Scattering of 192-Mev Electrons from the Deuteron*†

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Elastic scattering of 192-Mev electrons from deuterium has been studied using both solid (CD_2) and high-pressure gas targets. The results obtained by these two methods are in agreement and yield the following conclusions: (1) The range of the neutron-proton potential is between 1 and 4×10^{-13} cm if a square potential well is assumed and if the deuteron wave functions predicted by the binding energy of the deuteron are used. (2) It is, however, impossible with the present data to eliminate the possibility of other charge distributions for deuterium such as uniform or Gaussian.

Inelastic electron scattering from deuterium was also investigated, and promises to give independent information of the deuteron structure.

I. INTRODUCTION

THE reduced wavelength of electrons with energy in the neighborhood of 200 Mev is about 10^{-13} cm. Consequently the scattering of a beam of such electrons from charged objects such as the various atomic nuclei which have these dimensions yields information about the size and structure of the nuclei. Investigations of this type have been carried out already for gold and lead nuclei which have radii of about 7×10^{-13} cm and it has been found possible to determine two parameters which specify the charge distribution in these nuclei.[1]

In considering what nuclei to study next, several factors were apparent which made the deuteron an attractive possibility. (1) The deuteron is the only nucleus in which the two-body character of nuclear forces is isolated for study. (2) The deuteron is large enough so that some information about its structure should be obtainable with the electrons available. (3) The "size" of the deuteron has been determined by one method only, i.e., by using the value of the binding energy.[2] While there is no reason to doubt the result obtained by that method, an independent check is desirable. (4) It was found some time ago[3] that scattering studies with deuterium as well as hydrogen were feasible with solid (e.g., CH_2) targets because of the significant change in scattered electron energy due to recoil of the struck nucleus. Thus, the electrons scattered from deuterium nuclei may be separated from most of the electrons scattered from the heavier carbon nuclei.

The results of scattering 192-Mev electrons from CD_2 targets are reported in this paper. Since absolute cross sections have not yet been obtained experimentally, the scattering from hydrogen was also studied in CH_2. Hydrogen was found to scatter essentially as a point charge.‡ Since calculations have been made for point-charge scattering,[4] it is necessary only to determine the ratios of deuterium to hydrogen scattering at various angles in order to obtain the absolute cross sections for deuterium.

Finally, scattering experiments were carried out by using D_2 and H_2 gas targets with pressures ranging from 137 atmospheres to about half this value.[5] The results from these experiments are in agreement with those obtained with the solid targets.

II. APPARATUS

Several improvements have been made in the apparatus previously described.[1,6] A CsBr(Tl) scintillator has been placed on the electron-beam axis beyond the scattering chamber. The scintillator is viewed with a telescope in the room where pulse counting is carried on, and the electron beam is then steered to its correct position on the axis. A remotely-controlled lead slit has also been installed at the entrance to the analyzing magnet; this slit width determines the spread in scattering angle accepted by the analyzing magnet. Finally, the high-pressure gas-target chamber mentioned above is now available for use. It can be filled and emptied from the remote counting room in about five minutes.

The CD_2 and CH_2 targets used for most of the runs were 0.200 in. and 0.115 in. thick, respectively. Thicker targets were also tried but were rejected, because of an indirect effect of the energy loss in the target. Since the incident electrons lose energy in the target and hence are incident at slightly different energies, the recoil energy taken away by the struck nucleus is

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[1] Hofstadter, Hahn, Knudsen, and McIntyre, Phys. Rev. **95**, 512 (1954); Yennie, Ravenhall, and Wilson, Phys. Rev. **95**, 500 (1954); D. G. Ravenhall and D. R. Yennie, Phys. Rev. **96**, 239 (1954).

[2] See, for example, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 52.

[3] Hofstadter, Fechter, and McIntyre, Phys. Rev. **91**, 422 (1953).

‡ *Note added in proof.*—Newer data on hydrogen obtained by R. S. McAllister and R. Hofstadter at larger scattering angles reveal the effect of the magnetic moment of the proton. Also, a slit correction in the analyzing magnet has been omitted in this paper since it is small compared to other uncertainties in the data (e.g. 10 percent at 90° and 20 percent at 120°).

