with respect to ϕ. The real difficulty in treating (C.24) is that of extracting the self-energy and Green's function renormalization from the last factor, without expanding the log in a Taylor series. We have checked that the first few terms are in agreement with the corresponding expressions obtained by expanding our previous results for the renormalization constants. We can recognize the Green's function renormalization as a multiplicative constant and the self-energy from its form

$$\exp[-i\delta m(t-t')].$$

It appears that the meson scattering can be obtained in closed form from the ϕ dependent terms, but we shall omit any further discussion of this matter.

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High-Energy Reactions and the Evidence for Correlations in the Nuclear Ground-State Wave Function*

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High-energy nuclear reactions which depend strongly on nucleon position correlations in the nuclear ground state are analyzed and shown to give evidence for the existence of marked correlation effects. The following high-energy experiments are considered: nuclear photoeffect, meson absorption in nuclei, deuteron pickup, proton-proton scattering in a nucleus, and meson production in proton-nucleus collisions. The corresponding cross sections depend on a nucleon momentum distribution which can be represented at high energies by a single function giving reasonable agreement with all the experiments considered. This momentum distribution differs substantially from that for the shell model of the nucleus and thus provides strong evidence for correlation in the nuclear ground-state wave function.

The transformation methods developed in previous papers are used to provide a unified theory of the above five processes. The momentum distribution predicted by this theory is estimated by two methods each of which gives close agreement with the experimentally determined function in the relevant energy ranges.

I. INTRODUCTION

In the last few years a considerable body of evidence has been accumulated which provides information about the ground state of nuclei. This evidence comes primarily from quite different types of experiment and contains, as we shall show, upon first examination apparent contradictions in the information given about the ground state. One type of evidence, that perhaps is best known, comes from the study of ground and low excited states of nuclei and is encompassed in the very successful shell model theory which has been useful and accurate in predictions and understanding of nuclear properties. We shall not attempt to summarize this evidence on the theory here; we only comment that the central feature of the shell model is the assumption that nucleons move in the independent particle states of a uniform potential. The success of the shell model as usually formulated is very intimately connected with this assumption since the existence of long mean free paths and independent particle motion are reflections of the absence of two-body interactions and of the absence of correlations in the ground-state wave function.

The second body of evidence which has direct bearing on the nuclear ground state comes from high-energy experiments. It is the purpose of this paper to summarize this evidence and show how it may be reconciled with the knowledge of nuclear structure derived from low-energy experiments. We consider the following reactions: deuteron pickup, meson capture, high-energy photonuclear effect, high-energy proton-nucleus collisions, and meson production in high-energy proton-nucleus collisions. These high-energy reactions are all similar in that they provide in effect a method of observation with great resolving power since they allow us to probe nuclear structure with particles of wavelength less than the typical nucleon spacing in a nucleus. Consequently we can expect to resolve details of the structure which are not accessible to us if we restrict ourselves to observations at low energy with particles of large wavelength. As we shall see, the information we obtain from the high-energy experiments is in contradiction with the shell model as usually formulated and requires a change in the interpretation of the low-energy nuclear phenomena and their relation to the ground-state wave function.

This new interpretation has been described in

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previous papers[1] and is further discussed in Sec. III of the present paper with particular reference to the high-energy experimental information which is described in Sec. II.

In the first part of Sec. III we develop the formalism required for relating the nuclear ground-state wave function to the shell model wave function. In the second part of Sec. II this formalism is utilized to give a unified development of the theories of the nuclear photoeffect, meson capture, and deuteron pickup. We find that the cross sections for these processes depend on a momentum distribution function in the same way as has previously been stated, for example by Chew and Goldberger,[2] provided that the energies involved are sufficiently high. However, our momentum distribution function differs in principle from that of Chew and Goldberger because of our use of a different form of the impulse approximation which appears to have more general validity (this method was first used by Heidmann[3]). Finally in Sec. III we note the cross section formulas for high energy, for proton-proton scattering in nuclei and meson production by protons bombarding nuclei, with particular reference to their dependence on a momentum distribution function.

In Sec. IV we apply the theory to the various experiments. Since our momentum distribution appears in the formulas for cross sections in the same way as that used by previous authors, we are able to make use of earlier analyses of experiments to determine the momentum distribution from experiment. It is found that these experiments can be fitted to reasonable accuracy by a suitable Gaussian momentum distribution in the energy range 50 to 100 Mev. We compare this experimental distribution with our theory in two ways, one of these involves the use of a Hulthén wave function for two nucleons, and the second is partly phenomenological in that we insert an experimental value for the two nucleon scattering matrix. It is found that both these methods give a momentum distribution in agreement with the experimental one in the relevant energy range.

Finally in Sec. V we summarize our conclusions from the evidence presented in this paper.

II. EXPERIMENTAL EVIDENCE

A. Deuteron Pickup

This process is the ejection by fast neutrons of fast deuterons from nuclei[4]; the original theory was developed by Chew and Goldberger[2] and later modified by Heidmann.[3] The phenomenon occurs in the following manner: a fast neutron (90 Mev, for example) in passing through a nucleus occasionally encounters a proton with such a momentum that the relative momentum of the neutron and proton can be accommodated in the deuteron wave function. When this occurs, it is possible for the neutron to "pickup" the proton and emerge as a deuteron. It is apparent that the probability of this process is a sensitive function of the momentum distribution of the proton in the ground-state wave function; consequently the empirical observations can be used to deduce properties of the wave function. The theory as it has been developed is only a Born approximation; the calculations of Chew and Goldberger and of Heidmann are somewhat different in form, but both agree on a simple dependence on the ground-state wave function and probably can be used to draw qualitative conclusions. The result given by Chew and Goldberger is that the cross section depends most strongly on a factor

$$N(\mathbf{k})=\left|\int\psi_I(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}\right|^2, \tag{1}$$

where $\psi_I(\mathbf{r})$ is the initial wave function of the picked up proton. Their analysis showed that the experiments were consistent with

$$N(\mathbf{k})=\frac{\alpha}{\pi^2}\frac{1}{(\alpha^2+\mathbf{k}^2)^2}, \tag{2}$$

with $\alpha^2/2m=(18)$ Mev. This distribution departs very markedly from a Fermi gas or from an independent-particle function in that a much stronger admixture of high-momentum components is predicted. Thus the pickup process is evidence for a strongly correlated wave function.

