new positive ions and electrons. According to Lehmann[3] and Brewer and Westhaver,[4] the number of new ions and electrons produced in the negative glow is directly proportional to $\gamma_i I/(1+\gamma_i)$ and to the cathode-negative glow potential difference. Only a fraction of these new electrons are needed, however, to carry the tube current. The ratio of the available current to the actual tube current will be roughly proportional to the cathode-negative glow potential difference, assuming that γ_i is approximately constant. Thus, the excess of ions and electrons in the negative glow and Faraday dark space is greater the higher the tube potential and current. Therefore, the higher the potential and current are, the greater is the tube length needed to reduce the ion and electron density to the point where a positive column occurs. In agreement with this argument, the length of the Faraday dark space increases as the tube current and potential are increased.[2]

Thus, the picture of the cathode region is the following. In the negative glow where intense ionization and excitation occur, the tube current is dominantly electron current and the field and net space charge are very low. Near the cathode, the tube current is dominantly positive ion current, and this current contributes a large positive space charge. Between the negative glow and the cathode, the current is partly positive ion and partly electron but the net space charge is positive. Therefore, from Maxwell's law, the field increases steadily from nearly zero at the negative glow to a large value at the cathode.[2] The exact dependence of the field upon position is determined by the location of the ionization and the motion of the positive ions in the cathode region.

The Probable Location of the Ionization in the Cathode Region

From visual and photomultiplier observation of the intensity of the negative glow as a function of position, it is noticed that the part of the negative glow nearest the cathode seems to have a sharp edge, i.e., the intensity rises quite rapidly as the negative glow is approached from the cathode.[2] For most gases the probability for excitation by electron collision as a function of electron energy, especially for optically allowed transitions, is of approximately the same form as the probability for ionization. Therefore, since the amount of excitation appears to increase rapidly at the cathode edge of the negative glow, the amount of ionization should do likewise.

This rather abrupt increase in excitation and ionization can be understood, for most of the electron collisions in the Crooke's dark space and some of them in the negative glow involve electrons which have come directly from the cathode with generally few collisions

[2] R. W. Warren, preceding paper, Phys. Rev. **98**, 1650 (1955).

[3] J. F. Lehmann, Proc. Roy. Soc. (London) **A115**, 624 (1927).

[4] A. K. Brewer and J. W. Westhaver, J. Appl. Phys. **8**, 779 (1937).

and few if any collisions involving large energy loss. Therefore, if an ionizing collision does occur, the kinetic energy of the impacting electron may easily be several hundred electron volts, and thus the new electron will have an energy of about 40 ev and an isotropic velocity distribution. If these new electrons are formed in the high-field cathode region, they will be accelerated by the field and thus gain energy. As a result of this high energy, further collisions are not likely to result in ionization or excitation, and elastic collisions are likely to result in extremely small angular deflections. If, however, the new electrons are formed in the negative glow where the field is low, because of their isotropic velocity distribution, many will move *towards the cathode* and *against the weak field* until the field begins to rise at the cathode edge of the negative glow, where the electrons will be repelled. Since these low energy electrons are experiencing collisions, and since the angular deflections of low energy electrons upon collisions are distributed nearly isotropically, each electron will bounce back and forth in the gas, instead of speeding directly down the axis of the tube as in the Crooke's dark space. It will eventually cause further ionization and excitation at some point in the negative glow up to its cathode edge where the field rises. Since electrons formed by ionization have a distribution of energies, the sharpness of the cathode edge of the negative glow is only approximate. The recent work of Pringle and Farvis[5] on electron energy distributions in the negative glow of glow discharges in helium, as measured with a refined probe, agrees remarkably well with these ideas.

