Superfluidity and Heat Transport in the Unsaturated Helium-II Film*

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The mobility of unsaturated He-II films has been investigated by heat transport measurements as a function of thickness and temperature. The results confirm data from previous flow measurements that at a given temperature a certain number of layers are immobile, changing smoothly from ~2 at 1.3°K to ~10.5 at 2.0°K. All additional layers flow in a temperature gradient with about the same drift velocity, this velocity being of the same order of magnitude as that derived from the transfer rate of the saturated film. Films of thickness between 20–80 layers show superfluidity at and even slightly above the λ temperature of the bulk liquid.

INTRODUCTION

SOME experiments on heat transport by superfluid flow in thin films of adsorbed helium in the He-II region (below 2.19°K) have previously been reported by Bowers, Brewer, and Mendelssohn (BBM),[1] and a preliminary report of the experiments to be discussed in this paper has been given by the present authors.[2]

The experiments deal with thin films of adsorbed helium in an open wide tube with a chamber at each end, one provided with a heater and the other in good thermal contact with a heat reservoir, with sensitive thermometers attached to the tube so that temperature gradients along the conduction path may be determined. The tube and heater chamber are insulated by high vacuum. If helium gas is then admitted to the system, a film of adsorbed He is formed, the thickness being determined by the temperature and pressure. If heat is now applied to the heater chamber, and *if* superflow is possible under the prevailing conditions, the film (or part of it) will flow toward the source of heat, evaporate there, and the vapor will return via the open tube to the cool end, where it condenses to film, thus completing the heat transport cycle.

The film coverage in the cell is determined at each temperature by the saturation P/P_0, where P is the pressure of the vapor in equilibrium with the adsorbed phase and P_0 is the vapor pressure of the bulk liquid. The preliminary work of reference 2 showed that for a given film thickness the superfluid contribution to the heat transport appears abruptly at a certain temperature (the "onset" temperature). A film of a given thickness will not therefore exhibit superflow above this temperature. A plot of P/P_0 *vs* the "onset" temperatures is in agreement with the results of method I of earlier flow measurements of the present authors,[3] and is consistent with extrapolations to zero of the heat transport data reported by BBM (reference 1).

Since an apparatus can easily be constructed to have a very small thermal conductance in the absence of superflow, and since the heat transport by superflow in this system involves the heat of vaporization of the film in addition to the thermomechanical heat, the method provides a sensitive means of investigating the superfluid behavior of these thin films.

APPARATUS

The heat transport cell is shown schematically in Fig. 1; it resembles qualitatively the arrangement of reference 1. A copper-nickel alloy tube of 4.7 mm o.d. and 0.127 mm wall thickness was provided at the top with a copper chamber E, on which a manganin heater was wound, and at the bottom with another copper chamber D, immersed in the liquid He-II bath which surrounded the vacuum can A. Sensitive carbon thermometers[4] Th-1, Th-2, and Th-3 were sealed by glyptal or Corex varnish into heavy copper holders soldered at the top, middle, and bottom of the tube; the distance between the top and bottom thermometers was 10 cm, with the middle thermometer mounted halfway between.

The insulating vacuum in the can A was pumped through the tube P, which was provided with a radiation trap (not shown). The electrical connections are not shown; the thermometers were connected in series, with a common current lead, and with potential leads arranged so that the resistance of each thermometer could be measured independently. The thermometer leads were of No. 36 manganin wire, to provide connections of very low thermal conductance; the heater leads were of No. 40 copper wire, to minimize heating effects. All electrical leads were connected to kovar-glass terminals soldered to the bottom plate B of the vacuum

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[1] Bowers, Brewer, and Mendelssohn, Phil. Mag. **42**, 1445 (1951).

[2] E. Long and L. Meyer, Phys. Rev. **87**, 153 (1952). In the discussion of this note in the review by E. Long and L. Meyer, Phil. Mag. 44, Supplement 2, 18–22 (1953), the values of the heat current shown in Fig. 13 are too high by a factor of ten, due to an error in reproduction.