B. Meson Capture from Low Bohr Orbits

This process was first observed by Panofsky[5] and has since then been extensively studied. The theoretical analysis[6] shows that it is possible, from a study of the capture of π mesons at rest in light elements, particularly in hydrogen, deuterium, and carbon, to derive some striking information about nuclear structure. The interesting observed feature of the meson capture is that while the reactions in deuterium,

$$\pi^-+\mathrm{D}\rightarrow 2n,\quad \pi^-+\mathrm{D}\rightarrow 2n+\gamma,$$

occur with a ratio of about 2:1; in carbon, the ratio of the cross sections for the two processes

$$\pi^-+\mathrm{C}\rightarrow \text{star},\quad \pi^-+\mathrm{C}\rightarrow \text{star}+\gamma$$

has changed to a number greater than 65. It is theoretically expected that the γ emission, since it occurs

[1] Brueckner, Levinson, and Mahmoud, Phys. Rev. **95**, 217 (1954); K. A. Brueckner, Phys. Rev. **96**, 508 (1954) and Phys. Rev. **97**, 1353 (1955); K. A. Brueckner and C. A. Levinson, Phys. Rev. **97**, 1344 (1955); R. J. Eden and N. C. Francis, Phys. Rev. **97**, 1366 (1955).

[2] G. F. Chew and M. L. Goldberger, Phys. Rev. **77**, 470 (1950).

[3] J. Heidmann Phys. Rev. **80**, 171 (1950).

[4] The experimental evidence is that obtained by J. Hadley and H. F. York, Phys. Rev. **80**, 345 (1950); K. A. Brueckner and W. Powell, Phys. Rev. **75**, 1274 (1949).

[5] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).

[6] Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951), referred to as BSW in the text.

essentially as a one nucleon process

$$\pi^- + p \rightarrow n + \gamma$$

will be only weakly affected by the other nucleons. It is possible therefore to reach conclusions on the relative transition rates for the nonradiative capture in deuterium and carbon (or in other light and medium heavy nuclei).

The theory of the capture next is used to show that the nonradiative capture rate is a sensitive measure of the probability of finding two closely associated nucleons in the ground state of the nucleus, since the capture involves large momentum transfers and consequently can take place only through the cooperative effect of at least two nucleons. Qualitatively it is obvious that if in heavy nuclei the nucleons were randomly distributed, the probabilities of finding two nucleons close together is small, and the meson capture rate would be small, in contract to the effect observed. The result of a quantitative analysis of the various phenomena affecting the radiative and nonradiative capture showed that the nucleons are indeed highly correlated.

According to BSW, the transition rate depends on a factor $P(z_{\text{Av}})$ which is the probability of finding two nucleons with a separation $z_{\text{Av}} \approx \hbar/\Delta p$, where Δp is the momentum transferred to the two nucleons. For an uncorrelated nucleus, $P(z_{\text{Av}})$ is just the nuclear density $1/v$, where v is the total volume $(4/3)\pi r_0^3 A$. Thus it is useful to define a correlation factor f by the equation

$$P(z_{\text{Av}}) = f \cdot [(4/3)\pi r_0^3 A]^{-1}.$$

The analysis of BSW showed that $f \approx 35$ and thus that the wave function for the nuclear ground state must depart very markedly from that for an uncorrelated system. We will return to an alternative formulation of this result later in Sec. IV.

C. Photonuclear Effect at High Energy

The cross section for the photoejection of high-energy (50–200 Mev) protons from nuclei has been known for some time to be much larger than would be given by an independent-particle model. The presence of a large high-energy component of fast protons indicates quite unambiguously the existence of high-momentum components in the ground-state wave function. The process has been analyzed by Levinthal and Silverman[7] and by Levinger,[8] all in the dipole approximation. The former authors show that the observed cross section is in good agreement with the Chew-Goldberger[2] momentum distribution for the ground-state wave function. The analysis of Levinger[8] uses a somewhat different method of calculation to which we will return later, but he also concludes that the experiment shows the marked departure at high-momentum values of the wave function from that for an uncorrelated system.

[7] C. Levinthal and A. Silverman, Phys. Rev. **82**, 822 (1951).
[8] J. S. Levinger, Phys. Rev. **84**, 43 (1951).

D. Proton-Nucleus Scattering and Meson Production in Proton-Nucleus Collision

These processes,

$$p + \text{nucleus} \rightarrow \text{nucleus} + p',$$
$$p + \text{nucleus} \rightarrow \text{nucleus} + \pi,$$

can be used to give quite quantitative indications of the departures of the ground-state nuclear wave function from that of an independent particle model. In these cases a contribution to the cross section comes from each component of the nucleus ground-state momentum distribution. Thus if the cross section for the elementary process involving free nucleons is known, then the observed cross section is a function only of the momentum distribution. The analyses of Henley[9] and of Wolff[10] show that the momentum distribution is determined with fair accuracy, and is fitted well by a Gaussian

$$|\psi(\mathbf{p})|^2 = \frac{1}{\alpha^3 \pi^{\frac{3}{2}}} \exp(-\mathbf{p}^2/\alpha^2), \tag{3}$$

where α^2 is such that the mean kinetic energy is 19.3 Mev. This distribution does not depart as strongly from an uncorrelated wave function as does the wave function used by Chew and Goldberger[2]; the difference, however, is still very marked, as is shown in Fig. 1.

E. Summary of Experimental Results

All the experiments discussed in this section are similar in that phenomena are observed which depend strongly on strong correlation effects in the coordinate space wave function or, equivalently, on an appreciable admixture of high momentum components in the momentum space wave function. Thus it is evident that the nuclear ground-state wave function cannot describe nucleons moving as independent particles, and reinterpretation of the independent particle model is necessary. In the next section we shall show how this can be done in the general case, using the techniques and concepts we have developed for treating the nuclear ground state. We shall also make detailed applications to specific cases of particular interest.

III. GENERAL THEORY

A. General Description of Method

The processes considered in the previous section have an important common feature; an appreciable contribution to the cross sections comes from an initial nuclear ground state in which at least one nucleon has a high momentum. We make the physical assumption that this high momentum is the result of strong inter-

[9] E. M. Henley, Phys. Rev. **85**, 204 (1952); this paper lists the relevant experimental references.
[10] P. A. Wolff, Phys. Rev. **87**, 434 (1952); see also the experimental results given by Cladis, Hess, and Mayer, Phys. Rev. **87**, 425 (1952).