This discussion has demonstrated that a self-consistent explanation can be found for the visual appearance of many glow discharges on the assumption that the ionization in the cathode region is low until it increases abruptly on the cathode side of the negative glow, very near the position in the discharge where the field first approaches zero. This does not rule out the possibility that discharges can occur in which much of the ionization takes place in the central part of the high-field region. One would expect to achieve the greatest possible amount of ionization in this region for discharges operating at the lowest possible voltages and the longest possible cathode-negative glow spacings for a given pressure. Such conditions appear in what is called a normal glow discharge. Observations[2] there show that the cathode edge of the negative glow is visibly much less abrupt for normal discharges than for abnormal ones, indicating the presence of more ionization and excitation well into the Crooke's dark space.

Some rather inconclusive and indirect investigations[3,4,6–8] have been devised to evaluate the amount of ionization by electrons occurring in the cathode fall region. In most cases, it is believed that for discharges at low potentials, from 1 to 10 ionizing events take place between the cathode and the negative glow, while at high potentials, only 0.1 to 1 such event takes place.

An interesting calculation can be made using values of the normal glow discharge potential and data given by Lehmann for the average energy expended by electrons in forming a new electron-positive ion pair. By dividing the first of these by the second, a number F results which equals the maximum value for the average production of positive ions and electrons in the negative glow per incident electron if the electron is assumed to pass from the cathode through the high-field region without ionization. F is about five for most gases and cathode surfaces. Therefore, γ_i must be at least 0.2, and probably considerably more, in order that $\gamma_i GF=1$. This is a high value for γ_i. Accordingly, in the case of the normal discharge, the assumption that no ionization occurs in the high-field region is probably incorrect. Instead, some electrons are produced there which then gain energy from the field, producing much more total ionization once they reach the negative glow than the same number of electrons all produced in the negative glow.

In developing a theory for the cathode region of an abnormal discharge then, it will be assumed that no appreciable ionization occurs in the cathode region but that instead, all of the production of new electrons and positive ions needed to satisfy the current continuity condition occurs in the negative glow up to a rather abrupt edge on the cathode side of the glow.[2] This edge occurs near the position in the cathode region at which the field has just approached zero.[2] Positive ions formed in the negative glow drift across the edge with low energy. From the edge on to the cathode the tube current must be carried almost entirely by positive ions. The exact form of the field in the cathode-negative glow spacing then depends only upon the law governing the motion of the positive ions. Positive-ion diffusion to the walls in this space can be ignored, for positive ions and electrons must be lost to the walls in equal numbers at all points. Since the electron current is so small, even if it were all lost to the walls with an equal positive-ion current, the fraction of the positive-ion current lost would be negligible.

A complete theoretical solution to the cathode region, giving the cathode-negative glow separation and potential, would involve an exact knowledge of both γ_i and the behavior of the negative glow. Because of ignorance concerning each, a complete solution is not achieved, but instead, a partial solution which predicts the form of the variation of the field with position, pressure and current is possible. This partial solution depends upon the law governing the motion of the positive ions. However, this motion is so complicated that different laws hold in different parts of discharges and for different

[5] D. H. Pringle and W. E. J. Farvis, Phys. Rev. **96**, 536 (1954).
[6] M. J. Druyvesteyn and F. O. Penning, Revs. Modern Phys. **12**, 87 (1940).
[7] P. L. Morton, Phys. Rev. **70**, 358 (1946).
[8] G. W. Johnson, Phys. Rev. **73**, 284 (1948).

operating parameters. It appears, however, that for large portions of many discharges a single law will hold. A theory, therefore, is developed for this case, and for comparison purposes, another theory is developed for the limiting case where the positive ions suffer no collisions in drifting from the negative glow to the cathode.