[4] N. F. Mott, Proc. Roy. Soc. (London) **A135**, 429 (1932); W. A. McKinley, Jr., and H. Feshbach, Phys. Rev. **74**, 1759 (1948).

[5] Hofstadter, McAllister, and Wiener, Phys. Rev. **96**, 854 (1954).

[6] Hofstadter, Fechter, and McIntyre, Phys. Rev. **92**, 978 (1953).

different if the scattering occurs at the front than if it occurs at the back of the target. Hence, the elastically-scattered electrons at a given angle exhibit this energy difference. The targets, therefore, had to be thin enough so that this energy difference would not be larger than the spread of energy in the incoming beam of electrons. The target thicknesses so determined yielded negligible energy spreading due to collision straggling and bremsstrahlung. For most of the experiments reported here, the spread in energy in the electron beam was set at $\frac{1}{2}$ percent and the slit at the exit of the analyzing magnet was set for $\frac{1}{2}$ percent energy acceptance band. Again, for most of the experimental runs, the slit at the entrance of the analyzing magnet which determines the solid angle subtended by the magnet was set at $\frac{1}{4}$ in. wide (about one-third maximum) by $1\frac{1}{4}$ in. high (maximum). Selection of the small width was dictated by a recoil effect: *viz.*, electrons scattered at different angles give up different energies to the recoil nucleus. Thus, if the scattered electrons detected are to have less than a given spread in energy, the spread in angle accepted by the analyzing magnet must be limited.§ The energy spread in electrons scattered was about 1.5 Mev due to the different effects mentioned. This small spread insured that inelastically-scattered electrons in the disintegration of the deuteron would not be accepted as elastically-scattered electrons.

The gas targets for both deuterium and hydrogen were used with a maximum pressure of 137 atmospheres. At this pressure, none of the "thick-target" effects discussed above was important. However, the effects of multiple scattering in the walls of the pressure chamber must be considered. These effects have been found to be less than 5 percent.[7]

The beam was monitored as before with a helium-filled ionization chamber. Such a chamber is known to become nonlinear at sufficiently high beam currents. For these experiments this nonlinearity was at most 5 percent. The nonlinearity was determined by using a secondary-electron-emitter monitor of a type developed by Fechter and Tautfest[8] and built for our use by A. W. Knudsen and B. R. Chambers.

III. PROCEDURE

When taking data the analyzing magnet was set at a particular angle and the number of scattered electrons of a particular energy was determined for a fixed number of beam electrons. The analyzing-magnet field was then changed and the number of scattered electrons of a different energy determined. Such determinations yield curves such as that shown in Fig. 1 for electrons scattered through an angle of 80° from a CH_2 target. The C and H peaks are due to elastically-scattered electrons from carbon and hydrogen, respectively. The other peaks are due to scattered electrons which have lost energy by exciting levels in carbon nuclei.[9] The electrons in the H peak have lost energy because of recoil of hydrogen nuclei. To obtain the elastic scattering from hydrogen alone, the scattering from a pure carbon target is measured and a subtraction is made. It is evident that the subtraction is only of the order of 10 percent and so should not contribute more than a few percent error.

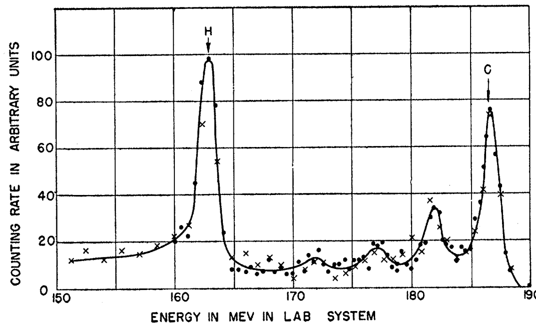

FIG. 1. Scattering from a CH_2 target at 80°. The C and H peaks are due to electrons scattered elastically from carbon and hydrogen, respectively. The remaining peaks are due to electrons scattered inelastically from carbon.

