deuteron, say. Brueckner and Watson, in their discussion of the nuclear-force problem for *ps-pv* theory, have suggested that the fourth-order interaction is better approximated by the term $\mathbf{V}^{(4b)}$, in which case the repulsion, mentioned above, is turned into an attraction. However, this procedure is difficult to justify in the light of our previous discussion of the neutral scalar theory [immediately following Eq. (50)].

Finally, we need to consider the nonadiabatic corrections of order μ/M with respect to the second-order potential (77a). For this purpose, we can essentially take over the results of our earlier calculation for the neutral scalar theory [Eq. (51) ff.] suitably modified for the *ps-pv* theory. The final result is to find[35]

$$(\mathbf{p}_1\lambda_1,\,\mathbf{p}_2\lambda_2|\mathbf{V}^{(2)}|\,\mathbf{p}_1+\mathbf{k}\lambda_3,\,\mathbf{p}_2-\mathbf{k}\lambda_4)$$
$$=-\frac{1}{2}\left(\frac{g}{\mu}\right)^2\left(\frac{1}{2\pi}\right)^3\left\{\frac{1}{\omega^2(k)-[E(p_1)-E(\mathbf{p}_1+\mathbf{k})]^2}+\frac{1}{\omega^2(k)-[E(p_2)-E(\mathbf{p}_2-\mathbf{k})]^2}\right\}$$
$$\times u^\dagger(\mathbf{p}_1\lambda_1)(\boldsymbol{\sigma}\cdot\mathbf{k})\tau_\alpha u(\mathbf{p}_1+\mathbf{k}\lambda_3)$$
$$\times u^\dagger(\mathbf{p}_2\lambda_2)(\boldsymbol{\sigma}\cdot\mathbf{k})\tau_\alpha u(\mathbf{p}_2-\mathbf{k}\lambda_4).\quad(86)$$

It is clear from (86) that the nonadiabatic corrections of order μ/M vanish so that the nonrelativistic potential for *ps-pv* theory is given by the fixed-source calculation [(77a) and (85)].[31]

[35] This result has been obtained previously by L. Van Hove, Phys. Rev. **75**, 1519 (1949).

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Effect of Renormalization on Meson-Nucleon S-Scattering*

MAURICE M. LÉVY†
Physics Department, University of Rochester, Rochester, New York

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Renormalization prescriptions are given for the covariant integral equations of meson-nucleon scattering, taking into account the difficulties of overlapping divergences. The covariant wave equations, corresponding to the iteration of second-order irreducible processes, are solved approximately and renormalized in closed form (in the case of the pseudoscalar theory with pseudoscalar coupling). The S-phase shifts corresponding to the states of isotopic spin 1/2 and 3/2 are computed, and their variation with energy is compared with experiment. The only parameter which can be adjusted is the meson-nucleon coupling constant G. It is found that a good agreement with experiment is obtained when $G^2/4\pi=7.5$. The possibility of this agreement being purely coincidental cannot be ruled out, but other interpretations of this result are discussed.

I. INTRODUCTION

IN a foregoing paper,[1] a covariant treatment of meson-nucleon scattering has been presented, which permits, in principle, the elimination of special renormalization difficulties arising in this problem. The main result was that—once the wave integral equation corresponding only to the finite processes is solved—it is possible to express and to renormalize in closed form all the remaining contributions to the scattering cross sections.[2]

The renormalization prescriptions which have to be applied to the closed expressions yielded by the theory were, however, incorrectly stated in that paper,[3] mainly because the difficulties coming from the so-called "overlapping" divergences[4] were not properly taken into account. Fortunately, it is possible to reformulate those prescriptions without losing the advantage of having a closed expression for the corresponding part of the renormalized S-matrix elements. This reformulation is given in Sec. II of the present paper.

Once this formal work has been done, however, there still remains the problem of actually calculating the scattering differential cross sections, in order to compare them with experiment. The first difficulty, here, is that the kernel of the partial wave-equation corresponding to the finite processes is still expressed as a series of powers of the large coupling constant G. This series does not seem easy to sum, and its first few terms do not appear to yield a good approximation. However,

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† On leave of absence from the University of Paris, Paris, France.

[1] M. Lévy, Phys. Rev. **94**, 460 (1954). This paper will be, in the following, referred to as (I). References to its equations will be given as Eq. (I,···).

[2] This applies, of course, only to the special divergences mentioned above. The "normal" radiative corrections must be handled by means of the well-known methods of Feynman and Dyson.

[3] The correct results were stated without proof in a note added in proof to paper (I). Most of the results contained in Sec. I of the present paper have already been reported in a letter from the author to Prof. N. M. Kroll, which has been reproduced, together with the answer from N. M. Kroll, in an Appendix to the *Proceedings of the Fourth Annual Rochester Conference on High Energy Physics* (University of Rochester, Rochester, 1954). Our special thanks are due to Prof. N. M. Kroll for this interesting correspondence.

[4] A. Salam, Phys. Rev. **82**, 217 (1951).

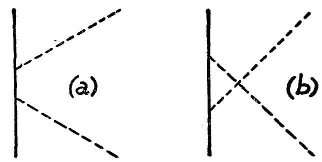

FIG. 1. The two basic diagrams of meson-nucleon scattering to the second order in the coupling constant.

the two basic diagrams corresponding to the second order in G (Fig. 1) have been the object of a detailed study without renormalization,[5] the intrinsically divergent terms being either discarded or arbitrarily cutoff at high frequencies. It seems therefore interesting to investigate to what extent a coherent renormalization scheme affects the results deduced from these processes. It must be emphasized, however, that—even if a good agreement with experiment is obtained—there is no reason to suppose that higher order irreducible terms in the expansion in G^2 will not largely contribute to the results. The interest of a calculation limited to order G^2 is mainly a methodological one, since the difficulties which arise in obtaining a finite solution are, in this case, sufficiently representative of what happens in general. Furthermore, it is possible that higher order terms, although important, affect mainly the value of the effective coupling constant without changing very much the qualitative behavior of the various phase-shifts as functions of energy.[6]