Various Theories for the Field in the Cathode Region

In an early theory of Ryde,[9] an attempt was made to describe accurately the dependence of the electric field upon position in the cathode region by using the aforementioned general theory with the special assumption that the positive ions move from the negative glow to the cathode without collisions of any kind. Under these conditions, the field in the cathode region can be calculated from a solution of the two equations $\nabla\cdot\mathbf{E}=d^2V/dx^2=4\pi\rho$ and $eV=Mv_+^2/2$. V and x, the potential and the position in the cathode region, are measured from the cathode edge of the negative glow. v_+, the speed of the positive ions of mass M moving towards the cathode is assumed to be zero at the cathode edge of the negative glow. v_-, the speed of the electrons, is always much greater than v_+ even at the cathode. ρ_+ and ρ_- are the positive-ion and electron-charge densities; J_+ and J_- are the positive-ion and electron-current densities; and ρ and J are the total charge and current densities. Now,

$$\rho=\rho_+-\rho_-=\frac{J_+}{v_+}-\frac{J_-}{v_-}=J_+\left(\frac{1}{v_+}-\frac{\gamma_i}{v_-}\right)=\frac{J}{1+\gamma_i}\left[\frac{1}{v_+}-\frac{\gamma_i}{v_-}\right],$$

and since $1/v_+\gg\gamma_i/v_-$,

$$\rho\simeq\frac{J}{(1+\gamma_i)}\frac{1}{v_+}.$$

However, $v_+=(2eV/M)^{\frac{1}{2}}$ so that

$$\frac{d^2V}{dx^2}\simeq\frac{4\pi J}{(1+\gamma_i)}\left(\frac{M}{2eV}\right)^{\frac{1}{2}}.$$

The solution of this last equation is

$$V=\left[\frac{9\pi J}{(1+\gamma_i)}\left(\frac{M}{2e}\right)^{\frac{1}{2}}x^2\right]^{\frac{2}{3}},$$

and

$$E=\left[\frac{8\pi J}{1+\gamma_i}\left(\frac{3Mx}{2e}\right)^{\frac{1}{2}}\right]^{\frac{2}{3}},$$

where the boundary conditions are $E=V=0$ at $x=0$. This will be called the "free fall" solution.

The resulting fields and potentials do not appear to agree with past measurements. It also seems probable that many positive-ion collisions occur in the cathode region since the cathode-negative glow separation corresponds to about thirty ionic mean free paths. Therefore, a more appropriate treatment in which collisions play a dominant role must be used. However, the "free fall" solution yields a useful limit since *under no circumstances can a legitimate solution to Maxwell's equation be found which involves a field versus position curve which falls beneath the "free fall" field curve* as long as $J=(1+\gamma_i)J_+$. This fact is not as obvious as it may at first appear, but may be proven to hold by some rather subtle reasoning involving the relations that the slope of any field curve is inversely proportional to the positive ion velocity at that point (from Maxwell's equation), and $e\int_0^{x'}Edx$ of any field curve must be greater than, or equal to, the energy of the positive ions at $x=x'$.

The recent work of Hornbeck and Wannier,[10] Hornbeck,[11] Hornbeck and Molnar,[12] and Varney[13] shows conclusively that the average velocity of positive ions in gases is determined by collisions with gas molecules, and for field to pressure ratios greater than about one hundred volts per cm-mm Hg, the velocity varies as $(E/p)^{\frac{1}{2}}$, i.e., $v_+=k'(E/p)^{\frac{1}{2}}$, where k' is a constant for each gas called the mobility. The upper limit to the E/p values permitted in these measurements was about 1000 volts cm-mm Hg. By assuming the above dependence to hold for all values of E/p, and using the general theory outlined above, it is possible to show that

$$\frac{dE}{dx}=\frac{4\pi J}{(1+\gamma_i)v_+}=\frac{4\pi Jp^{\frac{1}{2}}}{k'E^{\frac{1}{2}}(1+\gamma_i)}.$$

This has the solution

$$E=\left[\frac{6\pi Jp^{\frac{1}{2}}x}{k'(1+\gamma_i)}\right]^{\frac{2}{3}},$$

where the boundary conditions are $E=V=0$ at $x=0$. According to Wannier,[14] the constant k' is given in most cases by $k'=1.147(ep\lambda_i/M)^{\frac{1}{2}}$, where λ_i is the ionic mean free path which is in general about one third of a molecular mean free path. Thus

$$E=\left[\frac{6\pi Jx}{1.147(1+\gamma_i)}\left(\frac{M}{e\lambda_i}\right)^{\frac{1}{2}}\right]^{\frac{2}{3}}.$$

This is approximately the relationship originally formulated by Weizel, Rompe, and Schoen.[15] It will be called the "mobility" solution.