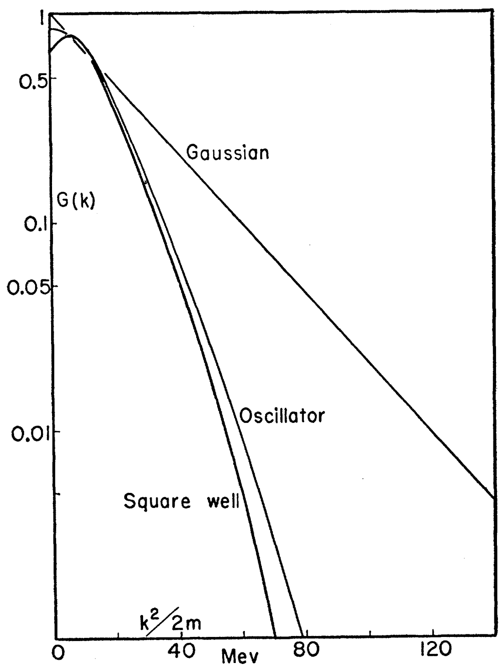

FIG. 1. Momentum distribution $G(k)$ of 8 neutrons and 8 protons in the independent-particle states of a square well with infinite walls and of a harmonic oscillator well. For comparison the Gaussian distribution of Eq. (3) is also given.

actions between a pair of nucleons; indeed the experiments[11] seem to verify this assumption. Consequently the ejection of a fast nucleon by the photoprocess, by meson capture or as a member of the deuteron in deuteron pickup, will usually be associated with the ejection of another fast nucleon which was originally paired with the directly ejected particle. Thus the process corresponds to ejection of a fast pair of nucleons from the ground state, with the residual nucleus only weakly excited. This assumption (or a stronger assumption) is explicit or implicit in the theory of all the high-energy processes we have considered.

It is at this point that the usefulness of the high-energy processes in the study of nuclear structure is particularly apparent. As we will see in the following development of the theory, if the ground-state function is weakly correlated as for a Fermi gas or an independent particle model, then the matrix elements will vanish in the former case or be very small in the latter case. Since the predominant low-momentum components in the wave function make very little or no contribution to the matrix elements, the importance of the high momentum components is greatly enhanced and hence it is possible to get detailed information about this aspect of the wave function. Before proceeding to the theory of these processes, we shall first make some brief remarks on the nature of the ground-state wave function and in particular on our interpretation of the shell model and its reconciliation with the simultaneous success of the shell model and with the high-energy phenomena which interest us.

It is well known from both experiment and theory that the nucleon-nucleon interactions are strong and short ranged. Consequently if the same forces act when nucleons are immersed in a many-body medium, one will very naturally expect to observe under appropriate experimental conditions very appreciable correlations in the nuclear wave function. On the other hand, the success of the shell model has often been assumed to indicate that the two-body forces in nuclear matter are in fact much weaker and long-ranged and can lead in an excellent approximation to a uniform Hartree field acting on the nucleons. The origin of this effect might be, for example, a strongly nonlinear behavior of the meson fields so that a very large damping effect modifies and smooths out the forces in nuclear matter. This effect can arise from many-body forces or from a nonlinearity in the meson field equations. In either case the effective potential felt by one nucleon would not have the rapidly varying spatial dependence which would result if the two-body forces remained strong, and a uniform potential would be a good approximation. A direct consequence would then be that the nuclear wave function would be weakly correlated, in disagreement with the high-energy experiments we are analyzing. It is also perhaps worth commenting here that the theoretical expectations for the character of the two-body and many-body interactions also strongly suggest that the strong two-body forces are still effective in a medium of nuclear density.

Thus we must adopt a picture of strong two-body nuclear potentials and modify our views of the shell model. The concepts and techniques which we have developed are discussed in detail in other papers[1]; the essential points may be summarized in the following way. We require that the shell model (or independent particle) wave function be a description not of nuclear motion but of a "collective particle" motion, the actual nucleon wave function being generated from the "shell model" wave function by a transformation. This transformation has (among other effects) the effect of introducing correlations and hence high momentum components into the wave function. Under certain conditions the behavior of the shell model "particles" is very nucleon-like, but this approximate identification is not generally valid. We see, in fact, a complete breakdown of the approximate description in the region of strong correlations or high momenta, where the departure of the simple shell model states from actual nuclear states becomes particularly marked. Stated in other terms, a consequence of our description of the nucleus is that the departure of shell model states from nuclear states is not very appreciable if observations of the state are

[11] Byfield, Kessler, and Lederman, Phys. Rev. 86, 17 (1952); see also reference 16. The effect of correlations involving more than two nucleons will become apparent only in the high-energy tail of the spectrum of the ejected nucleons, this will not influence appreciably the total cross sections of the processes we consider.

made which depend only on averages over space or time intervals which are large compared with characteristic correlation distances or fluctuation frequencies in the state. In the other extreme, observations at high frequencies or short wavelengths readily detect the departure of the ground state from that for an uncorrelated system.

We next shall summarize the necessary formalism which we need in the following discussions. For further details of notation and explanation the reader should refer to previous papers.[1] The nuclear ground-state wave function $\Psi_0(A)$ is in general a complicated function of the coordinates of the nucleons and contains marked correlations as a result of the strength of the two body interactions. It is related to the "shell-model" or "independent particle" wave function $\Phi_0(A)$ by a transformation function or "model operator" F, i.e.,

$$\Psi_0(A)=F\Phi_0(A). \tag{4}$$

Since $\Phi_0(A)$ is a weakly correlated function (a degenerate Fermi gas for the lowest energy state), F has the effect of introducing correlations into the wave function. Clearly, therefore, explicit knowledge of the transformation F and thus of the wave function $\Psi_0(A)$ is necessary in our problem, depending as it does on the correlations in the wave function. This is in marked contrast to determination of the ground-state energy, for example, where only the transformed Hamiltonian (the independent particle Hamiltonian) need be known and there are no departures of the particle (not nucleon) motion from independent particle motion.

The explicit form of the transformation, which has been used successfully in the considerations of other ground-state properties, is given by the following set of coupled equations:

$$F=1+\frac{1}{e}\sum_{ij} I_{ij}F_{ij}, \tag{5}$$

$$F_{ij}=1+\frac{1}{e}\sum_{lm\neq ij} I_{lm}F_{lm}, \tag{6}$$

where the "energy denominator" e is

$$e=E_0-\sum_i T_i-V_c, \tag{7}$$

and the energy eigenvalue E_0 is determined by

$$(E_0-\sum T_i-V_c)\Phi_0(A)=0. \tag{8}$$

The quantities I_{ij} and V_c are simply related to the two-body scattering operators t_{ij} which are defined by the equations:

$$t_{ij}=v_{ij}+v_{ij}(1/e)t_{ij}, \tag{9}$$

where v_{ij} is the potential between nucleons i and j. The operators I_{ij} are those parts of the t_{ij} which are nondiagonal with respect to the nuclear states; finally, the the uniform potential V_c is defined by

$$V_c=\tfrac{1}{2}\sum_{ij} t_{cij}, \tag{10}$$

where t_{cij} is the diagonal part of t_{ij}.

Some features of these results and of the transformation F are easily seen. The incoherent or nondiagonal operators I_{ij} cause transitions from the uncorrelated independent-particle state $\Phi_0(A)$; the effect is closely analogous to an inelastic scattering of particles a pair at a time out of the Fermi gas to excited states. Consequently the departures of the wave function $\Psi_0(A)$ from $\Phi_0(A)$ are very closely related to the details of the inelastic scattering of nucleon by nucleon and thus to the strength and range of the two-body potentials.