In comparing the results yielded by the pseudoscalar meson theory with pseudoscalar coupling with recent pion-nucleon scattering experiments[7] at low or moderate energies, the study of the S-wave appears of special interest. In a previous paper,[8] Lévy and Marshak have calculated the phase-shifts α_1 and α_3 (corresponding to isotopic spin 1/2 and 3/2 respectively) using the unrenormalized nonadiabatic Tamm-Dancoff equation. The result was that the magnitude, sign and variation with energy of α_3 are, on the whole, in reasonable agreement with experiment, but that the properties of α_1 are completely unacceptable. It was remarked, however, that α_1 is precisely the only S phase-shift which is likely to be greatly modified by renormalization. The solution of the noncovariant integral equations in meson momentum space was, in both cases, an approximate one, based on the replacement of the true kernel $K(\mathbf{p},\mathbf{p}')$ by a product for $K_1(\mathbf{p})K_2(\mathbf{p}')$. It was verified by iteration that this approximation is adequate at low energies. Unfortunately, a similar approximation cannot be used here for the solution of the wave equation for ψ_b, because of its noncovariant form. It is, indeed, essential that ψ_b be expressed covariantly before it can be introduced in the remaining renormalized expression of ψ_a. Since an exact solution of the ψ_b integral equation seems out of question, it is therefore necessary to find a covariant method of approximation. This method, which is given in Sec. III, is also based on the transformation of the kernel into a product form, by modifying the expression of the nucleon propagation function. It can be verified that this approximation is quite good for S-scattering at not too high energies, because it amounts to neglecting only terms of order $(\epsilon/M)^2$, where ϵ is the meson energy and M the nucleon mass.[9] It becomes, however, very bad for P-scattering, as indeed all approximations which transform the kernel of the integral equation into a product[10] (one would obtain, for example, no scattering at all in the states corresponding to $J=3/2$).

By solving the ψ_b equation approximately, one introduces, however, another difficulty. The renormalization prescriptions given in (I) and corrected in Sec. II of this paper are valid only *if the solution of the finite equation is exact.* A straight application of the prescriptions to an approximate solution will not, in general, lead to finite results. This means that the renormalization prescriptions *have to be adapted* to each special method of solution. This can, however, be done without too much difficulty (see Sec. III) by using Salam's general prescriptions[11] for obtaining finite integrals in relativistic field theories.

The resulting expressions of α_1 and α_3 as functions of energy are compared with experiment in Sec. IV. It is found that for a value of the coupling constant corresponding to $G^2/4\pi=7.5$, the magnitude, sign, and variation with energy of *both* phase-shifts agree reasonably well with the experimental data. The only discrepancy remains in the value of $(k)^{-1}|\alpha_1-\alpha_3|$ at zero energy, which does not seem to agree with Panofsky's measurement of the absorption cross section of slow π mesons in hydrogen.[12] Possible interpretations of this discrepancy are discussed at the end of Sec. IV. It might be of interest to note that the calculations given in this paper involve only one parameter, namely the coupling constant G. No cut-off momentum or critical minimum radius are introduced.

II. RENORMALIZATION PRESCRIPTIONS FOR THE "OVERLAPPING" VERTEX OPERATORS AND NUCLEON PROPAGATION FUNCTION

We use the same notations as in (I). The calculation of the ψ_a part of the wave function (Eq. I,10), involves two vertex operators $\Gamma_5^{(\alpha)}$ and $\Gamma_5^{(\beta)}$ and one modified

[5] G. F. Chew, Phys. Rev. **89**, 591 (1953); S. Fubini, Nuovo cimento **10**, 564 (1953); N. Fukuda, Proceedings of the International Conference of Kyoto, 1953 (unpublished); Fukuda, Goto, Okubo, and Sawada, Progr. Theoret. Phys. **12**, 79 (1954); see also reference 8.

[6] See the discussion of Sec. IV.

[7] Barnes, Angell, Perry, Miller, Ring, and Nelson, Phys. Rev. **92**, 1327 (1953); Bodansky, Sachs, and Steinberger, Phys. Rev. **93**, 918 (1954); Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953); J. Tinlot and A. Roberts, Phys. Rev. **95**, 137 (1954).

[8] M. Lévy and R. Marshak, Nuovo cimento **11**, 366 (1954).

[9] We take, in the following, $\hbar=c=1$.

[10] The *iterated* approximation should, however, be good in P-states. This is being investigated at the University of Paris.

[11] A. Salam, Phys. Rev. **84**, 426 (1951).

[12] Panoksky, Aamodt, Hadley, and Phillips, Phys. Rev. **80**, 94 (1950).

nucleon propagation function K_N'. The renormalization of the vertex operators can be done by means of two methods: one can start directly from the integral equations which these operators satisfy (a method similar to the one used by Edwards[13]); or one can follow the prescriptions of Salam[11] for obtaining finite integrals in relativistic field theories. These prescriptions being rather difficult to carry out in detail, we have used here the two methods simultaneously, in order to verify that we were following Salam's procedure correctly, since the renormalization of K_N' can only be done by means of Salam's subtraction method.

1. Renormalization of $\Gamma_5^{(\alpha)}$ and $\Gamma_5^{(\beta)}$ by the Method of the Integral Equation

We write, for $\rho=\alpha, \beta$,

$$\Gamma_5^{(\rho)}(\xi_i;x,y)=\tau_i\gamma_5\delta(x-\xi)\delta(y-\xi)+\Lambda_5^{(\rho)}(\xi_i;x,y), \quad (1)$$

and put, furthermore:

$$\Lambda_5^{(\alpha)}(\xi_i;x,y)=\tau_i\gamma_5\Lambda^{(\alpha)}(\xi;x,y)$$

and

$$\Lambda_5^{(\beta)}(\xi_i;x,y)=\Lambda^{(\beta)}(\xi;x,y)\gamma_5\tau_i. \quad (2)$$

$\Lambda^{(\alpha)}$ and $\Lambda^{(\beta)}$ obey integral equations which can be written, in momentum space [see Eq. (I,17)], as follows:

$$\Lambda^{(\alpha)}(p,q)=\Lambda_0^{(\alpha)}(p,q)+iG^2(2\pi)^{-4}\int\Lambda^{(\alpha)}(p,\,p-k) \times K_N(p-k)K_M(k)\gamma_5K_N(q-k)\gamma_5dk, \quad (3)$$

and

$$\Lambda^{(\beta)}(p,q)=\Lambda_0^{(\beta)}(p,q)+iG^2(2\pi)^{-4}\int\gamma_5K_N(p-k) \times\gamma_5K_M(k)K_N(q-k)\Lambda^{(\beta)}(q-k,\,q)dk, \quad (4)$$

where we have put

$$\Lambda_0^{(\alpha)}(p,q) =iG^2(2\pi)^{-4}\int K_N(p-k)K_M(k)\gamma_5K_N(q-k)\gamma_5dk \quad (5)$$

and

$$\Lambda_0^{(\beta)}=\gamma_5\Lambda_0^{(\alpha)}\gamma_5.$$

Let us write first[14]:

$$\Lambda^{*(\alpha)}(p,q)=\Lambda^{(\alpha)}(p,q)-\Lambda^{(\alpha)}(p_0,p_0'), \quad (6)$$

where $i\gamma p_0+M=0$, and $(p_0-p_0')^2+\mu^2=0$. $\Lambda^{*(\alpha)}$ obeys the equation

$$\Lambda^{*(\alpha)}(p,q)=[1+\Lambda^{(\alpha)}(p_0,p_0')]\Lambda^*{}_0^{(\alpha)}(p,q) +iG^2(2\pi)^{-4}\int\Lambda^{*(\alpha)}(p,\,p-k)K_N(p-k)K_M(k) \times\gamma_5K_N(q-k)\gamma_5dk-iG^2(2\pi)^{-4}\int\Lambda^{*(\alpha)}(p_0,\,p_0-k) \times K_N(p_0-k)K_M(k)\gamma_5K_N(p_0'-k)\gamma_5dk. \quad (7)$$