The "mobility" solution is valid only if enough collisions occur so that the positive ions are in equilibrium or near equilibrium with the impressed field. If the field were instantaneously to increase from a low value to a high one, the positive ions could not have an

[9] J. W. Ryde, Phil. Mag. **45**, 1149 (1923).

[10] J. A. Hornbeck and G. H. Wannier, Phys. Rev. **82**, 458 (1951).

[11] J. A. Hornbeck, Phys. Rev. **80**, 297 (1950); **83**, 374 (1951); **84**, 615 (1951).

[12] J. A. Hornbeck and J. P. Molnar, Phys. Rev. **84**, 621 (1951).

[13] R. N. Varney, Phys. Rev. **88**, 362 (1952); **89**, 708 (1953).

[14] G. H. Wannier, Bell System Tech. J. **32**, 170 (1953).

[15] Weizel, Rompe, and Schoen, Z. Physik **112**, 339 (1939).

average velocity given by $v_+=k'(E/p)^{\frac{1}{2}}$ until they had gained sufficient energy by traveling a distance in the field direction corresponding to a few mean free paths. Thus, if the field changes at a rate of less than about thirty percent per ionic mean free path, the relationship $v_+=k'(E/p)^{\frac{1}{2}}$ is approximately true. Obviously the field gradient far exceeds this rate near the origin, and thus, the solution depending upon the mobility law must be in error there. This failure near the origin is also evident from the behavior of the "mobility" curve of E *versus* position which violates the previous rule by falling beneath the "free fall" curve.

Further investigation shows that the failure of the simple "mobility" theory occurs for only a small fraction of the total cathode-negative glow spacing—that same fraction for which several other approximations of these simple theories also become invalid. Rough calculations indicate that the effects of the different approximations tend to cancel and that, in any case, the error introduced in the simple "mobility" theory is minor, to be of importance only very near the negative glow.

Various further modifications of the simple "mobility" theory have been attempted, mainly by assuming ionization by electrons to occur in the cathode region. A fairly complete account of these investigations, their assumptions, and their resulting theories is given in an excellent article by Druyvesteyn and Penning.[6] The main effect upon the field curves of ionization in the cathode region is an over-all decrease in the field values, being especially noticeable near the negative glow. If the ionization is intense enough, it can lead to anomalous field curves that are concave upwards rather than downwards near the cathode. Under all conditions of ionization, however, the positive ion current density *at the cathode* is related to the total current density by $J=J_+(1+\gamma_i)$. Therefore, if $v_+=k'(E/p)^{\frac{1}{2}}$,

$$\frac{dE}{dx}=\frac{4\pi J}{v_+(1+\gamma_i)}=\frac{4\pi J p^{\frac{1}{2}}}{k'E^{\frac{1}{2}}(1+\gamma_i)},$$

and therefore

$$E^{\frac{1}{2}}\frac{dE}{dx}=\frac{4\pi J p^{\frac{1}{2}}}{k'(1+\gamma_i)}$$

at the cathode, no matter how much ionization occurs at various places in the tube. Thus, at the cathode in any case, k' can be calculated if γ_i is known. Use will be made of this relationship later.