[3] E. Long and L. Meyer, Phys. Rev. **79**, 1031 (1950); Phys. Rev. **85**, 860 (1952).

[4] These thermometers were nominal 100-ohm, ½-watt radio resistors made by the Speer Resistor Corporation, St. Mary's, Pennsylvania. Their resistance increased from 100 ohms at 300°K to 500–1000 ohms at 1.2°K. The sensitivity ranged from 100 to 300 ohms per degree in the liquid helium region. Their use offers a number of advantages for work such as this, although it was felt that calibration was required each time the apparatus was cooled to helium temperatures.

can, thus providing electrical connections of minimal thermal leakage from temperatures higher than that of the surrounding bath.

The helium gas was admitted to the conductance cell through the stainless steel capillary C, which was connected to a large buffer volume at room temperature, to ensure that heating of the chamber H did not increase the gas pressure in the cell.

The gas pressure P in the cell and the bath vapor pressure P_0 were measured using Octoil-S manometers and a Wild cathetometer of precision 0.02 mm.

A rather rigorous experimental procedure was required to ensure proper performance of the apparatus. The cell had such a low thermal conductance (in the absence of exchange gas in chamber A, or of a superfluid He film inside the cell) that the apparatus could not readily be cooled to working temperature without introducing exchange gas into chamber A. This then entailed a prolonged pumping period to ensure a vacuum in the insulating space sufficiently low that heat conduction by the gas across chamber A did not contribute to the heat transport in any appreciable way. Since it was considered necessary to calibrate the carbon thermometers, using exchange gas, each time the apparatus was cooled to helium temperatures, it was thus standard practice to calibrate[5] the thermometers during one working day, and then to pump the system overnight before proceeding with the heat transport experiments. In this manner insulating vacua below 10^{-6} mm Hg, usually $1-2\times10^{-7}$, were obtained, as measured on a liquid nitrogen-trapped RCA 1949 ion gauge mounted in the external vacuum system.

The resulting thermal conductance of the cell, without the contribution of the superfluid film, was 10–15 microwatts per degree, although no attempts were made to accurately measure this quantity, since it was sufficiently small to be ignored, even at the lowest heat transport levels observed with the superfluid film.

Because of the precision desired, it was necessary to maintain the temperature of the cryostat to a drift of 1 millidegree per hour or less during the measurements.

EXPERIMENTS

A series of experiments was carried out in which the heat transport was measured at a number of temperatures from 1.3°K to 2.0°K, with a number of film coverges being investigated at each temperature, all showing superfluid behavior.

The data for the temperatures and film coverages measured are summarized in Table I. P is the pressure measured in the system, P_0 the saturation vapor pressure of bulk liquid at the temperature of the bath; P/P_0 represents therefore the saturation at the bottom

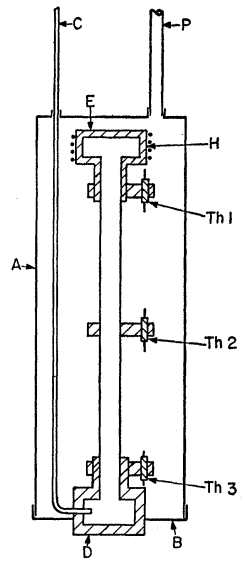

FIG. 1. Schematic drawing of the heat transport apparatus.

of the tube. $\dot{Q}$ is the heat input in microwatts, and ΔT is the measured temperature difference in millidegrees between the top and bottom thermometers, Th-1 and Th-3, separated by a 10-cm distance along the tube.

It should be remembered that the experiments are made at constant P and constant T of the reservoir and therefore that the values quoted for P/P_0 are for the bottom reservoir only. P/P_0 decreases towards the warm end with increasing ΔT and is thus somewhat in error. The error is small, except at the higher ΔT's, with which this work is not mainly concerned.

The heat current $\dot{Q}$ is a function of T, ΔT, and P/P_0. Figure 2 is qualitatively representative of the data at all temperatures and unsaturated film coverages in these measurements and gives $\dot{Q}$ in microwatts plotted against ΔT in millidegrees at constant temperature (1.700°K) of the thermal reservoir for different values of P/P_0.