Consequently, if we define

$$\Lambda_c^{(\alpha)}(p,q)=\Lambda^{*(\alpha)}(p,q)/[1+\Lambda^{(\alpha)}(p_0,p_0')], \quad (8)$$

$\Lambda_c^{(\alpha)}$ is a convergent function which obeys the equation

$$\Lambda_c^{(\alpha)}(p,q)=\Lambda^*{}_0^{(\alpha)}(p,q)+iG^2(2\pi)^{-4}\int\Lambda_c^{(\alpha)}(p,\,p-k) \times K_N(p-k)K_M(k)\gamma_5K_N(q-k)\gamma_5dk-iG^2(2\pi)^{-4} \times\int\Lambda_c^{(\alpha)}(p_0,\,p_0-k)K_N(p_0-k)K_M(k) \times\gamma_5K_N(p_0'-k)\gamma_5dk. \quad (9)$$

Equation (8) can be written in terms of $\Gamma^{(\alpha)}=1+\Lambda^{(\alpha)}$ and is equivalent to

$$\Gamma_c^{(\alpha)}=\Gamma^{(\alpha)}(p,q)/[\Gamma^{(\alpha)}(p_0,p_0')].$$

The infinite constant $\Gamma^{(\alpha)}(p_0,p_0')$ is just the Z constant introduced by Edwards.[13] A similar relation is, of course, found for $\Gamma_c^{(\beta)}$, the infinite constant being, this time, $\Gamma^{(\beta)}(p_0,p_0')$.

2. Renormalization of the Vertex Operators by the Method of Salam

In this method, the subtraction of partial divergences from a diagram of a given order yields expressions which are proportional to the contributions of lower order diagrams, multiplied by infinite constants. It is therefore always convenient to start subtracting partial divergences from the highest order term, and move progressively down to the lowest order. As an example, we shall apply the subtraction method to the sum of the first four terms of $\Gamma^{(\beta)}=1+\Lambda^{(\beta)}$. With notations similar to those of Salam, let us put:

$$F(p,t_1)=iG^2(2\pi)^{-4}K_N(p-t_1)K_M(t_1)$$

and

$$G(p,t_1,t_2)=\gamma_5K_N(p-t_1-t_2)\gamma_5.$$

We have, consequently,

$$\Gamma^{(\beta)}(p,t_1)=1+\int G(p,t_1,t_2)F(p,t_2)dt_2 +\int G(p,t_1,t_2)F(p,t_2)G(p,t_2,t_3)F(p,t_3)dt_2dt_3 +\int G(p,t_1,t_2)F(p,t_2)G(p,t_2,t_3)F(p,t_3)G(p,t_3,t_4) \times F(p,t_4)dt_2dt_3dt_4+\text{etc}\cdots. \quad (10)$$

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[13] S. F. Edwards, Phys. Rev. **90**, 284 (1953).

[14] Note a difference with the corresponding equation (I,18), where two *equal* free momenta were used in the substracted vertex function. The prescription used in the present paper is equivalent to the standard method of F. J. Dyson [Phys. Rev. **75**, 1736 (1949)], after summation over a certain number of graphs. It is different both from that of N. Kroll and M. Ruderman, Phys. Rev. **93**, 233 (1953), and of Deser, Thirring, and Goldberger, Phys. Rev. **94**, 711 (1954). The variety of prescriptions in the renormalization of vertex operators is related to the well-known ambiguity in the definition of meson charge-renormalization.

Let us consider first the fourth term of the right-hand side of Eq. (10). We have to subtract from it the partial divergences over t_4, over t_3 and t_4, over t_2, t_3, and t_4. They are all logarithmic. All the other integrations, or groups of integrations, are—according to Salam's terminology—"superficially convergent." Before doing the subtraction, it is convenient to expand $\Lambda^{(\beta)}(p_0,p_0')$, the infinite constant (which is just a *number*) introduced in the preceding subsection, in powers of the coupling constant

$$\Lambda^{(\beta)}(p_0,p_0')=\sum_n\Lambda_{0,\,2n}{}^{(\beta)}, \tag{11}$$

where $\Lambda_{0,\,2n}{}^{(\beta)}$ is proportional to G^{2n}. Subtracting the divergence over t_4 in the fourth term of Eq. (10) yields an additional term:

$$-\int G(p,t_1,t_2)F(p,t_2)G(p,t_2,t_3)F(p,t_3)\times G(p_0,\,p_0-p_0',\,t_4)F(p_0,t_4)dt_2dt_3dt_4$$

which is equal to the third term of Eq. (10) multiplied by $\Lambda_{02}{}^{(\beta)}$. Similarly, subtracting the divergence over t_3 and t_4 amounts to multiplying the second term of (10) by $(1-\Lambda_{04}{}^{(\beta)})$, etc$\cdots$. After subtracting therefore all partial divergences from the fourth term of Eq. (10), we are led to the new expression:

$$\begin{aligned}\Gamma_2{}^{(\beta)}(p,t_1)=&\;(1-\Lambda_{06}{}^{(\beta)})\\&+(1-\Lambda_{04}{}^{(\beta)})\int G(p,t_1,t_2)F(p,t_2)dt_2\\&+(1-\Lambda_{02}{}^{(\beta)})\int G(p,t_1,t_2)F(p,t_2)G(p,t_2,t_3)F(p,t_3)dt_2dt_3\\&+\int G(p,t_1,t_2)F(p,t_2)G(p,t_2,t_3)F(p,t_3)\\&\qquad\times G(p,t_3,t_4)F(p,t_4)dt_2dt_3dt_4+\text{etc}\cdots.\end{aligned} \tag{12}$$