COMPARISON OF THEORY AND EXPERIMENT

The foregoing "mobility" solution will be tested against field measurements made by the author for abnormal glow discharges in a ten cm diameter tube filled with helium, hydrogen, nitrogen, or argon at pressures between 0.03 and 1.0 mm Hg, and for currents between 0.1 and 10 ma.[2,16]

The field measurements used were accurate to within a few percent, but only for pressures greater than about 0.1 mm Hg was the effect of the walls negligible, so that the theoretical treatments given here could be tested directly. For lower pressures, the equipotential surfaces in the discharge tube departed considerably from planes because of positive charges which accumulated on the walls. Therefore, a direct comparison of the measurements and such one-dimensional theories as those aforementioned might result in discrepancies of the order of 10 percent.

The previous paper contains three graphs for each gas presenting field measurements at three different pressures—1.0, 0.3, and 0.1 mm Hg for helium and hydrogen, and 0.3, 0.1, and 0.03 mm Hg for nitrogen and argon. Each graph contains curves of $E/I^{\frac{2}{3}}$ *versus* the distance from the negative glow, plotted for a number of different tube currents, I. Each graph also contains a curve of $[(8\pi/A)(3Mx/2e)^{\frac{1}{2}}]^{\frac{2}{3}}$ (where A equals the cathode area), which should equal $E/I^{\frac{2}{3}}$ according to the "free fall" theory, and a curve of $[6\pi p^{\frac{1}{2}}x/k'A]^{\frac{2}{3}}$ which should equal $E/I^{\frac{2}{3}}$ according to the "mobility" theory. This is done so that if either the "free fall" or "mobility" theory is exactly correct, all curves taken for a given gas at a given pressure but for different currents will fall together and coincide with either the "free fall" or "mobility" curve. By reducing all of the field measurements in this way, most of the important characteristics of the measurements can be made obvious.

For instance, especially for the light gases, the curves of $E/I^{\frac{2}{3}}$ at a given pressure but different currents are so similar that they all fall within a very small region on the graphs. Again, especially for the light gases and at high pressures, the $E/I^{\frac{2}{3}}$ curves fall well above the "free fall" curves indicating that collisions are playing an important part in the motion of the positive ions. At low enough pressure, however, the experimental curves fall well beneath the "free fall" curve, which can only occur if ionization is taking place in the cathode-negative glow space or if the cathode emission has increased enormously. From the extent to which the experimental curves fall beneath the "free fall" curves, it can be estimated that, in some cases, about 9/10 of the tube current in the cathode region is carried by electrons. From the parameters of such discharges, it seems highly unlikely that the new electrons can be formed through ionization by electrons, although the anomalous shape of the field curves leads one to expect a volume production of new electrons. From the work of Parker,[17] Varney,[18] Hagstrum,[19] and Berry and

[16] R. W. Warren, Rev. Sci. Instr. (to be published).
[17] J. H. Parker, Phys. Rev. **93**, 1148 (1954).
[18] R. N. Varney, Phys. Rev. **93**, 1156 (1954).
[19] H. D. Hagstrum, Phys. Rev. **89**, 244 (1953).

Abbott,[20] it is concluded that $(1+\gamma_i)\simeq 1$ and that energetic neutral particles cannot be contributing a very large additional γ. Other cathode mechanisms can, similarly, be ruled out. It thus seems most probable that the phenomenon accounting for the high electron production is none of these but instead, the ionization of gas molecules upon the impact of high-energy positive ions and neutral molecules.

According to the work of Rostagni[21] and Varney,[18] the probability of ionization of gas molecules is of about the same order for collisions with high-energy positive ions or neutral molecules, with values actually favoring collisions with molecules. Thus, in crossing the cathode-negative glow distance of about thirty ionic mean free paths, a positive ion will have collisions with molecules and the recoil neutral molecules will have collisions with other molecules. At high and medium pressures, very few of these collisions will involve energies greater than 100 volts. At low pressures, however, there will occur a total of perhaps 100 or so ion-molecule and molecule-molecule collisions involving energies greater than 100 electron volts for each positive ion leaving the negative glow. With ion and molecule energies of about 100 ev, the probability for the ionization of a gas molecule upon a collision is about 1 percent. Therefore, on the average, each positive ion will form of the order of one new positive ion and electron in its passage from the negative glow to the cathode. These new particles may themselves cause further ionization. In consequence, it is believed that this ionization can account for both the high electron production and an upcurving of the field curves near the cathode.