Figure 2 shows that the heat current is not linearly dependent on the temperature gradient, but resembles qualitatively the cube root dependence which occurs in the heat conduction of liquid He-II.[6] Indeed, an analysis of the film data shows a fairly good agreement with a cube root law at low values of ΔT.

However, the resemblance can be only superficial: The heat current in liquid He-II can be described by

$$\dot{Q}=\text{const}\,(dT/dx)^{\frac{1}{3}},$$

where the constant comprises the geometry of the current and a factor representing the "conductivity." ΔT then remains a linear function of x, whatever the exponent of dT/dx may be.

In order to investigate this distribution of the temperature gradient, Th-2, mounted half-way along the tube, was used. A typical result is shown in Fig. 3, in

[5] All calibrations were based on the vapor pressure of liquid helium, as tabulated in the 1949 temperature scale—see H. van Dijk and D. Shoenberg, Nature **164**, 151 (1949). Although later evidence indicates that this scale is somewhat in error—see R. A. Erickson and L. D. Roberts, Phys. Rev. **93**, 957 (1954)—the deviations are not relevant to this experiment.

[6] Keesom, Saris, and Meyer, Physica **7**, 817 (1940).

TABLE I. Amount of heat in microwatts carried by superfluid films as a function of temperature T, saturation P/P_0 and temperature difference ΔT.

T, °K	P/P_0	$\dot{Q}$, watts	ΔT, °K×10³
1.313	0.598	3.97	0.9
		4.30	2.7
		5.72	17.1
	0.813	15.9	0.7
		22.9	1.3
		26.8	3.0
		31.1	5.2
		35.7	22.9
	0.905	15.9	0.5
		40.6	3.3
		51.4	12.7
		57.3	
1.503	0.704	0.64	1.2
		0.99	3.3
		1.43	
	0.774	1.43	0.2
		3.97	1.1
		4.30	2.9
		5.72	10.4
	0.846	10.2	0.1
		12.9	1.7
		15.9	11.7
	0.912	10.2	0.5
		15.9	0.8
		31.1	2.1
		35.7	4.6
		40.6	22.4
	0.955	31.1	0.3
		40.6	0.3
		63.5	1.6
		70.1	4.3
		76.8	10.0
1.700	0.892	4.3	1.1
		5.0	2.8
		5.7	9.2
0.932	5.0	5.0	0.4
		5.7	0.6
		7.8	0.6
		15.9	2.2
		17.2	3.6
		19.2	14.6
1.700	0.955	10.2	0.1
		15.9	0.2
		22.9	0.7
		31.1	2.7
		32.9	4.3
		35.7	12.3
	0.978	31.1	0.2
		40.6	0.6
		53.5	2.8
		70.0	5.7
		73.7	10.6
1.900	0.963	1.42	0.2
		2.55	3.3
		2.80	9.7
		3.35	29.3
	0.979	2.55	0.2
		5.7	0.5
		10.2	1.2
		12.9	5.0
		15.9	~25
	0.988	10.2	0.4
		15.9	0.5
		22.8	1.2
		26.8	2.9
		31.1	5.4
		35.7	11.7
	0.991	22.8	0.4
		31.1	0.5
		40.6	2.9
		45.9	5.7
		51.4	12.1
2.000	0.981	0.64	0.0
		1.43	3.3
		1.94	12.2
		2.54	0.0
	0.995	10.2	0.0
		19.2	0.1
		35.7	1.6
		45.9	3.5
		57.3	14.5

which the solid curve represents the ΔT between top and bottom of the tube (Th-3—Th-1), while the dashed curve is that between center and bottom (Th-2—Th-1). The temperature was 1.785°; the saturation P/P_0 was 0.927.

This temperature gradient along the lower half of the tube also obeys roughly a cube root law, within the accuracy of the measurements; however, the agreement may be fortuitous. It was suggested earlier (see reference 2) that if this finite but very small ΔT is real, then perhaps the film transport cannot be regarded as a true superfluid process. Unfortunately, the effect is so small, and the establishment of equilibrium between the thermometers and the flowing film so correspondingly uncertain to the required accuracy, that no definite conclusions can be drawn.