We shall in the following parts of this section make explicit applications of these principles to the high-energy phenomena we have summarized in the previous section. Where the development follows the work done by other authors, we shall abbreviate the discussion where this can be conveniently done.

B. Application to Specific Phenomena

(1) *Nuclear Photoeffect*

The cross section for the production of high-energy protons by γ radiation of a nucleus in dipole approximation is obtained from the matrix element H_{0f} given by

$$H_{0f}=\left(\Psi_f(A),\sum_{i=1}^{Z} e\mathbf{r}_i\cdot\mathbf{A}\Psi_0(A)\right), \tag{11}$$

where $e\mathbf{r}_i$ is the charge moment for the ith proton. We approximate to the final state wave function by making use of our physical assumption that two of the nucleons have high momenta so the wave function is approximately separable, i.e., we assume

$$\Psi_f(A)=\Psi_f(1,2)\Psi_f(A-2). \tag{12}$$

We suppose that nucleon "1" is a proton; the associated nucleon "2" must then be a neutron since the contribution from two protons is zero in the dipole approximation. The assumption of a product wave function is equivalent to the neglect of corrections due to antisymmetrization between the high-momentum and low-momentum particles, but $\Psi_f(1,2)$ and $\Psi_f(A-2)$ will be separately antisymmetrized. With a final wave function of this form we can neglect contributions to H_{0f} from terms not involving the position $\mathbf{r}_1$ associated with proton 1.

We may use the method of partial closure to eliminate the wave function $\Psi_f(A-2)$ from the cross section provided that the rest of the matrix element depends only weakly on the energies of excitation E_f of the residual nucleus. This will be the case if the energies of the ejected particles are high; it is also possible to correct approximately for the error made in the closure sum by altering the energy conservation law to include a mean excitation energy $\bar{E}_f$; i.e., we can set

$$E_0+E_\gamma=k_1^2/2m+k_2^2/2m+\bar{E}_f, \tag{13}$$

where $\bar{E}_f$ can either be estimated or regarded as a phenomenological parameter. Making this approximation, we find

$$\sum_f |H_{0f}|^2=ZN\int d\mathbf{r}_3\cdots\int d\mathbf{r}_A \times\left|\int\int\Psi_f^*(1,2)e\mathbf{r}_1\cdot\mathbf{A}\Psi_0(1,2,\cdots,A)d\mathbf{r}_1d\mathbf{r}_2\right|^2, \tag{14}$$

the factor of NZ coming from the number of ways of choosing the protons (1) and neutrons (2). This result also neglects interference terms which is consistent with our assumptions about the separability of the final state wave function. In this matrix element the final wave function can be determined with considerable accuracy since it is probably safe to assume that this is simply given by the solution to the two-body problem at high energy neglecting the effects of the nuclear medium. Thus only the initial wave function is unknown and its specifications largely determines the matrix element and the cross section.

It might be pointed out at this point that in the assumptions we have just made in treating the final states, the possible attenuation by nuclear collision of the outgoing proton wave have been neglected. One consequence of this assumption is that the calculated cross section will depend on the total number of protons in the nucleus, i.e., that it is a volume effect. The best evidence for this assumption comes from the photoeffect itself[8] and also from the process of meson capture in nuclei as studied by Byfield, Kessler, and Lederman.[11] We shall in this section and also in discussing the remaining phenomena neglect the possibility of absorption of the outgoing particles and as a consequence overestimate the cross sections by a factor depending on the process considered. The appropriate correction factor can usually be estimated without difficulty.

The wave function $\Psi_0(A)$ defined by Eqs. (4) and (5) to (10) is extremely difficult to give explicitly because of its very complicated dependence on the incoherent operators I_{ij}. If, however, we are willing to assume that the necessary correlation is largely a result of two-body interactions, then we can approximate to the function F by expanding it to first order in I_{ij}. Consequently we cannot expect fully quantitative knowledge of the wave function in the important high-momentum region, although the error introduced is probably not enough to alter the order of magnitude of the result. In this approximation, we take

$$\Psi_0(1,2,\cdots,A)=\left(1+\frac{1}{e}\sum t_{ij}\right)\Phi_0(1,2,\cdots,A). \tag{15}$$

We have replaced I_{ij} by t_{ij} in F since here only off-diagonal matrix elements contribute to the cross section. The wave function $\Phi_0(1,2,\cdots,A)$ can be written (treating neutrons and protons separately):

$$\frac{1}{\sqrt{(ZN)}}\sum_{lm}(\phi_l(1)\phi_m(2))\Phi_0^{lm}(3,\cdots,A) \tag{16}$$

with $\Phi_0^{lm}(3,\cdots,A)$ normalized to unity. Only the terms involving $\mathbf{r}_1$ and $\mathbf{r}_2$ will make any important contribution to the cross section, hence after integrating over the variables $\mathbf{r}_3, \cdots, \mathbf{r}_A$ we get

$$\sum_f |H_{0f}|^2=ZN\sum_{lm}\left|\frac{1}{\sqrt{(ZN)}}\int\int\Psi_f^*(1,2)e\mathbf{r}_1 \cdot\mathbf{A}F_{12}\phi_l(1)\phi_m(2)d\mathbf{r}_1d\mathbf{r}_2\right|^2, \tag{17}$$

where l is summed over proton and m over neutron states, and

$$F_{12}=1+(1/e)t_{12}. \tag{18}$$

We define a quantity $\chi_{lm}'(1,2)$ by the equation

$$\chi_{lm}'(1,2)=F_{12}\phi_l(1)\phi_m(2) \tag{19}$$

(the prime distinguishes this from a symmetrized form used later). Then $\chi_{lm}'(1,2)$ will give the high-momentum components for the nucleons 1 and 2 in the nucleus. It is important to note that our approximation method is based on the fact that only high-momentum components will give large contributions to the matrix element so that it has been possible to neglect terms in Eq. (15) which do not refer to particles 1 and 2. For these reasons it would be misleading to regard $\chi_{lm}'(1,2)$ as a kind of two-body wave function in the nucleus.[12] We get now

$$\sum_f |H_{0f}|^2=\sum_{lm}\left|\int\int\Psi_f^*(1,2)e\mathbf{r}_1\cdot\mathbf{A}\chi_{lm}'(1,2)d\mathbf{r}_1d\mathbf{r}_2\right|^2. \tag{20}$$

This can be compared (see Levinger[8]) with the deuteron photoeffect, for which

$$|H_{0f}^D|^2=\left|\int\int\Psi_f^*(1,2)e\mathbf{r}_1\cdot\mathbf{A}\Psi_D(1,2)d\mathbf{r}_1d\mathbf{r}_2\right|^2, \tag{21}$$

where $\Psi_D(1,2)$ denotes the deuteron wave function.