We now start subtracting the partial divergences from the third term of Eq. (12), namely those over t_3, and t_2 and t_3. This yields a new expression $\Gamma_4{}^{(\alpha)}$, from which we finally subtract the partial divergence over t_2 of the second term. The final expression of $\Gamma^{(\alpha)}$ can be written as

$$\begin{aligned}\Gamma_6{}^{(\beta)}(p,t_1)=&\;1-\Lambda_{06}{}^{(\beta)}-(1-\Lambda_{02}{}^{(\beta)})\Lambda_{04}{}^{(\beta)}\\&-[1-\Lambda_{04}{}^{(\beta)}-(1-\Lambda_{02}{}^{(\beta)})\Lambda_{02}{}^{(\beta)}]\Lambda_{02}{}^{(\beta)}\\&+[1-\Lambda_{04}{}^{(\beta)}-(1-\Lambda_{02}{}^{(\beta)})\Lambda_{02}{}^{(\beta)}]\int G(p,t_1,t_2)\\&\times F(p,t_2)dt_2+(1-\Lambda_{02}{}^{(\beta)})\int G(p,t_1,t_2)\\&\times F(p,t_2)G(p,t_2,t_3)F(p,t_3)dt_2dt_3\\&+\int G(p,t_1,t_2)F(p,t_2)G(p,t_2,t_3)F(p,t_3)\\&\qquad\times G(p,t_3,t_4)F(p,t_4)dt_2dt_3dt_4+\text{etc}\cdots.\end{aligned} \tag{13}$$

A rapid inspection of (13) shows that this is nothing but the expansion, up to G,[6] of the expression

$$\Gamma_c{}^{(\beta)}(p,t_1)=\Gamma(p,t_1)/(1+\sum_n\Lambda_{0,\,2n}{}^{(\beta)}), \tag{14}$$

which is the same result as Eq. (9).

3. Renormalization of K_N' by Means of Salam's Method

Now that we have verified that we know how to use Salam's procedure properly, we can apply the same method to $\Sigma(p)$, the sum of the irreducible parts of K_N', defined as follows:

$$K_N'(p)=K_N(p)/[1-3\gamma_5\Sigma(p)\gamma_5K_N(p)]. \tag{15}$$

We therefore consider the first four terms of the expansion of $\Sigma(p)$ and write:

$$\begin{aligned}\Sigma(p)=&\int F(p,t_1)dt_1+\int F(p,t_1)G(p,t_1,t_2)F(p,t_2)dt_1dt_2\\&+\int F(p,t_1)G(p,t_1,t_2)F(p,t_2)G(p,t_2,t_3)F(p,t_3)dt_1dt_2dt_3\\&+\int F(p,t_1)G(p,t_1,t_2)F(p,t_2)G(p,t_2,t_3)F(p,t_3)\\&\qquad\times G(p,t_3,t_4)F(p,t_4)dt_1dt_2dt_3dt_4+\text{etc}\cdots.\end{aligned} \tag{16}$$

One difference with the calculation of the previous paragraph is that we now have divergences at both ends of each term. Furthermore, all the partial divergences are logarithmic, but the final one is linear. We consequently subtract first the partial logarithmic divergences, with the same method as in the previous paragraph. This yields the result

$$\Sigma_1(p)=\Sigma(p)/[1+\Lambda^{(\alpha)}(p_0,p_0')][1+\Lambda^{(\beta)}(p_0,p_0')]. \tag{17}$$

Subtracting the remaining linear divergence gives the renormalized expression of $\Sigma(p)$:

$$\Sigma_c(p)=\Sigma_1(p)-\Sigma_1(p_0)-(p-p_0)\left[\frac{\partial}{\partial p}\Sigma_1(p)\right]_{p=p_0}. \tag{18}$$

In obtaining the expression (17), advantage has been taken of the fact that $\Lambda^{(\alpha)}(p_0,p_0')$ and $\Lambda^{(\beta)}(p_0,p_0')$ are just numbers, since the operators γ_5 and τ_i have been subtracted [see Eqs. (1) and (2)], and use has been made of the relations $i\gamma p_0+M=p_0^2+M^2=0$ and $(p_0'-p_0)^2+\mu^2=0$. The relatively simple result contained in Eqs. (17) and (18) is *not* true in general of any overlapping self-energy graph. It arises from the following simple property of the particular series of graphs under consideration: these graphs contain partial divergences *only at the ends* (on both sides), but not in the middle, where the partial integrations are always "superficially convergent."

4. Renormalized Expressions of the Wave-Function and the S-Matrix Elements

The results yielded by the application of prescriptions (9), (17) and (18) to the expressions of K_a, ψ_a and of the $R=S-1$ matrix elements, can be stated immediately, using the same notations as in (I). Equation (I,34) becomes:

$$K_a(x,\xi_i;\, y,\eta_j)=\frac{-iG^2}{\Gamma_0{}^\alpha\Gamma_0{}^\beta}\int K_b(x,\xi_i;\, \xi',\xi_k')\gamma_5\tau_k K_N' \times(\xi',\eta')\gamma_5\tau_l K_b(\eta',\eta_l';\, y,\eta_j)d\xi' d\eta', \quad (19)$$

where we have put, to simplify, $1+\Lambda^{(\rho)}(p_0,p_0')=\Gamma_0{}^\rho$. Similarly, the expression (I,35) of ψ_a can be written now as

$$\psi_a(x,\xi_i)=\frac{-iG^2}{\Gamma_0{}^\alpha\Gamma_0{}^\beta}\int K_b(x,\xi_i;\, \xi',\xi_k')\gamma_5\tau_k K_N'(\xi',\eta') \times\gamma_5\tau_l\psi_b(\eta',\eta_l')d\xi' d\eta'. \quad (20)$$

Separating out the motion of the center-of-mass [Eqs. (I,50, 51 and 58)], transforming into momentum space [Eq. (I,64)], and projecting on the states of total isotopic spin 1/2 and 3/2, we obtain, for the Fourier transform of the part of the wave function corresponding to the relative motion:

$$\phi_a(p;\, P)=\frac{-iG^2}{\Gamma_0{}^\alpha\Gamma_0{}^\beta}T_a(2\pi)^{-4}\int K_b(p+P,\, -p;\, q+P,\, -q) \times dq\gamma_5 K_N'(P)\gamma_5\phi_b(0) \quad (21)$$