It is clear from these results that the observed temperature distribution must be a consequence of a change of the constant along the tube, i.e., the geometry and/or the "conductivity." As ΔT is increased at constant P, the value of P/P_0 at the warm end decreases; consequently the film becomes thinner in this region and its capacity to carry heat decreases correspondingly. Real superfluidity could even under these circumstances not produce a temperature gradient, only instability. We have therefore to assume that as soon as the thermal load exceeds a certain critical value heat resistance sets in rather sharply in the film. The continuity of the heat current then requires a considerable temperature gradient in this region of lower conductivity.[7] The re-

[7] If the film thickness can be represented by the isotherm $\ln(P/P_0)=k/v^3$ as shown by R. Bowers, Phil. Mag. 44, 487 (1953)

duction in film thickness can finally go so far that the moving part of the film evaporates completely before reaching the chamber H itself, and a small region of the tube below chamber H takes over the heat transport. Thus a short but finite length of the tube practically determines the resistance to the heat transport, with a very considerable local temperature gradient. This is the "runaway" phenomenon observed by BBM (reference 1); it may be seen clearly in Fig. 2 as the end of the curve which becomes almost parallel to the T axis.[8,9]

An analysis of these data and of the earlier flow data of reference 3 requires that values of the film thickness be derived from the saturations P/P_0 of the experiments, using the adsorption isotherms of He. The adsorption of He on this particular Cu—Ni tube surface is not known; however, the fact that the "onset" temperatures for superflow are the *same* in this arrangement[2] as in the flow measurements previously quoted lead to some confidence in the coverages derived below.

The adsorption measurements of Strauss,[10] on iron oxide, in fairly good agreement with those of Bowers[11] on metallic aluminum, are used here for this purpose.

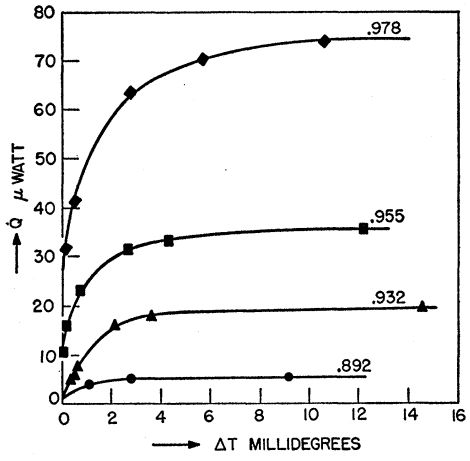

FIG. 2. Amount of heat in microwatts carried by films as a function of the temperature difference in millidegrees for different saturations P/P_0 at 1.700°K.

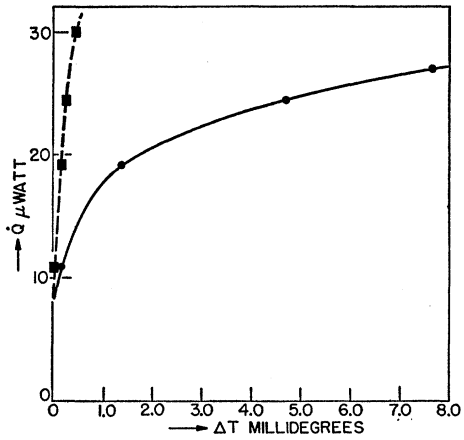

FIG. 3. Temperature difference between top and bottom thermometer (solid curve) and temperature difference between center and bottom thermometer (dashed curve) as functions of the heat input in microwatts.

In order to compute the average number of statistical layers adsorbed on a surface from the total quantity of adsorbate, it is inevitable that some assumptions be made concerning the layers nearest the wall. In the calculation made here, the quantity v_m (the amount adsorbed in the first completed monolayer) was that derived by Strauss from his isotherms, using the Brunauer-Emmett-Teller theory.[12] The quantity $\frac{1}{2}v_m$ was taken for the adsorption in the second monolayer, and then 0.28 cm^3/m^2 surface area was used for all subsequent layers. The number of layers derived in this way from the data of Strauss is consistently about two layers less than the values given by Bowers for the same saturation P/P_0. However, this is within the error, since Bowers, using a microbalance, was unable to measure the absolute values of the adsorption on his surfaces below $P/P_0 \sim 0.15$, and had to assume the amount adsorbed below this value, whereas Strauss measured these quantities in direct determination. Moreover, the consistency of the treatment is more important than the absolute magnitude.