Thus a comparison of the experimentally determined matrix elements for the photodisintegration of the nucleus (with fast proton ejection) and of the deuteron can be used to give us rather directly a relation between the deuteron and ground-state wave functions. This technique is particularly useful if we do not wish (or are unable) to give explicitly the final two-particle wave function $\Psi_f^*(1,2)$ since this enters very nearly as a

[12] A function analogous to our $\chi_{lm}^{(1,2)}$ is introduced on physical grounds by Levinger (reference 8) following a method of Heidmann (reference 3).

common factor into both matrix elements [Eqs. (20) and (21)].

(2) *Meson Capture*

The operator describing the absorption of a meson by a nucleon can be written[6,13]

$$a\phi(1)+b\sigma_1\cdot\nabla\phi(1), \tag{22}$$

where $\mathbf{r}_1$ denotes the coordinate of the nucleon which absorbs the meson, and $\phi(1)$ denotes the meson field at $\mathbf{r}_1$. If we assume the meson to be in an S state, $\phi(1)$ will be sufficiently slowly varying over the nucleus that we can replace it by some mean value ϕ_0, and similarly replace $\nabla\phi(1)$ by a mean gradient written $\nabla\phi_0$.

The matrix element for meson absorption in the nucleus is given by

$$H_{0f}=\sum_i(\Psi_f(A),\,\{a\phi_0+b\sigma_i\cdot\nabla\phi_0\}\Psi_0(A)). \tag{23}$$

We assume as with the photoeffect that two nucleons in the final state have high momenta and hence their wave function is separable from that for the residual (A-2) nucleons. Then

$$\Psi_f(A)=\Psi_f(1,2)\Psi_f(A-2). \tag{24}$$

Write θ_0 for the operator $a\phi_0+b\sigma_1\cdot\nabla\phi_0$, and let

$$\Psi_f(1,2)=F_{12}\Phi_f(1,2), \tag{25}$$

where $\Phi_f(1,2)$ denotes a plane wave state for the two outgoing nucleons. Then, using partial closure to eliminate $\Psi_f(A-2)$ and transforming $\Psi_0(A)$, we get for the nucleon pair (1,2):

$$\sum_f|H_{0f}|^2=\int d\mathbf{r}_3\cdots\int d\mathbf{r}_A\left|\int\int\Phi_f^*(1,2)(F_{12}{}^\dagger\theta_0)\times F_{12}\Phi_0(1,2,\cdots,A)d\mathbf{r}_1d\mathbf{r}_2\right|^2. \tag{26}$$

Hence

$$\sum_f|H_{0f}|^2=\frac{2}{A^2}\sum_{lm}\left|\int\int\Phi_f^*(1,2)(F_{12}{}^\dagger\theta_0)\chi_{lm}(1,2)d\mathbf{r}_1d\mathbf{r}_2\right|^2, \tag{27}$$

where the $\chi_{lm}(1,2)$ is properly antisymmetrized, i.e.,

$$\chi_{lm}(1,2)=(1/\sqrt{2})F_{12}\{\phi_l(1)\phi_m(2)-\phi_m(1)\phi_l(2)\}. \tag{19a}$$

It is useful to compare this with the corresponding process for meson absorption in the deteron for which

$$|H_{0f}{}^D|^2=\left|\int\Phi_f^*(1,2)(F_{12}{}^\dagger\theta_0)\Psi_D(1,2)d\mathbf{r}_1d\mathbf{r}_2\right|^2. \tag{28}$$

One result is that it is possible, if we wish to interpret the operator $(F_{12}{}^\dagger\theta_0)$ as a phenomonological meson absorption operator, to use the experiment to deduce relationships between the nuclear wave function and the deuteron wave function $\Psi_D(1,2)$. This method was used by BSW[6] in their analysis.

(3) *Deuteron Pickup*[14]

The Born approximation matrix element for this process is

$$H_{0f}=(\Psi_f(0,1,2)\Psi_f(A-2),\,V(\mathbf{r}_0-\mathbf{r}_1)e^{i\mathbf{k}_0\cdot\mathbf{r}_0}\Psi_0(A)). \tag{29}$$

Then following the same techniques as those used in the previous paragraphs, we can bring this to the form

$$\sum_f|H_{0f}|^2=\int d\mathbf{r}_3\cdots\int d\mathbf{r}_A\left|\int\Psi_f^*(0,1,2)V(\mathbf{r}_0-\mathbf{r}_1)\times e^{i\mathbf{k}_0\cdot\mathbf{r}_0}F_{12}\Phi_0(1,\cdots,A)d\mathbf{r}_0d\mathbf{r}_1d\mathbf{r}_2\right|^2. \tag{30}$$

If we assume

$$\Psi_f(0,1,2)=e^{i\mathbf{K}(\mathbf{r}_0+\mathbf{r}_1)/2}\phi(\mathbf{r}_0-\mathbf{r}_1)e^{i\mathbf{k}_2\cdot\mathbf{r}_2}, \tag{31}$$

where

$$\Psi_0(0,1)=e^{i\mathbf{K}(\mathbf{r}_0+\mathbf{r}_1)/2}\phi(\mathbf{r}_0-\mathbf{r}_1) \tag{32}$$

is the deuteron wave function, we obtain

$$\sum_f|H_{0f}|^2=\frac{2}{A^2}\sum_{lm}\left|\int e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}e^{-i\mathbf{k}_2\cdot\mathbf{r}_2}\chi_{lm}(1,2)d\mathbf{r}_1d\mathbf{r}_2\right|^2\times\left|\int\phi^*(\mathbf{r})V(\mathbf{r})e^{i(\mathbf{k}-\mathbf{K}/2)\cdot\mathbf{r}}d\mathbf{r}\right|^2. \tag{33}$$

The first factor corresponds to the momentum distribution for nucleons 1 and 2 in the nucleus. In our notation nucleon 1 is picked up and goes off as part of the deuteron $\Psi_0(0,1)$ and nucleon 2 is a recoil nucleon which we have represented by a plane wave. The second factor can be evaluated by substituting an explicit form for the relative coordinate part $\phi(\mathbf{r})$ of the deuteron wave function and using the Schrödinger equation to rewrite $V(\mathbf{r})$ in terms of the binding energy and momenta.

(4) *Proton-Proton Scattering in Nuclei and Meson Production by Protons Bombarding Nuclei*

For a high-energy incident proton ejecting a second proton from a nucleus it is a good approximation to suppose that the incident proton collides only with the one nuclear particle.[10] The same approximation can be made in considering the production of mesons by a proton colliding with a nucleus.[9] It is worth noting that both these processes are possible if the second proton (nucleon) was not bound in the nucleus but was free. This has the effect that for a limited range of the momenta of the outgoing particles a large contribution to the cross section will come from considering the

[13] See, for example, K. A. Brueckner and K. M. Watson, Phys. Rev. **83**, 1 (1951).

