TABLE I. p-p differential cross sections for unpolarized protons of 417-Mev average energy.

Barycentric angle	$(d\sigma/d\omega)$unpolarized mb/sterad
90°	3.42±0.13
80°	3.56±0.23
65°	3.34±0.19
54°	3.23±0.12
54°	3.18±0.21
43°	3.74±0.21
28°	3.41±0.20

These results do not indicate an increase of cross sections at small angles as was observed at 437 Mev,[4] but on the other hand, have larger statistical errors. Differential p-p cross sections measured at 460 Mev [5] seem to show a rise at somewhat larger angles than those of 437 Mev.[4] It may be that the angular distribution is changing rapidly in this energy region. We plan to measure again with unpolarized protons.

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Nuclear Moments of Ac^{227}†

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PRELIMINARY values for the nuclear magnetic dipole and electric quadrupole moments of Ac^{227} have been derived from hyperfine structure patterns previously used to obtain 3/2 as the nuclear spin.[1] Intervals were obtained for nine lines of Ac II, of which two were simple triplets involving transitions from $sp\,{}^3P_1$ and $ds\,{}^3D_1$ to $J=0$ terms. From the splitting factors measured for these two terms the line patterns involving the remaining ds terms were calculated and found to agree with experiment. One more step then gave the intervals of the dp terms. This somewhat indirect procedure was followed because in most cases the hfs patterns were incomplete, only the diagonal components being observed. The splitting factors thus found for the dp terms were roughly 20 percent of the corresponding sp terms and are a measure of the interaction between these overlapping configurations.

The values derived for the moments from the conventional treatment of hfs in intermediate coupling are $+1.1$ nm and -1.7×10^{-24} cm^2. The experimental error is believed to be less than 10 percent, but it is difficult to estimate the total error because of the configuration interaction and the large relativity corrections. No correction for closed shell distortion was made.

It is hoped that improved values can be obtained, but meanwhile it appears useful to offer the present results. We should like to acknowledge helpful discussions with Dieter Kurath and R. E. Trees.

† Based on work performed under the auspices of the U. S. Atomic Energy Commission.

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Radiative Corrections to Electron Scattering

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THE author's recent paper[1] on radiative corrections to electron scattering unfortunately contains an error. The infrared divergence in one term of the elastic cross section (coming from the first-order mass operator) was not handled correctly. Equation (4.12) should be replaced by

$$\Gamma_g{}^I=-8\int_0^1 dv\lambda^2(\mathbf{Q}\cdot\mathbf{k}-4\gamma)\eta^{-1}u^{-1}\times\left\{\left(l+\frac{(\epsilon/m)^2}{u^2}\right)^{-1}-v^2\left(l+\frac{(\epsilon/m)^2}{u^2}\right)^{-2}\right\},\quad(1)$$

$$l=1+\lambda^2(1-v^2),$$

which yields, in addition to the previous result,

$$\int[dk]\int_0 du\Gamma_g{}^{\mathrm{add}}=-8\pi(\theta^{-1}-1)\lambda^2(\tfrac{1}{2}G+F_1).\quad(2)$$

Since this vanishes in the limit $\lambda\to0$, the nonrelativistic result, Sec. 5, is unchanged. In the high-energy limit, however, there is a term to be added to (6.1) to (6.9):

$$\Gamma_g{}^{\mathrm{add}}\sim8\pi(1-\theta^{-1})(\log2\lambda)^2,\quad(3)$$

in the sense of those equations, i.e., neglecting constant terms as $\lambda\to\infty$. This will precisely cancel the $[\log2(p_0/m)\sin\frac{1}{2}\vartheta]^2$ term in (6.10) and produce no other changes in that equation.

The effect of the absence of the $[\log(p_0/m)]^2$ term at high energies upon the subsequent discussion is the following. The magnitude of the correction at 100 Mev and $\vartheta=90°$ is now somewhat larger: $\delta_2\approx0.3$ for $\Delta E/E=0.01$, and $\delta_2\approx0.2$ for $\Delta E/E=0.1$ [all with $f(\vartheta)$ neglected]. The correction will no longer change sign at high energies, and the importance of the energy resolution, $\Delta E/E$, will not decrease; δ_2/δ_1 will no longer increase arbitrarily. Moreover, the interpretation of the $[\log(p_0/m)]^2$ term as an indication of a $[(\log p_0/m)]^n$ dependence in the nth Born approximation is now invalidated.

No statement concerning the shape dependence for high-energy scattering by extended nuclei can now be made. The shape factor of the $\log(\Delta E/E)$ term is, of course, the same as that of the uncorrected second Born approximation. If that term is to be considered as at least an order-of-magnitude indication of the correction, (6.12) is to be replaced by

$$\delta_2' = 4\alpha\pi^{-1} \, |\log(\Delta E/E)| \, \log[2(p_0/m)\sin\tfrac{1}{2}\vartheta]. \quad (4)$$

The author is greatly indebted to Dr. H. Suura of Cornell University for pointing out in conversation and correspondence that the result containing a $[\log(p_0/m)]^2$ dependence was likely to be in error.