In Fig. 4, $\dot{Q}$, again in microwatts, is plotted at constant ΔT (3 millidegrees) against the film thickness derived from P/P_0 and the adsorption isotherms in the manner described above, for each temperature of measurement. The 3 millidegree choice for ΔT is clearly arbitrary, but is one for which even at the warm end the film thickness should not deviate much from that derived for the lower end.

The intercepts with the abscissa in Fig. 3, i.e., the points for $\dot{Q}=0$, are not extrapolated from the points for finite $\dot{Q}$, but were calculated from the dependence of the "onset" temperatures for superflow upon the film

the apparent dependency of $\dot{Q}$ on $(\Delta T)^{\frac{1}{2}}$ can be a consequence of the isotherm, since a change of $\ln P$ is proportional to ΔT, in first approximation; v therefore decreases proportional to $(\Delta T)^{\frac{1}{2}}$.

[8] It appears, however, difficult to explain all observed temperature gradients by assuming complete superfluidity with no ΔT at all along the greater part of the tube, and the whole resistance concentrated in a short length of the tube at the warm end. In many cases the heat resistance is so small that already less than a millimeter of tube length would produce it. It seems doubtful that such a short length would not eventually be swamped by film flow, and would be able to show the extraordinary stability of resistance observed in these experiments.

[9] In order to rule out the possibility that the evaporation of the film might produce the observed resistance, the chamber H was replaced by a copper bloc which reduced the area available for evaporation by a factor of about 8. No influence on the results could be observed, which indicates that the evaporation is probably not the rate determining step. Likewise, the resistance of the gas flow down the tube can be neglected, since less than 10^{-6} mm Hg ΔP is required.

[10] See E. Long and L. Meyer, Advances in Phys. 2, 18–22 (1953).

[11] R. Bowers, Phil. Mag. 44, 487 (1953).

[12] Brunauer, Emmett, and Teller, J. Am. Chem. Soc. 60, 309 (1938).

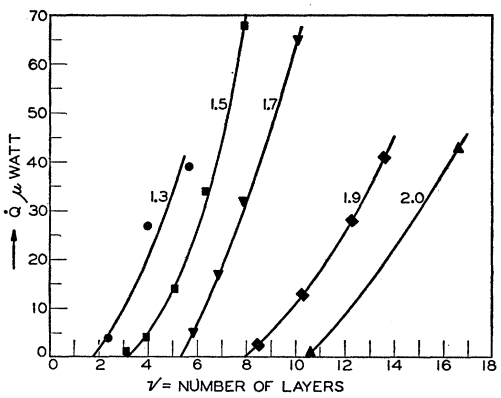

FIG. 4. Amount of heat in microwatts carried by superfluid films as a function of film thickness at constant temperature for different temperatures.

coverage as given in references 2 and 5. The consistency of the two methods in measuring the "onset" coverages for superflow is clearly shown in this treatment.

The slope of the curve for each temperature in Fig. 3, $\Delta\dot{Q}/\Delta\nu$ represents the differential amount of heat carried per added layer. A characteristic of the curves is that the slope does not change appreciably from the intercept $\dot{Q}=0$ to the highest values of $\dot{Q}$ measured for superfluid flow. Evidently the film behaves as though each further layer added to the film contributes about the same amount to the heat transport.