It might at first seem surprising that there should be more elastic scattering from hydrogen than from carbon since Coulomb scattering is proportional to Z^2. However, the carbon nucleus appears considerably different from a point charge at these electron energies, and because of this, carbon elastic scattering at large angles drops more than a factor of 18 $(=6^2/2)$ below point-charge scattering. At the smaller scattering angles, however, the C peak grows toward its Coulomb value of 18 times the H peak. Since the hydrogen recoil also becomes smaller at small angles, the H peak is not as well isolated from the C peak at these angles.

What has been said of hydrogen scattering from solid targets holds also for deuterium scattering. In addition, however there are two adverse effects: (1) The deuteron recoil is roughly half that of hydrogen so that the deuteron elastic peak is considerably closer to the carbon elastic peak. For example, at 80° it would appear at 174 Mev (compare to Fig. 1). Thus, at the smaller angles at least, where the recoil is small, there is a larger carbon background to be subtracted than for the hydrogen case. (2) The finite size of the deuteron manifests itself at the larger angles by a considerable decrease in the deuteron elastic scattering. Figure 2 shows how much smaller the deuterium scattering is than the hydrogen scattering at 110°. Naturally, the carbon subtraction for deuterium is more serious at 110° also. In addition, Fig. 2 shows deuterium scattering at a "small angle," 50°, where again the carbon subtraction is large. At angles between these values the carbon subtraction for deuterium is less serious.

§ This effect, of course, limits the thickness of the target also.

[7] Measurements made by R. W. McAllister and R. Hofstadter.

[8] G. W. Tautfest and H. R. Fechter, Phys. Rev. **96**, 35 (1954).

[9] Such inelastic peaks have already been reported for beryllium by McIntyre, Hahn, and Hofstadter, Phys. Rev. **94**, 1084 (1954). More recent data on carbon taken by J. Fregeau and R. Hofstadter with better resolution show additional carbon levels.

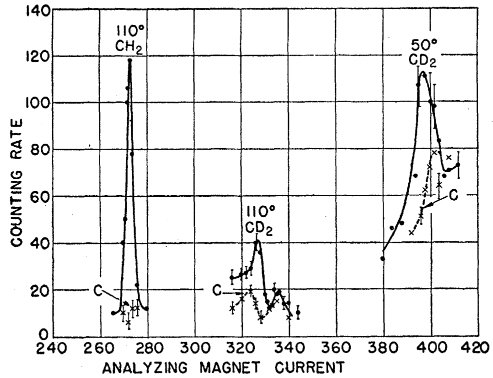

FIG. 2. Scattering from CH_2 and CD_2 targets at 110° and 50°. Scattering from a pure-carbon target is also shown. The ordinate scale for the two angles shown is different for each. Uncertainties shown for the ordinates are due to counting statistics alone.

The carbon-subtraction problem is eliminated entirely when the high-pressure gas target is used. However, the hydrogen scattering from the gas target is only about one-sixth that from the 0.115-in. CH_2 target, and the deuterium scattering is one-tenth that from the 0.200-in. CD_2 target. Thus, for deuterium, scattering from the gas target was not carried out for angles larger than 90°. Hydrogen data over a larger range of angles have been taken with the gas target; these will be reported in a subsequent publication.[10]

To compare experiment with theory, the elastic-scattering relative yields are plotted against center-of-mass scattering angle. For the Born approximation, and to first order in β, the scattering can be analyzed in the center-of-mass system in the usual way.[11] The deuterium scattering takes place at a center-of-mass energy of 175 Mev and the hydrogen scattering at 160 Mev for 192-Mev incoming electrons.

