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High-Energy Cross Sections. II. Nucleon-Nucleon Cross Section at Cosmic-Ray Energies*

ROBERT W. WILLIAMS
Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

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Cosmic-ray measurements are capable of yielding reliable results for the cross section of a nucleus for proton or neutron collisions involving a not too small energy transfer. This cross section should therefore be less than, or at most equal to, the true nonelastic cross section (reaction cross section). Results of recent cosmic-ray work are assembled and compared with the reaction cross sections measured at 1.4 Bev with the Brookhaven Cosmotron; it is found that the cosmic-ray cross sections are significantly *larger*, even for Pb. Assuming a nonuniform distribution of the density of nuclear matter, one can explain this surprising effect as the result of an increase in the elementary nucleon-nucleon cross section with energy. It is shown that the elementary cross section (the average of σ_{pp} and σ_{np}) must be $(120_{-20}^{+30})\times10^{-27}$ cm^2 in the neighborhood of 30 Bev.

INTRODUCTION

ALTHOUGH secondary cosmic radiation is an inconveniently heterogeneous beam to use as a source of high-energy particles for quantitative cross section work, a number of experimenters have refined the technique of one particular type of cosmic-ray measurement to the point where it is relatively straightforward and accurate. In the experiments we discuss, the quantity measured is the nonelastic or "reaction" cross section (usually called the collision cross section in cosmic-ray work) and the results should therefore be directly comparable with the Brookhaven reaction cross sections discussed in the preceding paper.[1] A systematic difference exists, however, and in a surprising direction: the cosmic-ray cross sections are *too large*. The experimental procedure for the cosmic-ray measurements is discussed in the Appendix, where it is pointed out that the only known important systematic error would make the cross sections *too small*.

ANALYSIS

In Table I the data of various experimenters[2–8] are collected, "best" values selected as discussed in the Appendix, and the results compared with the Brookhaven cross sections. It is seen that the light elements show the largest effect, but even lead—the most reliable of the cosmic-ray determinations—has a cross section at cosmic-ray energies $\sim$24 percent higher than its value at 1.4 Bev. In Table II some additional measurements on Pb are collected; these show (with rather low statistical precision) an even higher cross section at the highest average energies selected. Since lead is already 96 percent opaque (in the usual optical-model sense) to the 1.4-Bev neutrons of the Brookhaven experiment,[2] an increase in opacity is not enough to account for the effect. We are rescued from this dilemma by observing that a realistic nuclear density distribution, such as that used in the preceding paper[1] (hereafter called I), has appreciable transparency even for heavy elements, because the outer region of the nucleus has a low density and therefore a small absorption coefficient. The observed increase in cross section with energy is caused by an increase in the effective nucleon-nucleon cross section $\bar{\sigma}$. From the calculated opacity curve and the nuclear size determination described in I, one can determine the ratio of $\bar{\sigma}$ at about 30 Bev to $\bar{\sigma}$ at 1.4 Bev. Since $\bar{\sigma}$ should be approximately the average of σ_{np} and σ_{nn}, we have the possibility of determining the elementary nucleon-nucleon cross section, σ_{nucleon}, in a new energy region.

The approximations involved in the high-energy semiclassical calculation of the reaction cross section, σ_r, are discussed in I. The basic assumption of the model is that the nucleons may be treated as though they were independent; this assumption should be best for the outer, low-density region of the nucleus, which is just the region involved in the increase of opacity with $\bar{\sigma}$. Thus the general scheme of calculation is fairly reliable, and the principal uncertainty is the *shape* of the effective nuclear density distribution; we have only an approximate knowledge of the true distribution, and, as we have shown, this distribution is altered (though not greatly) by the effect of the finite range of interaction. We have therefore computed the opacity curve for five different shapes of the effective density, as follows:

(1) uniform
$$\begin{cases}\rho_u=\rho_0, & r<R\\ \rho_u=0, & r>R;\end{cases}$$

(2) Gaussian $\quad \rho_g=\rho_0\exp(-r^2/R^2)$;

(3) tapered
$$\begin{cases}\rho_t=\rho_0, & r\leqslant R\\ \rho_t=\rho_0(2r^3/R^3-9r^2/R^2+12r/R-4), & R\leqslant r\leqslant 2R\\ \rho_t=0, & r\geqslant 2R;\end{cases}$$

* This work was supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
[1] R. W. Williams, preceding paper [Phys. Rev. **98**, 1387 (1955)].
[2] Coor, Hill, Hornyak, Smith, and Snow, this issue [Phys. Rev. **98**, 1369 (1955)].
[3] Walker, Walker, and Greisen, Phys. Rev. **80**, 546 (1950).
[4] W. D. Walker, Phys. Rev. **77**, 686 (1950).
[5] H. W. Boehmer and H. S. Bridge, Phys. Rev. **85**, 863 (1952).
[6] Walker, Duller, and Sorrels, Phys. Rev. **86**, 865 (1952).
[7] K. Sitte, Phys. Rev. **77**, 714 (1950).
[8] R. R. Brown, Phys. Rev. **87**, 999 (1952).