[compare with (I,60 and 65)], where $T_a=3$ for the state of isotopic spin 1/2 and $T_a=0$ for the state of isotopic spin 3/2. Similarly, the expression (I,53) for the R_a-matrix elements becomes here:

$$(\mathbf{k}_1',\mathbf{k}_2',j\,|\,R_a\,|\,\mathbf{k}_1,\mathbf{k}_2,i)=[-iG^2(2\pi)^4/\Gamma_0{}^\alpha\Gamma_0{}^\beta]\bar{\rho}_b{}^{(k)}(0) \times\gamma_5\tau_k K_N'(P)\gamma_5\tau_l\phi_b{}^{(l)}(0)\delta(\mathbf{P}-\mathbf{P}')\delta(P_0-P_0'). \quad (22)$$

The special choice for the renormalization of the vertex-operators which has been made in the paragraph 2 of this section has the advantage that expressions (21) and (22) can now be greatly simplified by a direct evaluation of the ratios $(\Gamma_0{}^\alpha)^{-1}\bar{\rho}_b(0)$ and $(\Gamma_0{}^\beta)^{-1}\phi_b(0)$. Combining Eqs. (I,4), (I,13), (I,15), and (I,16) yields indeed the fundamental relations

$$\psi_b(x,x_i)=\int\Gamma^{(\alpha)}(y;\, x,z)\psi_0{}^i(z,y)dydz \quad (23)$$

and

$$\bar{\chi}_b(x,x_j)=\int\bar{\chi}_0{}^j(z,y)\Gamma^{(\beta)}(y;\, z,x)dydz. \quad (24)$$

Separating out the center-of-mass coordinates, and transforming into momentum space yields the following expression of the required ratios:

$$\phi_b(0)/\Gamma_0{}^\alpha\equiv\phi_b(0)/\Gamma^{(\alpha)}(P,k)=\phi_0(k,\, P-k), \quad (25)$$

$$\bar{\rho}_b(0)/\Gamma_0{}^\beta=\bar{\rho}_b(0)/\Gamma^{(\beta)}(k',P')=\bar{\rho}_0(k',\, P'-k'), \quad (26)$$

where use has been made of the relations which define P, P', k and k': $i\gamma k+M=i\gamma k'+M=0$, $(P-k)^2+\mu^2=(P'-k')^2+\mu^2=0$. Equation (22) becomes now:

$$(\mathbf{k}',\, -\mathbf{k}',\, j\,|\,R_a\,|\,\mathbf{k},\, -\mathbf{k},\, i)=-iG^2(2\pi)^4\bar{\rho}_0{}^{(k)}(k',\, P'-k') \times\gamma_5\tau_k K_N'(P)\gamma_5\tau_l\phi_0(k,\, P-k) \times\delta(\mathbf{P}-\mathbf{P}')\delta(P_0-P_0'). \quad (27)$$

The determination of the scattering phase-shifts is now entirely reduced to the solution of the ψ_b equation and to the determination of the renormalized nucleon propagation function $K_N'(P)$.

III. APPROXIMATE SOLUTION OF THE WAVE EQUATION

In this section, we first solve approximately the integral equation for ψ_b, and then apply this solution to the calculation of the renormalized expression of ψ_a, using an adaptation of the renormalization prescriptions to the special approximate form of the K_N' function resulting from ψ_b.

1. Solution of the Equation for ψ_b

After separating out the motion of the center-of-mass [Eq. (I,51)], transforming into the momentum representation and projecting on the states of total isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$, we obtain, for the part of the wave function corresponding to the relative motion, the following equation:

$$\phi_b(p;\, P)=(2\pi)^4\phi_0(k+P,\, -k)\delta(k-p) -\frac{iG^2}{(2\pi)^4}T_b K_N(p+P)K_M(-p)\gamma_5\int K_N(p+p'+P) \times\gamma_5\phi_b(p';\, P)dp', \quad (28)$$

where $T_b=2$ for the state of isotopic spin $\frac{1}{2}$ and -1 for the state of isotopic spin $\frac{3}{2}$. The whole problem has been reduced to the solution of Eq. (28), the kernel of which consists of the expression $\gamma_5 K_N(p+p'+P)\gamma_5=K_N\times(-p-p'-P)$. Now, it is worth remarking that p and p' are actually meson energy-momenta which, because of the weighting factor coming from the distribution function ϕ_b (which is centered around k), can be considered as "small" compared to P. In the non-relativistic region the kernel is therefore approximately equal to $(\gamma_4 P_0+M)^{-1}\sim(2M)^{-1}$. An approximate form of the kernel should be based on this property, which is due to the presence of the two γ_5 operators on each side of the K_N function. The brutal replacement of the kernel by $(2M)^{-1}$ would, however, introduce spurious divergences, coming from the fact that, although the distribution of the p'-momenta is centered around the "small" value k, the integration on the right-hand side of (28) extends over all values of p', large and small. A convenient approximation to $\gamma_5 K_N(p+p'+P)\gamma_5$ should therefore not assume that *both* p and p' are small but that only one of them, p or p', is small compared to P. Such an approximation exists, and can be written as follows[15]:

$$\gamma_5 K_N(p+p'+P)\gamma_5 \sim \gamma_5 K_N(p+P)K_N^{-1}(P)K_N(p'+P)\gamma_5. \quad (29)$$

Making this replacement in Eq. (29), we obtain the expression of $\phi_b(p;P)$ in the form

$$\phi_b(p;P)=(2\pi)^4\phi_0(k+P,-k)\delta(k-p) -iG^2 T_b K_N(p+P)K_M(-p)K_N(-p-P) \times K_N^{-1}(-P)C_b(P), \quad (30)$$

where $C_b(P)$ is, for a given energy, a constant defined by:

$$C_b(P)=\frac{1}{(2\pi)^4}\int K_N(-p-P)\phi_b(p;P)dp, \quad (31)$$

the expression of which can easily be obtained by introducing the value of $\phi_b(p;P)$, given by (30), into the right-hand side of (31):

$$C_b(P)=[1+(G^2/4\pi)T_b Q(P)]^{-1} \times K_N(-k-P)\phi_0(k+P,-k), \quad (32)$$

$Q(P)$, expressed as follows:

$$Q(P)=\frac{i}{4\pi^3}\int K_N^2(-P-p)K_N(P+p) \times K_M(-p)K_N^{-1}(-P)dp, \quad (33)$$

is a finite function of energy. Using Feynman's method of evaluation of integrals of this type, we obtain

$$Q(P_0)=\frac{(M+\gamma_4 P_0)}{4\pi}\int_0^1 \frac{xdx[M-\gamma_4 P_0(1-x)]}{D(x,P_0^2)}, \quad (34)$$

with:

$$D(x,P_0^2)=-P_0^2 x(1-x)+M^2 x+\mu^2(1-x). \quad (35)$$

From the expression (30) of $\phi_b(p;P)$, we can deduce easily the contribution of the (b)-type diagrams to the S-phase-shifts. Writing T_a, T_b and the complete S-phase-shift α as 2-row matrices (corresponding to $\frac{3}{2}$ and $\frac{1}{2}$ isotopic states), we split the expression for $\tan\alpha$ into two parts:

$$\tan\alpha=(\tan\alpha)_a+(\tan\alpha)_b. \quad (36)$$

[15] An even better approximation (which leads however to more complicated calculations) would be to write:

$$\gamma_5 K_N(p+p'+P)\gamma_5 \sim \gamma_5 K_N(p+k+P)K_N^{-1}(P+2k)K_N(p'+k+P)\gamma_5,$$

but the difference between the two approximations is negligible for low-energy S-scattering. The situation is different for P-scattering, where the more complicated approximation should certainly be preferable.

