by putting $k=0$ in the numerator of the integrand and in the central electron line. $M_{2B}+M_{2C}$ have an infrared divergence, $M_{2B}(\lambda)+M_{2C}(\lambda)=(\alpha/2\pi)\ln(m/\lambda)M_{20}$, and it is easily seen that

$$M_{2A}(\lambda)+M_{2B}(\lambda)+M_{2C}(\lambda)=aM_{20}.$$

In the cross section, $2|(M_{2A}+M_{2B}+M_{2C})M_{10}|$, this term cancels exactly with $b|M_{20}M_{10}|$ from the second half of the inelastic cross section.

The remaining parts of M_{2A}, M_{2B}, and M_{2C} are greatly simplified by putting $\lambda=0$. For $E\gg m$, a trivial but lengthy calculation gives the following contribution $\Delta\sigma_{2ABC}$ to σ_{21}:

$$\Delta\sigma_{2ABC}=\int d^3q\left[\left(\ln\frac{E}{m}\right)f_1+f_2\right].$$

The absence of the $\ln^2(E/m)$ term is again independent of the static potential. f_1 and f_2 contain only the field momentum q, E, and the scattering angle, but not the electron mass.

The author wishes to thank Dr. R. G. Newton, Dr. H. Suura, and Professor N. M. Kroll for helpful discussion.

* Part of this research was carried out at Stanford University, with support of the Office of Scientific Research, Air Research and Development Command.

1 R. G. Newton, Phys. Rev. **97**, 1162 (1955).
2 R. G. Newton, this issue [Phys. Rev. **98**, 1514 (1955)].
3 H. Suura, Phys. Rev. (to be published).
4 Weneser, Bersohn, and Kroll, Phys. Rev. **91**, 1257 (1953); R. Karplus and N. M. Kroll, Phys. Rev. **77**, 536 (1950).
5 J. Schwinger, Phys. Rev. **76**, 790 (1949).

Predicted 0+ Level in $_{40}Zr^{90}$*†

KENNETH W. FORD
Indiana University, Bloomington, Indiana
(Received March 21, 1955)

VERY many nuclear properties exhibit regularities which show clearly the effects of closing of major shells (4, 8, 20, 28, 50, 82, 126)—most notable of these are perhaps the first excited states of even-even nuclei.[1] However, few if any effects of the closing of intermediate subshells have been noted. On the contrary, a very regular variation of many nuclear properties has been noted between major shells, suggesting that the intermediate shell model states are filled in an irregular order, or that the nuclear states are mixtures of different subshell states. The recent discovery by Campbell *et al.*[2] of a high-energy isomeric transition in $_{40}Zr^{90}$ provided evidence that the subshell of 40 protons behaves like a closed shell. This leads, as described below, to the expectation that the first excited state of Zr^{90} might be a 0+ state. The predicted state has been discovered by other workers at this laboratory and is described in the following Letter.[3]

Beyond the closed shell of 28 nucleons, the $f_{5/2}$ and $p_{3/2}$ states seem to be nearly degenerate.[4] They fill at nucleon number 38 and are followed by the $p_{1/2}$ shell, filling at 40, and the $g_{9/2}$ shell, completing the major shell at 50. According to the evidence of the nucleus $_{39}Y_{50}{}^{89}$, the single-particle $g_{9/2}$-$p_{1/2}$ energy separation is 0.9 Mev, sufficient to make reasonable some clear-cut effect of the $p_{1/2}$ subshell closure for the nucleus $_{40}Zr_{50}{}^{90}$. The zero-order scheme for the low levels of Zr^{90} would then be as shown in the left half of Fig. 1: the states $(p_{1/2})^2$, $(p_{1/2}g_{9/2})$, and $(g_{9/2})^2$ separated respectively by about 0.9 Mev.

