Conservation of Heavy Particles and Generalized Gauge Transformations

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The possibility of a heavy-particle gauge transformation is discussed.

THE conservation laws of nature fall into two distinct categories: those that are related to invariance under space-time displacements and rotations, and those that are not. In the former category there are the conservation laws of momentum, energy, and angular momentum. In the latter category we find the conservation laws of electric charge, of heavy particles, and the approximate conservation laws of isotopic spin, and perhaps others.[1] We notice that the best known within this second category, the conservation of electric charge, is related to invariance under gauge transformations,[2] which expresses the nonmeasurability of the phase of the complex wave function of a charged particle.

We want to ask here whether similar gauge invariances should be related to all conservation laws of the second category. This question has been discussed in connection with the conservation of isotopic spin by Yang and Mills.[3] We wish here to discuss the problem in connection with the conservation of heavy particles.

If we take the conservation of heavy particles to mean invariance under the transformation

$$\psi_N \rightarrow e^{i\alpha}\psi_N, \quad \psi_P \rightarrow e^{i\alpha}\psi_P, \tag{1}$$

for the wave function of the heavy particles (neutrons and protons), a general gauge transformation (heavy-particle gauge transformation) is a transformation like (1) with the phase α an arbitrary function of space-time. Invariance under such a transformation means that the relative phase of the wave function of a heavy particle at two different space-time points is not measurable.

Such a gauge transformation is formally completely identical with the electromagnetic gauge transformation. Invariance under such a transformation therefore necessitates the existence of a neutral vector massless field coupled to all heavy particles. A nucleon would have a "heavy-particle charge" of $+\eta$ in such a field and an antinucleon would have a "heavy-particle charge" of $-\eta$. The force between two massive bodies therefore would contain a contribution from the Coulomb-like repulsion between such "heavy-particle charges." The total force including the gravitational attraction is:

$$\text{Force} = -G(M_1M_2/R^2) + \eta^2(A_1A_2/R^2). \tag{2}$$

Here M_1, M_2, A_1, and A_2 are the inertia masses and mass numbers of the two bodies. There should also be a magnetic-dipole-like interaction between individual nuclei because the nucleons are in constant motion in a nucleus. But in a macroscopic object the nuclear spins average out so that (2) is correct unless the two bodies are spinning at high speeds.

Now the packing fraction of various atoms differ so that M/A varies fractional-wise from substance to substance by $\sim 10^{-3}$. This means that the observed gravitation mass [which contains a contribution from the η^2 term in (2)] divided by the inertia mass would vary fractional-wise from substance to substance by $10^{-3}\eta^2/G(M_P)^2$, where M_P is the mass of the proton[4] Very careful measurements by Eötvös and co-workers. have shown this variation to be $<10^{-8}$. Therefore

$$\eta^2/G(M_P)^2 < 10^{-5}.$$

It may be remarked that since the packing fraction differs most between hydrogen and, say, carbon, Eötvös' experiment could yield a more sensitive detection of η^2 by a factor of 10 if repeated with a comparison of hydrogen and carbon.

The assumption that leads to the above line of reasoning and the force expression (2) is that the phase factor α in (1) should be space-time-dependent. It should be noticed that in addition the assumption has also been made that the transformation that generates the conservation of heavy particles is of the specific form (1).

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[1] See M. Gell-Mann and A. Pais, Proceedings of the Glasgow Conference, July, 1954 (to be published).

[2] W. Pauli, Revs. Modern Phys. **13**, 203 (1941).

[3] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).

[4] Eötvös, Pekár, and Fekete, Ann. Physik **68**, 11 (1922).