The amount of heat $\dot{Q}$ carried by a single layer in the film may be estimated by using an admittedly oversimplified picture: If it is assumed that the spacing of the He atoms in these adsorbed layers is equal to that in the bulk liquid, 3.6 A, then the number of atoms per cm circumference of the tube is $10^8/3.6$. If these atoms flow with a velocity v cm/sec, the number of atoms reaching the heat source per second and per cm circumference is $v\times10^{16}/(3.6)^2$. One mole of He carries ~100 joules (heat of adsorption plus thermomechanical heat) in the heat transport process. One atom then carries $100/(6.06\times10^{23})$ joules, and one statistical layer carries per cm circumference per second:

$$v\times10^{-5}/[(3.6)^2\times6.06] \text{ joules.}$$

The circumference of the tube used in these experiments is 1.5 cm, if surface roughness is neglected. By using this value, and taking the slopes near the intercepts of Fig. 3, the values of v, the flow velocity, are obtained as shown in Table II. The values for v derived from the slopes at the highest heat currents shown in Fig. 3 are about 30 percent higher than those of Table II.

The results presented here are completely consistent with those obtained in direct flow experiments by method *I* of reference 3, whereas the different ranges of superfluidity observed by method *II* of reference 3 did not appear in the heat conduction experiments. We intend to investigate the question whether these differences are due to the fact that in method *II* a combination of temperature and pressure gradients was acting on the film or that eventually metastable films were responsible. The discrepancy between the "onset" temperatures of superfluidity and the maxima in the specific heat found by Frederikse[13] and discussed already in detail in reference 10 also still remains.

The data of these experiments suggest that at a given temperature a certain number of layers are immobile, that above this number superflow sets in, and that the flow in all additional layers caused by temperature gradients shows about the same drift velocity, this velocity being of the same order or magnitude as that derived from the film transfer or the heat conductivity of bulk liquid.

Very approximately the following picture can be given: The first layer near the wall (v_m) is solid-like, followed by a region of immobile layers, and only the layers additional to these behave like He II. This picture has qualitatively some similarity to that given by Mastrangelo and Aston[14] and by Atkins,[15] who relate the properties of the film in the field of force of the wall to that of bulk liquid under pressure. Such a treatment overlooks the fact that bulk liquid under hydrostatic pressure is in a uniform field of force, whereas the forces of the wall act only in one direction: that perpendicular to the wall. The forces in the other two directions, parallel to the wall, are completely different and have to be considered as stresses (surface tension). As a consequence, for example, the Gibbs free energy G of a liquid under pressure is greater than that under saturation pressure, since $(\partial G/\partial p)_T=v$, whereas the free energy of the film is smaller than that of the bulk liquid, since $P<P_0$.

Furthermore, if the transition from immobile layers to superfluid layers in the field of force of the wall would be related in a simple way to the λ line of the bulk liquid, the curve of the number of immobile layers n *vs* temperature T, shown in Fig. 5, should show a break at 1.75°K, where the melting curve intersects the λ line, and below this temperature all but the solid first layer v_m should be superfluid. The smooth curve found for n as a function of T is inconsistent with such a relationship.

TABLE II. Drift velocity of unsaturated films derived from the heat transport as a function of temperature.

T, °K	v, cm/sec
1.3	48
1.5	44
1.7	38
1.9	25
2.0	21

[13] H. P. R. Frederikse, Physica **15**, 860 (1949).
[14] S. V. R. Mastrangelo and J. G. Aston, J. Chem. Phys. **19**, 1370 (1951).
[15] K. R. Atkins, Can. J. Phys. **32**, 347 (1954).

In reality the number of immobile layers n is roughly proportional to the fraction of normal fluid ρ_n/ρ of the bulk liquid. This proportionality is improved if one deducts from n the first "solid" layer, as shown in Table III.

The physical meaning of Table III is probably that the forces of the wall tend to increase the fraction of normal fluid; this effect becomes more and more pronounced the thinner the film. If ρ_n is very small at great distances from the wall, i.e., in the bulk liquid, then in a region close to the wall there is a borderline in which ρ_n/ρ reaches 1 and the film becomes immobile. If ρ_n/ρ is already approaching 1 in the bulk liquid, then perhaps the forces of the wall are able to raise ρ_n/ρ to 1 at a greater distance, and a greater number of layers becomes immobile.