The recently discovered isomeric transition[2] is most probably from a 5− level at 2.30 Mev to the 0+

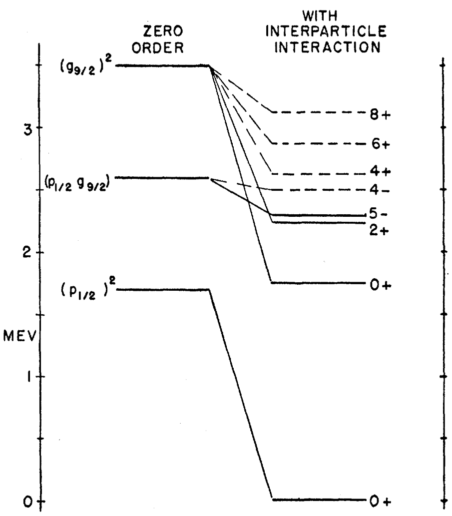

FIG. 1. Hypothetical level diagram of $_{40}Zr^{90}$. On the left are shown the zero-order states which should be important, as deduced from the neighboring nucleus $_{39}Y^{89}$. *Qualitative* level shifts due to a mutual interaction between the last two protons are shown, which are in agreement with shell calculations performed for other nuclei, and which are normalized to fit the known levels of Zr^{90}. A first excited 0+ state is expected because of the large mutual interaction of the $(g_{9/2})^2$ configuration.

ground state of Zr^{90}. The 5− state lives 0.8 sec. This discovery confirms the simple shell model description of the state in two ways. First, it rules out a 2+ level within the first 2 Mev or more of excitation, suggesting that the ground state is a closed shell configuration, e.g., $(p_{1/2})^2$ for the protons. Second, the 5− character of the isomeric state is readily attributed to the $p_{1/2}g_{9/2}$ proton configuration.

One then seeks to understand the 5−:0+ energy difference and to predict other levels of Zr^{90}. Detailed shell model successes elsewhere in the periodic table[5,6] suggest that a quantitative treatment of this nucleus would be profitable. We give here, however, only a very qualitative discussion. When a two-body attractive force is allowed to perturb the zero-order states, the spin 0 levels of the $(p_{1/2})^2$ and $(g_{9/2})^2$ configurations should be substantially depressed in energy—by roughly the order of 1.5 Mev. The $(g_{9/2})^2$ state is depressed by somewhat more than the $(p_{1/2})^2$ state from the diagonal energy contribution, but the extra effects of configuration interaction should act principally in the $(p_{1/2})^2$ state,[5] adding the order of 0.5 Mev to its depression. The two states of the $p_{1/2}g_{9/2}$ configuration are depressed by a much smaller amount due to the poor overlap of the single particle states, perhaps by a few tenths of an Mev. In the limit of short-range forces, the 4− state is not shifted,[5] and the 5− is shifted downward by a small amount. One concludes that the occurrence of the 5− isomeric state (below the 4−) and its excitation energy are in accord with simple shell theory. Also, one expects that the spin 0 state of the $(g_{9/2})^2$ configuration might lie below the 5− state, because of the much greater mutual interaction of two equivalent than of two inequivalent particles.

A hypothetical level diagram is presented in the right side of Fig. 1, adjusted to fit what is now known experimentally. In addition to the 5− state discussed above,[2] and the new 0+ state just discovered,[3] there has been observed in the decay of ${}_{41}Nb^{90} \rightarrow {}_{40}Zr^{90}$ a level at 2.23 Mev,[7] which appears not to be the 5− state because of its rapid decay to the ground state and because the energy difference is outside experimental error. This state could be interpreted as the 2+ state of the $(g_{9/2})^2$ configuration. The small 0–2 separation (~0.5 Mev) of this configuration must then be attributed to the configuration interaction of the 0+ state. In a usual even-even nucleus, the 0–2 spacing is substantially increased by configuration interaction. In this nucleus it may be decreased because of the unusual circumstance that there is another 0+ state below the one of interest. According to the semiempirical formulas of Moszkowski,[8] the 5− to 2+ 70-kev $E3$ transition should be slower by a factor of about 100 than the 5− to 0+ 2.3-Mev $E5$ transition. The observation of the ground-state transition only is therefore reasonable.