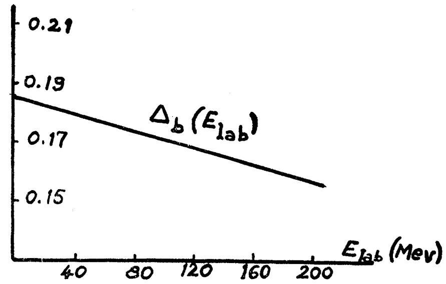

FIG. 2. Variation of the "damping" function Δ_b, as a function of the meson energy in the laboratory system.

Making use of the method given in Sec. IV of (I), we then obtain from the asymptotic form of $\phi_b(p;P)$:

$$(\tan\alpha)_b=-\frac{k}{2M}\left(\frac{G^2}{4\pi}\right)T_b\frac{(M+P_0)(E_k+M)}{4E_k P_0} \times\frac{1}{1+(G^2/4\pi)T_b\Delta_b(P_0)}, \quad (37)$$

where $\Delta_b(P_0)$ is obtained by putting $\gamma_4=1$ in $Q(P_0)$:

$$\Delta_b(P_0)=\left(\frac{M+P_0}{4\pi}\right)\int_0^1 \frac{xdx[-P_0(1-x)+M]}{D(x,P_0^2)}. \quad (38)$$

$\Delta_b(P_0)$ can be calculated analytically by standard methods. It is a slowly decreasing function of energy which is plotted in Fig. 2 for energies varying between 0 and 200 Mev (E_{lab} is the energy in the laboratory system).

2. Calculation of $K_N'(P)$ and $(\tan\alpha)_a$

Using Eqs. (15) and (I,30), we can write, after summation over the isotopic-spin indices,

$$3\gamma_5\Sigma(P)\gamma_5=-\frac{3iG^2}{(2\pi)^4}\int L_b(p;P)dp, \quad (39)$$

where $L_b(p;P)$ is defined by

$$L_b(p;P) =\frac{\gamma_5}{(2\pi)^4}\int K_b(p+P,-p;p'+P,-p')\gamma_5 dp', \quad (40)$$

and obeys the following integral equation:

$$L_b(p;P)=K_N(-p-P)K_M(-p) +\frac{iG^2}{(2\pi)^4}\gamma_5 K_N(p+P)K_M(-p)\gamma_5\int K_N(p+p'+P) \times\gamma_5 L_b(p',P)dp'. \quad (41)$$

Making again the approximation (29), we obtain

$$L_b(p;P)=K_N(-P-p)K_M(-p) \times\{1+(iG^2/4\pi)K_N(p+P)K_N^{-1}(P) \times[1-(G^2/4\pi)Q(-P)]^{-1}U(P^2)\}, \quad (42)$$

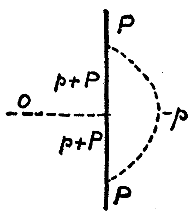

FIG. 3. Vertex diagram corresponding to the function $U(P^2)\gamma_5$.

where $U(P^2)$, given by the expression

$$U(P^2)=\frac{1}{4\pi^3}\int K_N(p+P)K_N(-p-P)K_M(-p)dp, \quad (43)$$

is a logarithmically-divergent integral, which is a function of P^2 only. $\gamma_5 U(P^2)$ corresponds to the simple vertex diagram represented on Fig. 3.

Integrating $L_b(p;P)$ over p, we finally obtain the expression

$$3\gamma_5\Sigma(P)\gamma_5=\Sigma^0(P)+3\left(\frac{G^2}{4\pi}\right)^2 K_N^{-1}(P)\left[1-\frac{G^2}{4\pi}Q(-P)\right]^{-1}U^2(P^2), \quad (44)$$

$\Sigma^0(P)$ corresponding to the lowest order self-energy diagram:

$$\Sigma^0(P)=-\frac{3iG^2}{(2\pi)^4}\int \gamma_5 K_N(p+P)\gamma_5 K_M(-p)dp. \quad (45)$$

It can be seen almost immediately that prescriptions (17) and (18) can no longer be applied to this approximate expression of $\Sigma(P)$. Instead, it is necessary to expand $\Sigma(P)$ in a power series of G^2 by means of the Neumann-Liouville expansion of integral equation (41), and to apply Salam's subtraction method to each term of the expansion. Summing back the renormalized expansion, one verifies—as it should have been expected—that the convergent expression of $\Sigma(P)$ is obtained by replacing $\Sigma^0(P)$ and $U(P^2)$ in (44) by their convergent forms Σ_c^0 and U_c respectively:

$$\Sigma_c^{(0)}=\Sigma^0(P)-\Sigma^0(p_0)-(P-p_0)\left[\frac{\partial}{\partial P}\Sigma^0\right]_{P=p_0}, \quad (46)$$

where p_0 is a free nucleon momentum ($i\gamma p_0+M=p_0^2+M^2=0$), and

$$U_C(P^2)=U(P^2)-U(-M^2). \quad (47)$$

This convergent form of $\Sigma(P)$ has the correct behavior when $P\rightarrow p_0$, $P^2\rightarrow -M^2$: $\Sigma(P)K_N(P)$ vanishes in this limit, since it is the square of the function $U(P^2)$ which appears on the right-hand side of (44).