Table III and Fig. 5 show furthermore that for $\rho_n/\rho=1$ in the bulk liquid, i.e., at the λ point, only about 20 layers should be immobile. Since it is known that the film can be more than 100 layers thick, the question arises whether or not films of more than 20 layers would exhibit superfluidity at or eventually even above T_λ. Heat conduction experiments were therefore performed at a temperature of the helium bath $1\frac{1}{2}\pm0.2$ millidegrees below the λ point, taking $P_\lambda=38.1$ mm Hg[16,17] corresponding to 2.183°K of the conventional scale and $dP/dT=0.094$ mm/Hg/10^{-2} degree. Without a contribution of superfluidity, a heating by 0.3 microwatt produces a steady warming rate of about 2 millidegree/minute, as confirmed by a run at $P/P_0=0.8$, corresponding to about 7 layers. Using then (*a*) $P/P_0=0.9996$ ($P_{\text{bath}}-P_{\text{system}}=0.3$ mm oil, $P_{\text{bath}}=560$ mm oil) yielding an estimated film thickness of about 30 layers, and (*b*) 5 percent and 15 percent more gas in the apparatus than necessary to make $P_{\text{system}}=P_{\text{bath}}$, estimated film thickness 40–50 and 50–60 layers respectively, lead to the following results.

Heat inputs up to 2μw did not produce any measurable temperature difference between the bottom and top thermometers up to a temperature of 2.185°K of the conventional scale, i.e., 2 millidegrees above the bulk liquid λ point. Higher heating rates up to 7.3μw did produce increasing temperature differences in a slow runaway phenomenon. However, the warming rate with these heat inputs was up to at least 2.195°K of the conventional scale only a fraction of that found without contribution of superfluidity. These results indicate that films more than 20 layers thick show superfluid behavior at and above the λ point of the bulk liquid, as already suggested by earlier thermodynamic considerations.[18]

TABLE III. Comparison of the number of immobile layers n with the fraction of normal fluid ρ_n/ρ in the bulk liquid for different temperatures.

T	$(n-1)$	ρ_n/ρ	$(n-1)\rho/\rho_n$
1.3	1.0	0.042	23.8
1.5	1.9	0.106	17.9
1.7	4.1	0.222	18.4
1.9	7.0	0.420	16.6
2.0	9.5	0.552	17.2

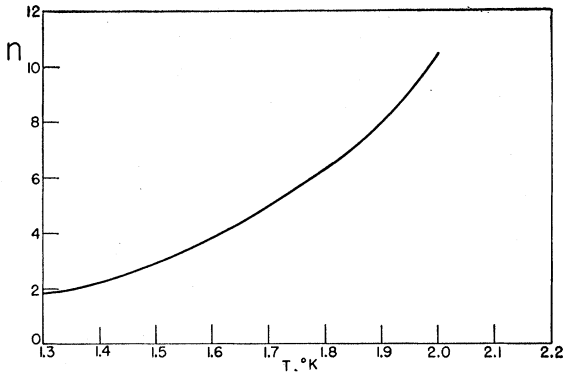

FIG. 5. Number of immobile layers n as a function of temperature.

In several cases the warm end of the apparatus cooled down from temperatures up to 2.192° (after switching off the heating current) much faster than a conductance of 10 microwatts/degree would allow, as would be expected for the case that the heat-carrying film is stable and in equilibrium. In several other cases, however, the cooling rate following a heating period dropped to the order of 10 microwatts/degree after a few minutes, showing that the stability range of the mobile film had been exceeded during the runaway phenomenon. However, the possibility cannot be completely excluded that the films showing superfluidity above T_λ are metastable.

Heat Transport in the Saturated Film ($P/P_0=1$)

It is hardly fruitful to make measurements for the condition $P/P_0=1$ as measured at the thermal reservoir, since the film thickness is reduced at the top by the gravitational field and as ΔT increases. The asymptotic infinity of the isotherm at $P/P_0=1$ produces extreme changes in thickness for minute changes in P/P_0 in this region,[19] so that the real thickness is indeterminable.