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Radiative Corrections to Electron Scattering*

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IN a recent paper[1] Newton discussed the possibility that the higher Born approximations (in the external potential) for the lowest order radiative corrections to electron scattering in a Coulomb field might contain higher powers of $\ln(E/m)$ than the first. (E,m = energy and mass of the scattered electron.) This would impose a serious restriction on the validity of such an expansion in powers of $Z\alpha$, if E becomes sufficiently large.

More recently[2] Newton was able to show that $\ln^2(E/m)$ terms do not occur in the second Born approximation and Suura[3] proved that this is true to all orders in a Coulomb field. Newton's calculations are rather involved, partly due to the simultaneous appearance of infrared divergences and so-called Coulomb divergences which arise from the infinite range of the Coulomb field.

This difficulty can easily be avoided in the second Born approximation by a method which removes all infrared (and ultraviolet) divergences before carrying out the potential integrations. Furthermore one can show that in this approximation $\ln^2(E/m)$ terms are absent not only in the somewhat academic Coulomb case, but for any static potential as well.

Figure 1 shows the graphs contributing to σ in order $(Z\alpha)^3\alpha$. M_{2A}, M_{2B}, and M_{2C} interfere with M_{1D}, and M_{1A} interferes with M_{2D}. In σ one has to include the inelastic cross section, i.e., cross terms between M_{2i} and M_{1i}, in order to cancel the infrared divergences in M_{2A}, M_{2B}, M_{2C}, and M_{1A}. M_{2i}' is not divergent and can be neglected for only slightly inelastic scattering ($\Delta E \ll E$). The ultraviolet divergences are taken care of by renormalization, i.e., by using the renormalized vertex[4] $\bar{\Gamma}_\mu^{(2)}$ in M_{2B} and M_{2A} and the renormalized propagator $\bar{S}_F^{(2)}$ in M_{2C}. M_{2A} is not divergent from large photon momenta.

M_{1A} terms.—M_{1A} contains a term $\sim\ln^2(E/m)$ and an infrared divergence $\sim\ln(m/\lambda)$ (λ = photon mass). These "critical" terms, denoted by $\bar{M}_{1A}$, turn out to be a numerical multiple of M_{10}:

$$\bar{M}_{1A} = aM_{10}, \quad a \text{ real}.$$

For the inelastic cross section, $|M_{1i}|^2$, one gets for $\Delta E \ll E$:

$$|M_{1i}|^2 = b|M_{10}|^2,$$

$$b = \frac{\alpha}{2\pi}\int_{k_0=0}^{k_0=\Delta E} d^4k \left(\frac{p}{k\cdot p} - \frac{p'}{k\cdot p'}\right)^2 \delta(k^2+\lambda^2).$$

b has been calculated by Schwinger.[5] Its critical part, $\bar{b}$ (partly contained in Schwinger's G function), is $-2a$. In $\sigma_{\mathrm{el}}+\sigma_{\mathrm{inel}}$, to order $(Z\alpha)^2\alpha$, the critical parts cancel and the net contribution to $\sigma_{11} = |M_{10}|^2$, the elastic cross section to order $(Z\alpha)^2$, becomes

$$\Delta^{\mathrm{rad}}\sigma_{11} = -\delta\sigma_{11}.^5$$

In the next higher order, $\sim(Z\alpha)^3\alpha$, the inelastic cross section is $2|M_{2i}M_{1i}| = 2b|M_{20}M_{10}|$. Obviously the critical part of $2|M_{20}M_{1A}|$ will be $2a|M_{20}M_{10}|$ and this will cancel with the critical terms of $b|M_{20}M_{10}|$, i.e., half of the inelastic cross section. The contribution of $2|M_{20}M_{10}| + |M_{2i}M_{1i}|$ to $\sigma_{21} = 2|M_{20}M_{10}|$, the elastic cross section to order $(Z\alpha)^3$, will be

$$\Delta_{1A}{}^{\mathrm{rad}}\sigma_{21} = -\delta\sigma_{21}.$$

M_{2A}, M_{2B}, and M_{2C} terms.—From M_{2A} one can split off a term $M_{2A}(\lambda)$ (it contains the infrared divergence)

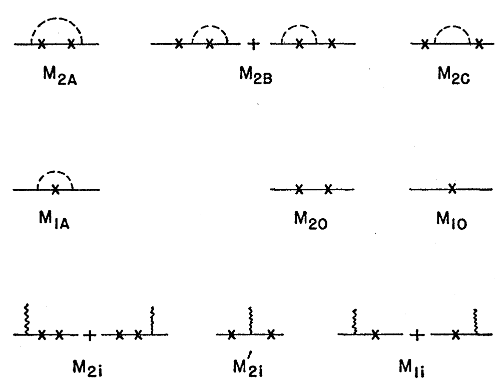

FIG. 1. Graphs contributing to the scattering cross section in order $(Z\alpha)^3\alpha$ (neglecting vacuum polarization).