All theoretical calculations of the scattering for deuterium and hydrogen are made for a Dirac electron scattering from a static charge. An error at the largest angles for hydrogen is made by this procedure chiefly due to neglect of the anomalous magnetic moment of the hydrogen nucleus. However, at the present time, the experimental data do not warrant a more detailed calculation. The Born approximation is used in all of the form-factor calculations for deuterium.[12]

IV. RESULTS

Figure 3 shows the angular distribution obtained for electrons scattered from hydrogen with the theoretical scattering from a point charge plotted for comparison. The fit is as good as the statistics warrant, and in the following, the hydrogen nucleus will be assumed to scatter as a point charge and all absolute cross sections quoted will be based on this assumption.‡

The results obtained by scattering from deuterium are given in Fig. 4. The hydrogen points in this figure were taken for normalization purposes during the deuterium runs and represent different data from the points in Fig. 3. Since only two hydrogen points were taken during each deuterium run, and since these two points were plotted to best fit the theoretical curve, the hydrogen data in Fig. 4 do not test the theory as well as the data in Fig. 3.

The deuterium points in Fig. 4 deviate from point-charge scattering by a factor of 10 at the large angles. Four theoretical curves[12] have been plotted to determine what information can be gained from the deuterium data. Each solid curve gives the angular distribution for scattering from the charge distribution of a proton that is bound in a square potential well with the binding energy of the deuteron. The range of the potential for each curve is noted in Fig. 4. The dotted curve gives the angular distribution for scattering from a uniform charge distribution with radius 2.1×10^{-13} cm. The root-mean-square radius of each charge distribution is also given in the figure. Of the four curves, only the zero-range deuterium curve can be eliminated as a possible fit. The other two deuterium curves roughly enclose the experimental points so that one can say that if the charge distribution of the deuteron is that expected from nuclear theory, then the radius of the potential well between neutron and proton is probably between 2 and 3×10^{-13} cm and certainly between 1 and 4×10^{-13} cm. If the predictions of nuclear theory are disregarded, however, a uniform charge distribution of radius 2.1×10^{-13} cm is also a possibility (a Gaussian charge distribution would fit equally as well). It is

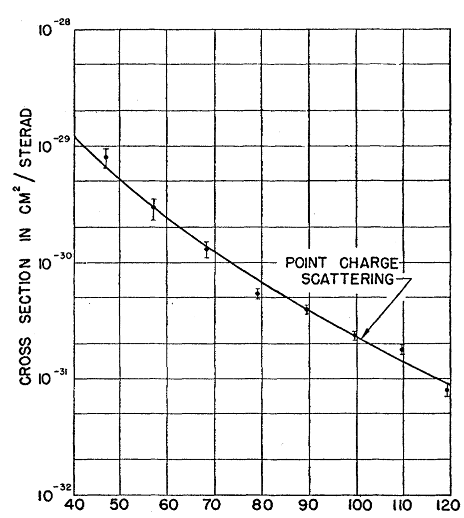

FIG. 3. Angular yields of elastic scattering from hydrogen with center-of-mass energy of 160 Mev. The theoretical curve is fitted to the experimental points and determines the ordinate cross-section scale. Uncertainties in the experimental points are statistical.

[10] Data taken by R. W. McAllister and R. Hofstadter.

[11] Here $\beta=v/c$, where v is the velocity of the nucleus in the center-of-mass system. For hydrogen at electron energies of 200 Mev, $\beta=0.175$.

[12] The scattering from deuterium for two square-well nuclear potential ranges has been calculated by J. H. Smith, Ph.D. dissertation, Cornell University, 1951 (unpublished). [A previous calculation by M. E. Rose, Phys. Rev. 73, 282 (1948) is in error.]

interesting that the root-mean-square radii of the deuteron charge distributions and the uniform charge distribution which fit the data can be as greatly different as 30 percent. Clearly the present data do not allow a very precise determination of the deuteron structure, but it is also clear from Fig. 4 that data of accuracy of 10 percent would give considerable information about the deuteron structure and the neutron-proton potential. Such data should be attainable without making major changes in the present apparatus. It should be noted in Fig. 4 that the root-mean-square radii are given in the barycentric coordinates of the deuteron while the nuclear potential ranges are given in the relative neutron-proton coordinates (twice the barycentric coordinates).