Therefore only a few measurements were made, between 1.7° and 2.1°K, with *excess liquid* in the cell. The resulting heat transport data were completely different from those of Table I and Fig. 2. After an initial heating rate at which no ΔT was detected within the accuracy of the experiment (0.0002°), a linear dependence of ΔT on $\dot{Q}$ occurred, until at a given critical heat input the ΔT rose exceedingly rapidly, and a "runaway" phenomenon occurred, as previously described. The

[16] E. Long and L. Meyer, Phys. Rev. **83**, 860 (1951).
[17] R. A. Erickson and L. D. Roberts, Phys. Rev. **93**, 957 (1954).
[18] L. Meyer and E. Long, Phys. Rev. **85**, 1035 (1952).
[19] See L. Meyer, Phys. Rev. **93**, 655–656 (1954).

ΔT's at which the critical heat inputs were observed were quite large (0.018° at 1.7°K), dropping to 0.005° at 2.1°K.

These critical heat inputs for the heat transport are proportional to the well-known transfer rate of the saturated film, as already reported by BBM (reference 1); however, the limiting initial heating rate (for which no ΔT was observed) was dependent on the amount of excess liquid in the cell. Further, the critical heat input observed was very high; at 1.7°K it was twenty-five times greater than that observed for the thickest film of Fig. 2. The experiments were not continued, since adequate interpretation seems difficult in view of the absence of a reliable estimate of the film thickness.

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Resonance Absorption of Sunlight in Twilight Layers

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Solar radiation must pass at least once through the absorbing layer to reach the twilight zone of a layer of atoms scattering resonance radiation. The attenuation of the solar beam is calculated for resonance absorption and the resulting scattered intensity compared with the transparent layer model intensity for the same thickness of material. It is shown that in the case of sodium, the reduction in intensity for layers thicker than 10^9 atoms/cm^2 is too serious to permit the deduction of layer thickness from a simple theory which neglects imprisonment of resonance radiation.

INTRODUCTION

FROM measurements of the time variation in intensity of the sodium D lines at twilight and at dawn the location, distribution, and vertical thickness of the scattering layer of sodium atoms has been deduced.[1–4] The basic assumption that the light observed is scattered sunlight is supported by strong evidence.[5–7] Careful account has been taken of such important factors in the interpretation of the observations as refraction and attenuation by the gases of the lower atmosphere which the incident sunlight must traverse.[3,4] Unfortunately, an uncertainty in the intensity incident at the bottom of the Fraunhofer lines makes for an uncertainty as large as an order of magnitude in the deduced sodium thickness. Values given range from 2×10^9 atoms/cm^2 to 2×10^{10} atoms/cm^2. The layer appears to be located somewhere between 70 and 115 km above the earth's surface.

With the correction mentioned for general atmospheric extinction—which is of course independent of the sodium layer thickness—the flux incident in the twilight scattering layer is taken in these calculations to be uniform throughout the region. This assumption, however, neglects the fact that the sunlight incident on the sodium layer after sunset must pass once completely through the layer and, in general, partly through it again. It is the purpose of this note to give the results of a calculation of the consequent attenuation of the incident light by resonance absorption for layer thicknesses in the range from 2×10^9 atoms/cm^2 to 2×10^{10} atoms/cm^2. Appreciable attenuation under these conditions would not only force a revision upward of the layer thickness, but, because this in turn would imply further attenuation, would seem to suggest an altogether different treatment of the problem of twilight excitation. This is particularly true since this treatment neglects reradiated resonance photons. Under such conditions these should contribute a non-negligible component to the density of excited atoms in twilight.[8]

The importance of an accurate knowledge of the sodium layer thickness for the determination of the altitude of the nightglow D line emission has been pointed out in connection with a calculation of the effect of resonance absorption on the variation with zenith distance of the radiation from an airglow layer.[9]

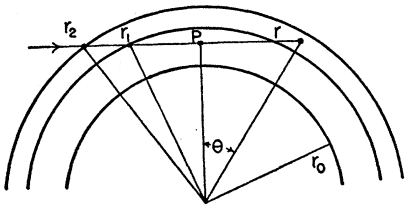

FIG. 1. Path of a photon through the absorbing layer to reach the point (r,θ). r_0 is the radius of the earth.

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