[14] The theory in this paragraph is similar to that of Heidmann (reference 3) except (1) we derive the correlations in the ground state wave function by transformation theory which allows for the effects of the nuclear medium whereas Heidmann obtains them as physical grounds, and (2) we use partial closure to eliminate the final state.

nucleus to be a Fermi gas, i.e., from taking the unit term in Eq. (5) which gives the transformation of the a Fermi gas, i.e., from taking the unit term in Eq. (5) which gives the transformation of the Fermi gas wave function to the actual nuclear wave function. However, if one looks instead at momentum regions which require a large momentum value for the proton initially in the nucleus, the principal contribution to these cross sections will come from matrix elements obtained by methods analogous to those considered in the previous sections. These involve the quantity $\chi_{lm}(1,2)$ and lead to a cross section which depends on a known factor (or one which can be found by comparison with experiment) and the momentum distribution. The analysis of p-p scattering in the nucleus has been given by Wolff,[10] and that of meson production in the nucleus by Henley.[9] We will not repeat these calculations using our methods as they would involve a lengthy analysis of approximations which are made by physical arguments. However, for comparison we note the formula for the relevant cross section given by Wolff and by Henley.

For proton-proton scattering in the nucleus, Wolff[10] obtains the cross section for an incoming proton of momentum p and final nucleons of momentum q and s:

$$\frac{d^2\sigma}{d_A dq}=\frac{4\pi M^2}{\hbar^4 p}\sum|a|^2\int\frac{d\mathbf{k}}{(2\pi)^3}N(\mathbf{k})\times\delta\left(p^2-q^2-(\mathbf{p}+\mathbf{k}-\mathbf{q})^2-\frac{2M}{\hbar^2}B_{if}\right)\cdot\frac{q^2}{(2\pi)^3}, \quad (34)$$

where $\mathbf{k}=(\mathbf{q}+\mathbf{s}-\mathbf{k})$ and $N(k)$ is the momentum distribution for a nucleon in the nucleus. The detailed form of $N(k)$ as predicted by our method will be discussed in the next section.

The cross section for the creation of a π meson of momentum q is given by Henley[9]:

$$\frac{d^2\sigma_A{}^0(\pi^+)}{dTd\Omega_q}=\frac{1}{v_0}\int\left[Z\frac{d^2\sigma(p-p)}{dTd\Omega_q}v_R+(A-Z)\frac{d^2\sigma(p-n)}{dTd\Omega_q}v_R\right]\rho(\mathbf{k})d\mathbf{k}, \quad (35)$$

where $\rho(\mathbf{k})$ is the momentum distribution for the struck nucleon in the nucleus, and $\sigma(p-p)$, $\sigma(p-n)$ denote the cross sections for collisions of free nucleons; v_0 and v_r represent the relative velocities of the target nucleus and nucleon with respect to the incident nucleon; T is the meson kinetic energy.

IV. APPLICATION OF THE THEORY TO EXPERIMENT

A. Dependence of the Theoretical Cross Sections on a Momentum Distribution in the Nucleus

In discussing the high-energy phenomena, we have shown that in each case the matrix element depends most strongly on the ground-state wave function through a term

$$f(\mathbf{k}_1,\mathbf{k}_2)=\frac{2}{A^2}\sum_{lm}\left|\int(2\pi]^{-3}e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}e^{-i\mathbf{k}_2\cdot\mathbf{r}_2}\times\chi_{lm}(\mathbf{r}_1\mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2\right|^2, \quad (36)$$

where k_1 and k_2 are the final momenta of the two fast ejected nucleons and χ_{lm} is, in the region of high momenta, an approximation to the initial wave function of the two nucleons. This result can be brought to a simpler form if we introduce relative and center-of-mass coordinates and further assume that

$$\chi_{lm}(\mathbf{r}_1,\mathbf{r}_2)=v^{-\frac{1}{2}}\chi_{lm}(\mathbf{r}_1-\mathbf{r}_2)e^{i(\mathbf{k}_l+\mathbf{k}_m)\cdot(\mathbf{r}_1+\mathbf{r}_2)/2}. \quad (37)$$

This is equivalent to the reasonable assumption that the center-of-mass moves with the momenta characteristic of the Fermi gas and that the departures from a Fermi gas occur only in the dependence on the relative coordinate. Making this approximation we find

$$f(\mathbf{k}_1,\mathbf{k}_2)=\frac{2}{A^2}\sum_{lm}\left|\int(2\pi)^{-\frac{3}{2}}e^{-i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}/2}\chi_{lm}(\mathbf{r})d\mathbf{r}\right|^2\times\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_l-\mathbf{k}_m), \quad (38)$$

where we have replaced the Kronecker delta function giving total-momentum conservation by the Dirac delta function, using the relation

$$(\delta_{\mathbf{k}_1+\mathbf{k}_2,\mathbf{k}_l+\mathbf{k}_m})^2=[(2\pi)^{\frac{3}{2}}v^{-\frac{1}{2}}]^2\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_l-\mathbf{k}_m), \quad (39)$$

a further simplification is possible if we assume (as we have previously) that k_1, $k_2 \gg k_l$, k_m and consequently that the momentum conservation gives approximately $\mathbf{k}_1=-\mathbf{k}_2$. In this approximation we can also relax the restrictions arising from momentum conservation and sum freely over l,m. This is related to the approximation made in using partial closure and as there we can introduce a correction by altering the energy conservation law to take account of a mean excitation of the residual nucleus.

We thus are led (dropping the delta function) to the final result

$$g(\mathbf{k}_1)=f(\mathbf{k}_1,-\mathbf{k}_1)\approx\frac{2}{A^2}\sum_{lm}\left|\int\frac{e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}}{(2\pi)^{\frac{3}{2}}}\chi_{lm}(\mathbf{r})d\mathbf{r}\right|^2. \quad (40)$$

The summation over the $A^2/2$ pairs of states lm is equivalent (with the factor $2/A^2$) to averaging over a Fermi gas. This result is now closely analogous to the results used by Chew-Goldberger, Levinthal-Silverman, Henley, and Wolff, who have all deduced a dependence of the cross section on a factor which they call the Fourier transform of the wave function of the *single* particle involved; i.e., they introduce a function $N(\mathbf{k})$

$$N(\mathbf{k})=\left|\int \frac{e^{-i\mathbf{k}\cdot\mathbf{r}}}{(2\pi)^{\frac{3}{2}}}\phi(\mathbf{r})d\mathbf{r}\right|^2, \qquad (41)$$

where $\phi(\mathbf{r})$ is assumed to be the particle wave function. Our result differs however in that $\chi_{lm}(\mathbf{r})$ is a function depending on the relative coordinate for a pair of nucleons and in that an average is to be carried out over the initial state of the two ejected nucleons.