Making the subtractions (46) and (47), and using again Feynman's method, we obtain

$$\gamma_5 K_N'(P)\gamma_5=\left\{1+\frac{3G^2}{4\pi}H(P)+3\left(\frac{G^2}{4\pi}\right)^2\left[1-\frac{G^2}{4\pi}Q(P)\right]^{-1}F^2(P^2)\right\}^{-1}\times K_N(-P), \quad (48)$$

where we have put

$$H(P)=\frac{M^2}{2\pi}\int_0^1\frac{x^2(1-x)dx}{D_0(x)}-\frac{(-i\gamma P+M)^{-1}}{4\pi}\times\int_0^1[-i\gamma P(1-x)+M]\log\left(\frac{D}{D_0}\right)dx, \quad (49)$$

with:

$$D_0(x)=D(x,M^2)=M^2x^2+\mu^2(1-x) \quad (50)$$

and

$$F(P^2)=\frac{1}{4\pi}\int_0^1\log\left(\frac{D}{D_0}\right)dx. \quad (51)$$

Introducing the expression (48) of $\gamma_5 K_N'(P)\gamma_5$ into the matrix elements (27) of R_a, and using again the method of Sec. IV of (I), we obtain the value of $(\tan\alpha)_a$:[16]

$$(\tan\alpha)_a=-\frac{kG^2}{(2M)4\pi}\frac{T_aM(E_k+M)}{P_0(M+P_0)}\frac{1}{1+\frac{G^2}{4\pi}\Delta_a(P_0)}, \quad (52)$$

where $\Delta_a(P_0)$ is equal to:

$$\Delta_a(P_0)=3h(P_0)+\frac{3G^2}{4\pi}\left[1-\frac{G^2}{4\pi}\Delta_b(P_0)\right]^{-1}F^2(-P_0^2), \quad (53)$$

$h(P_0)$ being the value of $H(P)$ when $\gamma_4=1$:

$$h(P_0)=\frac{M^2}{2\pi}\int_0^1\frac{x^2(1-x)dx}{D_0}-\frac{1}{4\pi}\int_0^1\frac{P_0(1-x)+M}{P_0+M}\log\left(\frac{D}{D_0}\right)dx. \quad (54)$$

IV. DISCUSSION OF THE RESULTS. COMPARISON WITH EXPERIMENT

Equations (36), (37), and (52), which define the complete S-phase-shift in our approximation, depend on only one parameter: the coupling constant G. Our problem is to determine whether it is possible to choose G so as to be able to reproduce the "experimental" behavior of the α_1 and α_3 phase-shifts. This "experimental" variation of α_1 and α_3 with energy is not too

[16] In calculating $(\tan\alpha)_a$, we have assumed that Eqs. (25) and (26), which are rigorous for the exact solution of the ψ equation, remain valid for the approximate solution. This is not quite true. However, the approximate expressions of the ratios which appear on the left-hand side of Eqs. (25) and (26) do not differ very much from their exact values, at least for S-states, as can be verified by a direct calculation.

well known,[17] but it is generally agreed that α_3 is negative and increases slowly in absolute value with increasing energy, whereas α_1 is positive and shows at first a rapid increase with energy, up to about 10° in the 40-Mev region; it remains afterwards more or less constant up to about 100 Mev, then decreases sharply and becomes negative. α_1 and α_3 are practically equal near 180 or 200 Mev (both of the order of $-20°$).

Let us first examine the behavior of the denominators in $(\tan\alpha)_a$ and $(\tan\alpha)_b$. The first term of $\Delta_a(P_0)$ defined by (53), is identical with the expression calculated by Brueckner, Gell-Mann, and Goldberger.[18] It is positive, and of the order of $3/4\pi$, so that the factor $[1+3(G^2/4\pi)\times h(P_0)]$, if it existed alone, would produce a "damping" of $(\tan\alpha)_a$ by a factor of the order of 3 or 4, if $G^2/4\pi$ is assumed to be of the order of 10. However, for such a magnitude of $G^2/4\pi$, the second term of (53) is negative, because $(G^2/4\pi)\Delta_b(P_0)$ is then appreciably larger than 1, and varies rather strongly with energy. However, at low energies, $1+(G^2/4\pi)\Delta_a$ is still positive, so that $(\tan\alpha)_a$, for the $T=1/2$ state, is negative. The observed positive value of α_1 can therefore be obtained only if $(\tan\alpha)_b$, for the $T=1/2$ state, is strongly positive. Since $T_b=-1/2$ in this case, this means that the denominator on the right-hand side of (37) must be positive, or that

$$\tfrac{1}{2}(G^2/4\pi)\Delta_b(P_0)<1. \tag{55}$$

$\Delta_b(P_0)$ being a slowly decreasing function of E_{lab}, it is sufficient to impose condition (55) at $E_{\mathrm{lab}}=0$, ($P_0=M+\mu$). This defines a maximum value of $G^2/4\pi$ which turns out to be

$$G^2/4\pi<10.8. \tag{56}$$

For values of $G^2/4\pi$ larger than 10.8, Eq. (37) would predict a resonance in the $T=1/2$ state at a finite energy, as was already obtained in the unrenormalized calculations.[8,19] Actually, $G^2/4\pi$ must be appreciably smaller than the limit of Eq. (56); otherwise, α_1 would take, at low energies, large positive values incompatible with the experimental data. It turns out that the value

$$G^2/4\pi=7.5, \tag{57}$$

gives an α_1 phase-shift of about 10° at 40 Mev. The rapid variation of α_1 with energy results from quick variations of the relative magnitudes of $(\tan\alpha)_a$ and $(\tan\alpha)_b$ in the $T=1/2$ state. At very low energies, $(\tan\alpha)_b$ predominates over $(\tan\alpha)_a$, and produces a sharply rising slope for α_1. However, because of the second (negative) term in Δ_a, $(\tan\alpha)_a$ increases rapidly at moderate energies and eventually changes the sign of α_1. There exists a region where the variations of the two terms compensate each other, giving rise to the observed broad maximum. On the other hand, $(\tan\alpha)_a$ does not contribute to the $T=3/2$ state and $T_b>0$, so that α_3 is a slowly varying negative function of energy. The variation of α_1 and α_3 as functions of k/μ (k is the momentum in the center-of-mass system: $P_0=E_k+\omega_k$), is represented in Fig. 4, together with points taken from the analysis of experimental results.[17] It is seen that the agreement is surprisingly good.[19a]

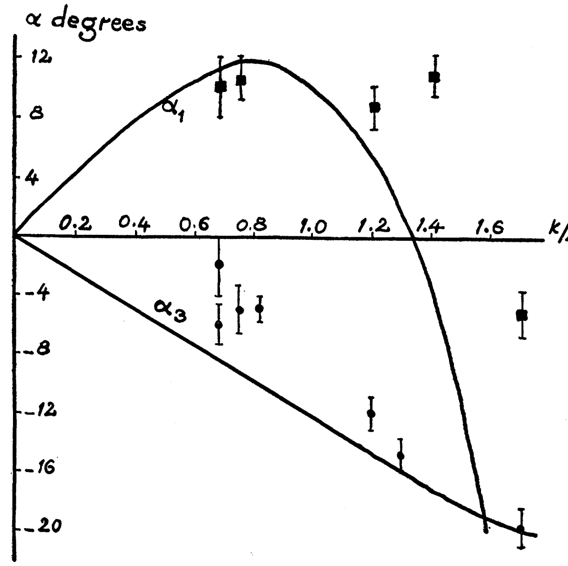

FIG. 4. Variation of the two S-phase-shifts, α_1 and α_3, as functions of k/μ, where k is the momentum in the center-of-mass system and μ the meson mass.