Additional studies, including those of the nature of canal rays,[2] indicate that the glow on the cathode of glow discharges is largely due to excitation by energetic ions and molecules. Therefore, the importance of energetic ions and molecules in the functioning of low-pressure discharges is further indicated.

In attempting to correlate the experimental field curves with the "mobility" theory, k' was allowed to assume various values so that the resulting "mobility" curves for each gas fitted the experimental curves as well as possible at the highest pressure. Once k' was fixed, new "mobility" curves were constructed for each gas at the lower pressures. If the "mobility" theory were exactly correct, all the experimental curves at all pressures should fall on these "mobility" curves. It was found that a good fit resulted for all gases at the high pressure and in addition for helium and hydrogen at the intermediate pressure and low E/p values. For lower pressures, as discussed above, ionization in the cathode-negative glow space precludes any comparison.

Since a fairly good agreement was found between the measurements and the "mobility" theory, especially for helium and hydrogen, it was felt that a comparison of the fitted k' values and the directly measured k' values of Hornbeck and Varney would be valid. Such a comparison depends upon the identification of the pertinent ion in each gas. However, little or nothing is known for the cathode region of glow discharges other than that many different ions can be present with k' values differing by a factor of perhaps two. In all cases where a comparison was possible, the experimental k' values differed from published values by a factor of about two or less. For instance, experimentally, $k'(\text{He})=8.2\times10^4$ cm/sec (cm-mm Hg/volt)$^{\frac{1}{2}}$, while Hornbeck gives $k'(\text{He}^+)=4.0\times10^4$ and $k'(\text{He}_2^+)=8\times10^4$. $k'(\text{He}_2^+)$ has been extrapolated from the low E/p values for which it was measured and may, therefore, be slightly in error. Probably the basic theory and the field measurements as affected by wall charges[2] are not accurate enough to permit much better agreement. Obviously, mass spectrographic analysis of the pertinent ion would be a great help in any other studies of this kind.

While what has gone before applies largely to the simple conditions at high pressures and fairly near the negative glow, the conditions near the cathode appear to be more complicated as originally indicated by Stein.[22] It was indicated above that k' could always be calculated at the cathode from $k'=4\pi Jp^{\frac{1}{2}}/(E^{\frac{1}{2}}dE/dx)$, no matter how much ionization was occurring. Such calculations give normal k' values at high pressures, values about three times too low at medium pressures, and values about ten times too low at low pressures. Several possible explanations may be suggested.

Since the gases used were only 99.5 percent pure, the pertinent ion might have changed from an ion of the main gas to one of the impurity, giving the observed low k' value near the cathode. However, in most cases, the probability for the reverse reaction is large enough to severely limit the impurity ion concentration. Impurity ions in nitrogen and argon would be likely to increase k' near the cathode which was not observed. Further tests with a purer gas showed no change in the field measurements, indicating the validity of the hypothesis that the pertinent ions were largely of the main gas, not of the impurity.

In mobility measurements in nitrogen, Varney found that near a critical E/p value, the nature of the positive ion changed back and forth between N_2^+ and N_4^+. Over a certain range of E/p near this critical value, the ion velocity and k' actually decreased with increasing E/p. It is not impossible that analogous changes occur in other gases giving a decreasing velocity and k' with increasing E/p, for E/p around 500.

Another possible explanation for the observed decrease in k' is more fundamental and more serious. The measurements of Hornbeck *et al.* were carried out in most cases for E/p values as high as 1000. It has been assumed that for higher values, k' remains constant,

[20] H. W. Berry and R. C. Abbott, Technical Report, Office of Naval Research from Syracuse University, June, 1954 (unpublished).