TABLE I. Reaction cross sections of nuclei for high-energy protons (σ_p) and neutrons (σ_n) in the cosmic-ray beam, in barns (reported in original sources as collision mean free paths). Errors quoted are standard deviations, and are obtained from the errors given in the original sources. We have assigned a larger error to the weighted means, in consideration of some possible systematic effects. The 1.4-Bev Brookhaven data are included for comparison; the 1.4-Bev values for S and Fe were obtained by interpolation.

Experiment	C σ_p	C σ_n	S σ_p	Fe σ_p	Pb σ_p	Pb σ_n
Walker *et al.*[a]	0.244±0.024	0.250±0.023			2.19±0.17	2.10±0.19
Walker[b] (average)					2.18±0.11	
Boehmer and Bridge[c] (average)		0.240±0.017				
Walker, Duller, and Sorrels[d]	0.262±0.028					
Sitte[e]					2.12±0.13	
Brown[f]	0.307±0.035		0.70±0.10	0.81±0.12		
Corrected mean values	0.256±0.015		0.68±0.10	0.79±0.12	2.15±0.10	
Brookhaven[g] 1.4-Bev neutrons	0.201±0.013		(0.44)	(0.67)	1.73±0.04	

[a] See reference 3.
[b] See reference 4.
[c] See reference 5.
[d] See reference 6.
[e] See reference 7.
[f] See reference 8.
[g] See reference 2.

(4) modified tapered

$$\rho_m=\int \rho_t(r')\frac{\exp(-|\mathbf{r}-\mathbf{r}'|\mu c/\hbar)}{4\pi(\mu c/\hbar)^2|\mathbf{r}-\mathbf{r}'|}d^3r';$$

(5) step

$$\begin{cases}\rho_s=\rho_0, & r<\tfrac{2}{3}R\\ \rho_s=\rho_0/5, & \tfrac{2}{3}R<r<R\\ \rho_s=0, & r>R.\end{cases}$$

The tapered density model (3) was shown in I to be the best[9] description of nuclear matter in medium and heavy nuclei; (4) is the extreme modification of this distribution, for a Yukawa type of interaction range, and is illustrated in Fig. 4 of I.

Opacity curves for the uniform, Gaussian, and tapered distributions are shown on a logarithmic plot in Fig. 1. The step-distribution curve lies between uniform and tapered; the modified-tapered curve lies between tapered and Gaussian; both have been omitted for the sake of clarity of the drawing.

The question now may be asked: by how much must the elementary cross section $\bar{\sigma}$ be increased in order that the opacities $\sigma_r/\pi R^2$ corresponding to Table I will fit the curve? Increasing $\bar{\sigma}$ decreases the mean free path in nuclear matter and moves the points to the left.

TABLE II. Some results of cross-section measurements on Pb, showing a trend toward still larger cross sections as the multiplicity of the detected event and therefore, loosely, the energy of the bombarding particle is increased. Cross section in barns, for protons only.

Experiment	Low	Cross section Multiplicity Medium	High
Walker[a]	1.9±0.1	2.3±0.1	2.3±0.15
Froehlich and Sitte[b]	2.1±0.1	2.3±0.15	2.6±0.2

[a] See reference 4.
[b] See reference 24.

A reasonable fit (see below) to the opacity curve for the tapered nucleus requires $\bar{\sigma}$ to be nearly three times the value at 1.4 Bev, or ~120 mb (1 mb$=10^{-27}$ cm^2). The cosmic-ray points are plotted for this value in Fig. 1. For comparison, the points are dotted in at the positions they would have if $\bar{\sigma}$ were 43 mb, the Brookhaven value.[10] It is clear that even the Gaussian distribution, which was shown in I to be much too radically nonuniform, would demand an increase in $\bar{\sigma}$ of at least a factor of two. The uniform model, as we have already pointed out, cannot fit the cosmic-ray cross sections for any value of $\bar{\sigma}$ and this proves to be true also for the step model.[11]