Regarding other nuclei in the neighborhood, we make two remarks. First, the nucleus ${}_{38}Sr_{50}{}^{88}$ does not show a double closed shell character. This may be attributed to the ease of exciting the $p_{1/2}$ state and/or a substantial $p_{3/2}$-$p_{1/2}$ mixing. Second, most odd-even nuclei in the region show a $g_{9/2}$-$p_{1/2}$ energy difference small compared to the 0.9 Mev in ${}_{39}Y_{50}{}^{89}$. This is not necessarily in contradiction to a simple shell picture, since the other nuclei have multiparticle configurations. For example, in ${}_{41}Nb_{50}{}^{91}$ the "$p_{1/2}$" and "$g_{9/2}$" states should more correctly be called $g_{9/2}{}^2p_{1/2}$ and $p_{1/2}{}^2g_{9/2}$, and these three-particle configurations should not have the same spacing as the single-particle levels.

Aside from its interest as a double closed shell nucleus, Zr^{90} calls to attention the way in which the beta- and gamma-decay selection rules can cause low-lying levels to go unobserved in conventional experiments. Shell theory predicts many low-lying states which have not been observed (see, e.g., reference 5). Experiments designed specifically to look for such levels in nuclei with simple shell configurations would be of great aid in theoretical analysis and in formulating a quantitative theory of nuclear structure.

I am indebted to Dr. E. C. Campbell and to Professor L. M. Langer for discussions of experimental information.

† Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] G. Scharff-Goldhaber, Phys. Rev. **90**, 587 (1953).
[2] Campbell, Peele, Maienschein, and Stelson, Bull. Am. Phys. Soc. **30**, No. 1, 33 (1955).
[3] Johnson, Johnson, and Langer, following Letter [Phys. Rev. **98**, 1517 (1955)].
[4] P. F. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952).
[5] D. E. Alburger and M. H. L. Pryce, Phys. Rev. **95**, 1482 (1955).
[6] J. P. Elliott and B. H. Flowers (to be published); C. Levinson and K. W. Ford (to be published).
[7] G. E. Boyd and H. Kitelle, Oak Ridge National Laboratory Report ORNL-1053, 1951 (unpublished).
[8] S. A. Moszkowski, Phys. Rev. **83**, 1071 (1951).

Evidence for a 0+ First Excited State in Zr^{90}†

O. E. Johnson, R. G. Johnson, and L. M. Langer
Indiana University, Bloomington, Indiana
(Received March 21, 1955)

EVIDENCE has been found for the existence of the 0+ level in Zr^{90} predicted by Ford.[1] By using a strong source of Y^{90} in the 40-cm radius of curvature magnetic spectrometer, an internal conversion line was observed corresponding to a transition of 1.75 Mev. The intensity of the line relative to that of the 2.26-Mev beta spectrum is 0.005 percent. The line was observed on repeated runs and its shape was determined with good statistical accuracy by recording 10^5 counts at each experimental point. The full width at half-maximum was 0.5 percent. A weak positron distribution, presumably arising, at least partly, from pairs, was also observed in the spectrometer. The maximum energy of the positron distribution was at about 0.8 Mev. The intensity of the positron distribution relative to the beta spectrum is 0.020±0.010 percent. A search for gamma radiation with a NaI scintillation spectrometer indicated that there is no gamma-ray line in the region of 1.75 Mev with an intensity of as much as 0.0005 percent. Measurements on the 0.3 percent gamma ray of Y^{91} at 1.2 Mev confirmed our estimates of the detectability of the apparatus. The 1.75-Mev transition is therefore assumed to be that of a monopole between two 0+ states of Zr^{90}.

If one assumes that the "unique" comparative half-life, $(W_0{}^2-1)ft$, is the same for the feeding of the 1.75-Mev level as for the beta transition to the Zr^{90} ground state, then one should expect transitions to the excited state with an intensity of 0.11 percent. However, if