B. Determination of the Function $g(\mathbf{k})$ from Experiment

We shall not attempt to make explicit calculations for the various phenomena; instead we shall make use of the empirical determinations of the function $N(\mathbf{k})$ which have been made. In the case of deuteron pickup,[15] proton-proton scattering in nuclei,[10] and meson production by protons as nuclei,[9] it has been shown that a function

$$N(\mathbf{k})=\exp(-k^2/\alpha^2)(\alpha^2\pi^{\frac{3}{2}})^{-1}, \qquad (42)$$

with $\alpha^2/2m\approx 14$ Mev gives a satisfactory fit to the experiments. We have also made an analysis of the photoproton data similar to that made by Levinthal and Silverman[7] using this Gaussian distribution. The results are shown in Fig. 2 which indicates that the shape of the predicted distribution agrees well with experiment. The magnitude of the predicted cross sections is considerably larger than the results of Levinthal and Silverman[7] and a factor of 10 has been introduced to renormalize the predicted cross section. The discrepancy is not so large, however, if comparison is made with the results of Keck[16] and Walker[17] which are a factor of 5 to 7 larger. Consequently we conclude that the Gaussian momentum distribution which is identical with that used in the other analyses also approximately fits the photoproton data, at least as well as the accuracy of the treatment would lead one to expect.

The remaining process of meson capture can also be easily brought into a form which allows comparison with the assumed Gaussian distribution. This is most simply done if we make use of the result of BSW[6] which shows that the ratio

$$\left|\int e^{-i\mathbf{p}\cdot\mathbf{r}}\phi(r)d\mathbf{r}\right|^2\Big/\left|\int e^{-i\mathbf{p}\cdot\mathbf{r}}\psi_D(\mathbf{r})d\mathbf{r}\right|^2\geqslant 2.39, \qquad (43)$$

where $\psi_D(\mathbf{r})$ is the deuteron wave function and p is the momentum carried off by one nucleon $[p\sim(M\mu)^{\frac{1}{2}}]$. Using for ψ_D a Hulthén wave function with standard

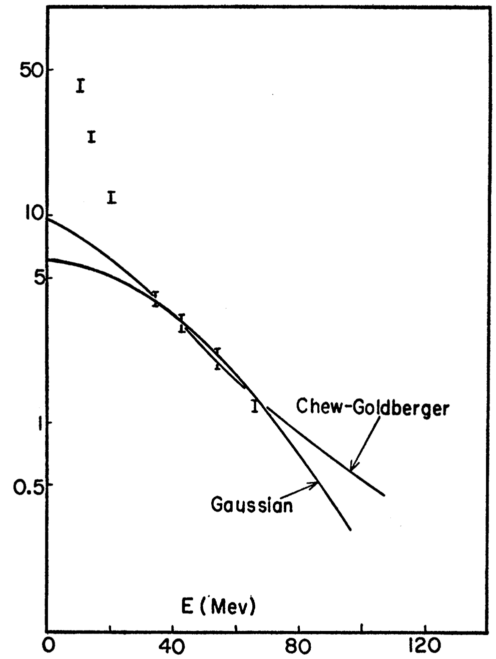

FIG. 2. Theoretical predictions for the photoproton process based on a Chew-Goldberger and on the Gaussian distribution of Eq. (3). The experimental data are taken from the paper of Levinthal and Silverman (see reference 7). The theoretical results are normalized to the experiments at 41.5 Mev by a factor of 0.75 for the Chew-Goldberger and 0.15 for the Gaussian.

parameters, and taking for $\phi(\mathbf{r})$ a normalized Gaussian $N_\beta\exp(-\beta^2r^2/2)$ it is possible to determine a value for $\beta^2/2M$ which agrees with the empirically determined ratio. The result is that $\beta^2/2M\geqslant 14.4$ Mev which agrees very well with the results of the other experiments.

We therefore can well represent the results of all of the experiments by the same momentum distribution. Returning to our definition of $g(k)$ [Eq. (40)], we thus find that if our theory predicts correctly the result,

$$\frac{2}{A^2}\sum_{lm}\left|\int \frac{e^{-i\mathbf{p}\cdot\mathbf{r}}}{(2\pi)^{\frac{3}{2}}}\chi_{lm}(\mathbf{r})d\mathbf{r}\right|^2=N_\alpha\exp(-p^2/\alpha^2), \qquad (44)$$

for high values of the momentum p (corresponding to energy per particle of 75 to 150 Mev), then we can also expect to find good agreement with the high-energy experiments.

C. Calculation of Momentum Distribution

In this section we shall attempt to determine the function $g(k)$ defined in Eq. (40). As we shall show, there are two rather different methods which lead to essentially the same result. Let us consider the function $\chi_{lm}(\mathbf{r})$, which is the wave function for the relative coordinate $\mathbf{r}$. We first approximate this in a manner similar to that used by Levinger and by Heidmann,

[15] The use of a Gaussian distribution to fit deuteron pickup has been tested by Henley (see reference 9).
[16] J. Keck, Phys. Rev. 85, 410 (1952).
[17] D. Walker, Phys. Rev. 81, 634 (1951).

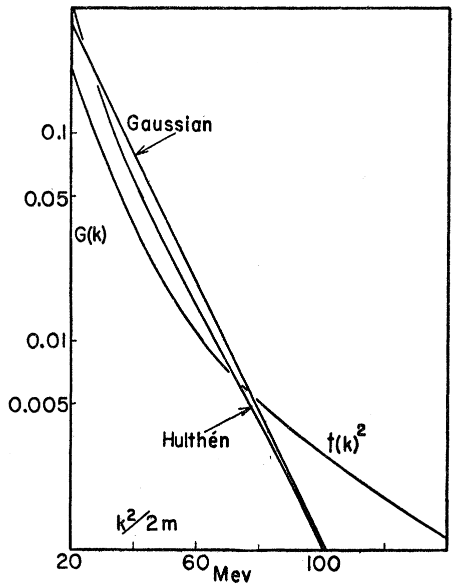

FIG. 3. Calculated momentum distributions for particles in the nucleus compared with the empirically derived Gaussian of Eq. (3). The results of the two methods discussed in the text are given, labelled Hulthén for the result of Eq. (48) and t^2 for the result of Eq. (52).