[21] A. Rostagni, Nuovo cimento **13**, 389 (1936).

[22] R. P. Stein, Phys. Rev. **89**, 134 (1953).

but this need not necessarily be so. As E/p increases, excitation and ionization by positive ions become more likely, and collision processes in general become more complicated. Most new complications tend to increase collision cross sections and decrease k'. Suggestions have even been made that in analogy to assumptions in solid state theory, for high enough E/p values, the ion velocity becomes independent of E/p, and thus k' decreases rapidly with increasing E/p. In any case, since two of the fundamental assumptions upon which the mobility law is based, i.e., elastic collisions and isotropic scattering upon collisions, fail at high E/p values, it is not surprising to find a theory based upon these assumptions also failing at high E/p. Some such drastic change in the mobility law must occur to explain the observed behavior of k' at the high E/p values (near 30 000) which occur in the measurements discussed here.

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Motion and Spectrum of Arc Cathode Spot in a Magnetic Field

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The velocity of the cathode spot of a mercury arc at the junction between liquid and metal in a transverse magnetic field has been measured for magnetic field strengths between 0 and 20 700 oersteds. The approximate doubling of retrograde velocity at about 11 000 to 15 000 oersteds was followed by an additional rapid rise of velocity at about 15 000 oersteds. Spectra of the arc showed Hg II and Hg III lines at the stronger magnetic fields. Radiation from the cathode spot showed mercury lines and a continuous spectrum which is especially intense at the lines. Some Hg lines are broadened symmetrically and others asymmetrically. If the broadening is due to a Stark effect, the electric field strength in the cathode spot region is greater than 6×10^6 volts/cm.

The arc mechanism previously proposed is extended to explain the rapid velocity rises with increasing magnetic field strength by associating them with the effect of Hg^{++} and Hg^{+++} ions.

INTRODUCTION

THE results of investigations on the motion of the cathode spot of an electric arc in transverse magnetic fields up to 12 000 oersteds were reported in an earlier article.[1] Both the direction and the magnitude of the velocity of the cathode spot are dependent upon the magnetic field strength, arc current, and pressure of an admixed inert gas.

With no inert gas in the arc tube and with a constant arc current, the velocity was retrograde or opposite to the force on the charged particles due to their motion through the magnetic field. The velocity changed as follows when the magnetic field strength was increased from 0 to 12 000 oersteds. First, the velocity increased rather linearly with field strength and then approached a saturation value. Finally, it rose rapidly to a value nearly double that at saturation. High-current arcs had greater velocities and exhibited the rapid velocity increases at lower magnetic field strengths than low-current arcs.

With an inert gas in the arc tube the spot velocity changed from forward (i.e., in the direction of the force due to motion through the magnetic field) to retrograde as the magnetic field strength was increased and an increase in arc current caused the velocity to increase in the forward direction.

With constant magnetic field strength and constant arc current and with a small amount of inert gas in the arc tube, the spot moved in the retrograde direction. Increasing the gas pressure caused the spot to slow down, stop, and finally move in the forward direction.

When an appreciable amount of inert gas was in the arc tube and conditions were such that the cathode spot was stationary, an increase in magnetic field strength caused the spot to move in the retrograde direction. This retrograde motion, however, could be overcome and the spot brought to rest again by either an increase in arc current or by an increase in gas pressure.

The spectrum of the radiation from the negative glow of the mercury arc in a very weak magnetic field showed Hg I lines and in a strong magnetic field showed both Hg I and Hg II lines. The spectrum of the radiation from the cathode spot was continuous with maxima of intensity at the Hg lines for all magnetic field strengths.

Smith[2] observed the arc motion and also the spectrum

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[1] R. M. St. John and J. G. Winans, Phys. Rev. 94, 1097 (1954).
[2] C. G. Smith, Phys. Rev. 83, 194 (1951); 84, 1075 (1951).