In addition to elastic scattering from deuterium, there is also inelastic scattering of electrons which have lost energy in disintegrating the deuteron. Figure 5 shows the energy spectrum of electrons scattered from deuterium and hydrogen gas at 70°. The hydrogen elastic peak represents scattering from protons at rest while the wider inelastic deuterium peak represents scattering from the moving proton in the deuterium nucleus. The shape of the inelastic peak, therefore, should yield information on the velocity or momentum distribution of the proton in the deuteron. Thus, the elastic scattering gives the wave function of the proton through its measure of the charge distribution while the inelastic scattering gives information about the wave function of the proton in the momentum representation through its measure of the velocity distribution. Calculations are now in progress at Stanford University by V. Jankus to investigate what information can be gained from the inelastic data.

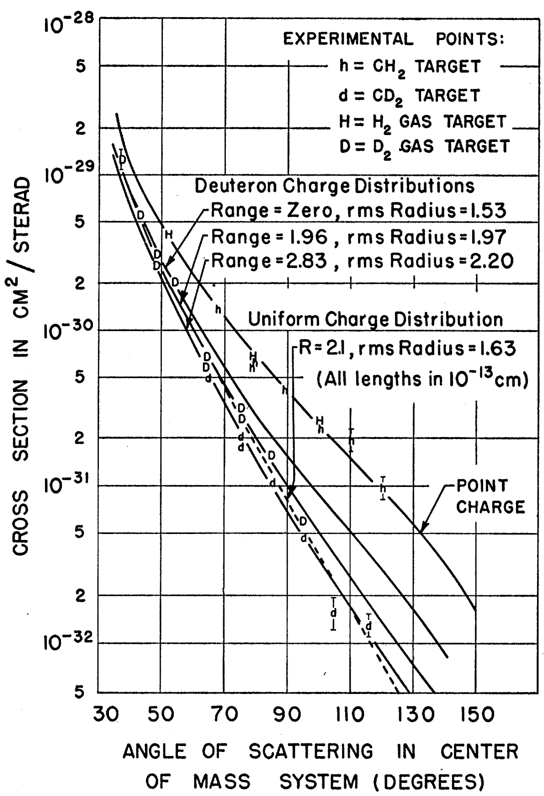

FIG. 4. Angular yields of elastic scattering from deuterium with center-of-mass energy of 175 Mev. Hydrogen points were taken for normalization. Data were taken in five runs, all runs being normalized by the hydrogen points. The ordinate scale is determined by assuming hydrogen scattering is point-charge scattering. The curves are all determined by the Born approximation. The two curves calculated from deuteron wave functions with nuclear potential ranges of 1.96 and 2.83×10^{-13} cm were calculated by Smith (see reference 12). "Range" in the figure means the range of a square-well potential in relative neutron-proton coordinates (neutron at rest), while the "rms radius" is given in barycentric coordinates of the deuteron (one-half the value of the relative coordinates). Statistical uncertainties in the experimental points are all smaller than the heights of the letters except where indicated.

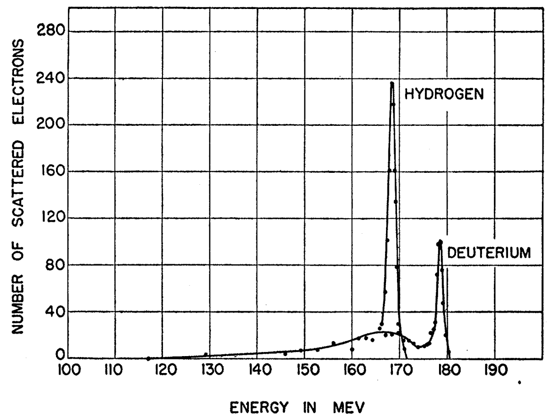

FIG. 5. Scattering from hydrogen and deuterium at 70° using gas targets.

V. CONCLUSIONS

Three conclusions may be drawn from the data presented:

(i) The range of the neutron-proton potential is between 1 and 4×10^{-13} cm if a square potential well is assumed and if the deuteron wave functions predicted by the binding energy of the deuteron are used.

(ii) The present data for deuterium are not accurate enough to eliminate the possibility of other charge distributions such as uniform and Gaussian.

(iii) Inelastic scattering from deuterium should also yield information about the internal dynamics of the deuteron.

VI. ACKNOWLEDGMENT

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