i.e., we write for the S-wave part of the function

$$\chi_{lm}(\mathbf{r})=\frac{1}{v^{\frac{1}{2}}}\left[\frac{\sin(kr+\delta)}{\sin\delta}-f(r)\right]\Big/\left[r(\alpha^2+k_{lm}^2)^{\frac{1}{2}}\right], \quad (45)$$

which is correctly normalized in the nuclear volume v. $1/\alpha$ is the scattering length; $f(r)$ is a function which is equal to one at the origin and depends in detail on the explicit choice of the potential. At small distances this can also be written as

$$\chi_{lm}(\mathbf{r})=\frac{1}{v^{\frac{1}{2}}}\left[e^{-\alpha r}-f(r)\right]/\left[r(\alpha^2+k_{lm}^2)^{\frac{1}{2}}\right]. \quad (46)$$

For our simple example we take for $f(r)$ the result appropriate to a Hulthén function

$$f(r)=e^{-\beta r}. \quad (47)$$

We use the Hulthén function for the following reasons: (i) the Hulthén function gives a cross section which is of approximately the correct magnitude at the high energies we consider, (ii) the major contribution to the cross section from the Hulthén potential comes from S-wave scattering, (iii) a repulsive core potential would require calculations for at least one higher partial wave since the S-wave is anomalously small in the energy region of interest. In addition it would lead to a result which would be very sensitive to the potential shape in the region of the core boundary where little is known about its detailed form.

Inserting the wave function of Eq. (46) into Eq. (44), we find

$$g(\mathbf{p})=\frac{2}{A^2}\sum_{lm}\frac{(4\pi)^2}{(2\pi)^3}\frac{A}{v}\left[\frac{(\beta^2-\alpha^2)}{(p^2+\beta^2)(p^2+\alpha^2)}\right]^2(\alpha^2+k_{lm}^2)^{-1}. \quad (48)$$

We approximate to the average over the Fermi gas by using Levinger's result $\langle(\alpha^2+k_{lm}^2)^{-1}\rangle_{\text{Av}}\approx 4/k_F^2$, where k_f is the maximum momentum in the Fermi gas. This result is given in Fig. 3 together with the Gaussian.

The second method which we use makes use to a greater extent of knowledge of the scattering cross sections. We go back to the original definition of $\chi_{lm}(r)$ and write

$$g(\mathbf{p})=\frac{A}{v}\frac{2}{A^2}\sum_{lm}\frac{1}{(2\pi)^3}\left|\int e^{-i\mathbf{p}\cdot\mathbf{r}}\left(r\left|1+\frac{1}{e}t\right|r'\right)\times\phi_{lm}(r')dr'dr\right|^2. \quad (49)$$

Using a relative momentum plane wave function for ϕ_{lm}, performing the integrations, and making use of the fact that the energy denominator "e" is simply $E-p^2/M$ if we neglect interactions of the final particles, we find

$$g(\mathbf{p})=\frac{A}{v}\frac{2}{A^2}\sum_{lm}\frac{1}{(2\pi)^3}\left[(\mathbf{p}|t|\mathbf{k}_{lm})^2/(E-p^2/M)\right]^2. \quad (50)$$

This can be evaluated in a very simple way if we assume that $(\mathbf{p}|t|\mathbf{k}_{lm})$ depends only on the momentum difference (as it would in Born approximation for example) and determine it experimentally. Making this assumption and making use of the relation between t and the scattering cross section,

$$t^2(\mathbf{p}-\mathbf{k}_{lm})=\left(\frac{4\pi}{M}\right)^2\frac{d\sigma(\mathbf{p}-\mathbf{k}_{lm})}{d\Omega}, \quad (51)$$

we find

$$g(\mathbf{p})=\frac{A}{v}\frac{1}{(2\pi)^3}\left(\frac{4\pi}{p^2}\right)^2\frac{d\sigma(\mathbf{p})}{d\Omega}, \quad (52)$$

where we have made use of $p^2/M\gg E$ and $k_{lm}\ll p$ to carry out the summation. To evaluate this result, we take the average differential scattering cross section at 90 degrees in the center-of-mass system for neutron-proton and proton-proton collisions, choosing an energy which gives the correct momentum transfer. The result is given in Fig. 3.

It is apparent that either of the methods we have used gives essentially the same results and that the agreement with the empirically derived Gaussian distribution is satisfactory over the energy range of primary interest. Therefore we can, as emphasized in the first part of this section, conclude that these methods will give correct order-of-magnitude predictions for the high-energy phenomena we have described.

V. CONCLUSIONS

We have analyzed evidence derived from a variety of high-energy experiments which has bearing on the problem of nuclear structure. This evidence is particularly significant since it is for these (or similar) processes that the possible departure of the nuclear ground-state wave function from an independent-particle wave function is most apparent. The result predicted uniformly by the group of quite diverse experiments which we have examined is that the nuclear ground-state wave function must have a very marked admixture of high-momentum components and hence must depart quite appreciably from an independent-particle-model wave function. Consequently it follows that the usual assumptions of the shell-model theory of the nucleus, that the particles move independently in a uniform potential, cannot be other than very approximately correct.

To investigate quantitatively the general conclusion drawn from experiment, we have made use of methods recently developed by us in other studies of the nucleus. These methods lead to a nuclear model which appears to agree well with many general details of the structure of the nucleus and also with the detailed properties predicted by the shell model, but does not assume the existence of the independent nucleon motion. An essential assumption of this method is that the nuclear forces acting between nucleons in dense nuclear matter are still very nearly the same as those acting between free nucleons and hence very strong and short ranged. An immediate consequence of this assumption is that the presence of marked correlation effects in the nuclear wave function is to be expected. Conversely, the experimental observation of such correlation effects implies directly that the strong two-body forces have not been "damped out" by nonlinear effects or cancelled by many-body effects.

In applying this nuclear theory to the study of the high-energy phenomena, we have made several simplifying assumptions to bring the theory into easily manageable form. The most important assumption made is that in the very-high-momentum region the effects which interest us are primarily due to the intimate association of pairs of nucleons. In this form our methods are analogous to those used by Heidmann and by Levinger although their origin and interpretation is rather different. The final expressions for the momentum distribution have been evaluated by making use of two quite different approximations which give results in reasonable agreement with each other and with the momentum distribution derived from experiment. It is to be emphasized that the theoretical predictions are very sensitive to the choice of the two-body interaction, which we have assumed to be identical with that acting on free particles. Thus we have shown not only that appreciable correlation effects are present but also that the presence of other nucleons in the dense nuclear matter cannot appreciably modify the two-body interactions.

In conclusion we would like to remark that although we have in this paper emphasized the departure of the nuclear wave function from an independent nuclear wave function, the great importance of the departure is manifested only in the specific high-energy processes (or similar cases) we have considered. As we have pointed out in other papers, the effects are not nearly so pronounced in many low-energy phenomena where the independent particles of the shell model can be identified more closely with nucleons.