We would like now to complete the discussion with a certain number of remarks:

(a) The value of $G^2/4\pi$ given in Eq. (57) is somewhat different from the values of 10, which is inferred from the study of nuclear forces,[20] or 15, used by Dyson *et al.*[21] in their computation of the meson-nucleon P phase-shifts by means of the Tamm-Dancoff method. This "discrepancy" should not, however, be taken too seriously. If the agreement with experiment which has been obtained, using the two lowest order diagrams, in the present work as well as in other investigations,[5,21] has any other significance than that of a pure coincidence, it means that higher order diagrams, although quite important, contribute mainly through an "effective" coupling constant, without changing appreci-

[17] See, for example, the discussion of H. A. Bethe in the *Proceedings of the Fourth Annual Rochester Conference on High Energy Physics* (University of Rochester, Rochester, 1954).

[18] Brueckner, Gell-Mann, and Goldberger, Phys. Rev. **90**, 476 (1953).

[19] See L. Sartori and V. Wataghin, Nuovo cimento **12**, 260 (1954) who get a resonance in the $T=1/2$ state by a renormalized calculation with a relatively low cut-off momentum.

[19a] The numerical discussion of the results was done with values of T_a and T_b equal to half of the correct ones (the values of T_a and T_b quoted in the text are correct). It is therefore necessary to replace $G^2/4\pi$ by half of its value (57) to obtain practically the same results. The only place where $G^2/4\pi$ does not occur multiplied by T_a or T_b is in the product $(G^2/4\pi)\Delta_a(P_0)$, on the right-hand side of Eq. (52). The difference, however, is insignificant, since the second term of the right-hand side of (53) becomes then positive and approximately doubles the value of Δ_a.

[20] M. Lévy, Phys. Rev. **88**, 725 (1952); S. D. Drell and K. Huang, Phys. Rev. **91**, 1527 (1953).

[21] Dyson, Ross, Salpeter, Schweber, Sunderesan, Visscher, and Bethe, Phys. Rev. **95**, 1644 (1954).

ably the energy variation of the scattering phase-shifts at low energies. It is then to be expected that this "effective" coupling constant would not be the same in different scattering states or, even more so, in different physical processes involving π mesons.

It should be noted that our results are very sensitive to the value of $G^2/4\pi$. For example the choice $G^2/4\pi=8$ would completely destroy the agreement with experiment.

(b) Although the agreement of our calculations with the experimental data on $\pi-p$ scattering is quite good, there is a definite discrepancy with the value of $k^{-1}\times|\alpha_1-\alpha_3|$ at zero energy, deduced from Panofsky's measurements of the absorption cross section of slow negative π mesons in hydrogen or from the photomesic production near threshold. From Eqs. (37) and (52), we find, using the value (57) of $G^2/4\pi$:

$$|(\alpha_1-\alpha_3)/k|\sim 25° \text{ per } 100 \text{ Mev}/c, \tag{58}$$

to be compared with the experimental value[22]

$$|(\alpha_1-\alpha_3)/k|_{\text{exp}}\sim 8° \text{ per } 100 \text{ Mev}/c. \tag{59}$$

The reasons for this discrepancy are that the slope of α_1 at low energy is still rather high, whereas α_3 varies linearly with k near $k=0$. It has been suggested, from the analysis of low-energy $\pi-p$ scattering[23] that α_3 should actually vary like k^ν, with $\nu>1$, near zero energy. This effect might come from the influence of meson-meson scattering or, perhaps, simply from higher order meson-nucleon diagrams. In this connection, it is worth remarking that the special choice of higher order diagrams which is made by iterating diagrams (a) and (b) of Fig. 1, violates the "symmetry theorem" of Gell-Mann and Goldberger,[24] which states that, in an exact calculation, α_1 and α_3 should be equal in the limit $k\to 0$ and $\mu/M\to 0$. In other words, the difference in slopes of α_1 and α_3 near the origin should only be a μ/M effect. When the limit $\mu\to 0$ and $k\to 0$ is made in our Eqs. (37) and (52), one finds, by putting

$$z=\frac{1}{2\pi}\left(\frac{G^2}{4\pi}\right), \tag{60}$$

the following values of α_1 and α_3:

$$\left.\begin{aligned}\frac{\alpha_1}{k}&=\frac{2\pi}{M}\frac{z(\frac{9}{4}z-1)}{(1-z)(\frac{3}{2}z+1)}\\ \frac{\alpha_3}{k}&=-\frac{2\pi}{M}\frac{z}{1+2z}\end{aligned}\right\}(k\to 0). \tag{61}$$

The two limits are equal when z is very small, but with our value of $G^2/4\pi$, we have $z=0.60$, so that a large part of the result (58) comes actually from effects which should be absent if all higher order diagrams had been taken into account. An investigation of higher order diagrams, selected in such a way as to respect the symmetry rule, is now in progress at the University of Paris.

(c) It can be seen that the approximation (29) is equivalent to introducing a kind of "natural" cut-off function in the integral equations (28) or (41), and that the corresponding cut-off momentum is of the order of $M+P_0\geq 2M+\mu$. This is a very high cut-off momentum, which agrees with considerations which have already been made[25] in the study of nuclear forces, where a central repulsive core at $r_c\sim 0.50\times 10^{-13}$ cm can be seen to result from a "natural" cut-off momentum of the order of $2M$.

V. ACKNOWLEDGMENTS

Most of the present work has been done during the summer of 1954, while the author was on a visit at the Physics Department of the University of Rochester. He therefore would like to thank Professor R. E. Marshak for his kind hospitality and for many stimulating discussions on this and related problems.

[22] This value results from measurements of the photomesonic production cross section near threshold, at the Massachusetts Institute of Technology and Cornell University: B. T. Feld and R. R. Wilson (private communications to R. E. Marshak).

[23] H. A. Bethe (private communication).

[24] M. Gell-Mann and M. Goldberger (to be published).

[25] M. Lévy, Proceedings of the International Conference of Kyoto, 1953 (unpublished); M. Lévy and R. Marshak, Proceedings of the Glasgow Conference of Nuclear Physics, 1954 (to be published).