One sees from Fig. 1 that a small rise in observed cross section, σ_r, corresponds to a large change in $\bar{\sigma}$; the reason is, of course, that most of the nucleons are located in the region of the nucleus that is already black, and the entire change must be due to those which are on the surface of the nucleus. We therefore must expect that the value of $\bar{\sigma}$ determined in this way has a large fractional error due solely to the statistical uncertainty in σ_r. Fitting the points by eye, one can see that they are consistent with a wide range of $\bar{\sigma}$, up to values so high as to be in definite conflict with the direct measurement of Walker, Duller, and Sorrels.[12] One must therefore conclude that the correct value of $\bar{\sigma}$ must not be far above the smallest acceptable value found in this analysis. In Table III we give this smallest value, for the "best" nuclear density distribution (3) and for the two which are more extremely nonuniform, (2) and (4). We arbitrarily set 10 percent probability

[9] According to the high-energy electron scattering, the best fit is obtained by a density which drops off slightly more sharply, but it is clear from Fig. 4 of I that the finite nuclear force range will cause the effective density distribution to be a little smoother.

[10] Strictly, the ratio ($\bar{\sigma}$ at 30 Bev)/($\bar{\sigma}$ at 1.4 Bev) is all that is determined; in I it is argued that $\bar{\sigma}$ at 1.4 Bev is unlikely to be as much as 20 percent lower than the true total nucleon-nucleon cross section, so that the values of $\bar{\sigma}$ given in Table III should not be much affected by this uncertainty.

[11] Of course, the effective density cannot actually be represented by a square well; as pointed out in I, the finite-range-of-interaction effect will always result in some rounding off.

[12] Walker, Duller, and Sorrels, reference 6. This seems to be the only direct measurement of CH_2-C difference which is in a comparable energy range. They report $\sigma_{pp}=54\pm18$ mb, but our analysis raises this to 80 mb (see Discussion).

as defining "smallest acceptable"—i.e., in each case, if the true value of $\bar{\sigma}$ were smaller than that in the table, the probability of obtaining the experimentally observed cross sections[13] for Pb, Fe, and S would be less than 10 percent (calculated from the chi-squared of the experimental points). For the tapered distribution, 5 percent and 30 percent limits are also shown.

From Table III, recalling that the evidence discussed in I favors the tapered distribution, we conclude that the lower limit for $\bar{\sigma}$ is about 100 mb; a value as large as 150 mb seems almost certain to have shown up in the CH_2-C experiments; we therefore take the most likely value as 120 mb, with limits of 100 mb to 150 mb. This is the nucleon-nucleon cross section, averaged over (np), (pp), and (nn) collisions, and over energy in the 15 Bev–50 Bev region. The median energy (discussed in the Appendix) is 30 Bev. In Fig. 2 the variation of the σ_{np} and σ_{pp} with energy is illustrated on a logarithmic energy scale from 0.05 to 30 Bev.

TABLE III. Illustrating the effect of statistics, and of the choice of nuclear density models, on the nucleon-nucleon cross section inferred from the rise in reaction cross sections. For each model, the "10 percent probability" cross section is given. This means that there is only 10 percent probability for the cross section to be less than the value quoted. In addition, 5 percent probability and 30 percent probability values are given for the preferred model (tapered density). "Best" value is ~120 mb (see text).

Model	Gaussian	Modified tapered	Tapered		
Probability limit	10%	10%	5%	10%	30%
Nucleon-nucleon cross section, in millibarns	86	103	108	116	160

DISCUSSION

Our indirect determination of σ_{nucleon} in the 20–50 Bev region shows it continuing the rise which starts in the 1-Bev region. From the attenuation of high-energy cosmic-ray protons in paraffin, Walker *et al.*[12] find by difference a (pp) cross section of 54 mb; however, they use for the carbon subtraction the cross section of Brown, which is 20 percent larger than our "best" cross section (Table I), and which would be quite far off the curve of Fig. 1. Using the carbon cross section of Table I, we find from Walker's paraffin data $\sigma_{pp}=80$ mb. This is still below our lower limit of ~100 mb. A possible explanation is offered in the Appendix, where an effect is pointed out which causes low-density materials (such as paraffin) to yield too small a cross section. Other cosmic-ray experiments using CH_2-C or D_2O-H_2O difference have been reported.[14] They indicate smaller cross sections but in general they refer to lower energies, in the few Bev region (see Appendix). The advantages of the present scheme are two: (1) the "difference" in our case is between one direct cosmic-ray measurement and a high-precision machine measurement, rather than between two cosmic-ray measurements; (2) the number of almost-free nucleons in a metal (on the periphery of the nuclei) per cubic centimeter is actually much greater than the density of H in CH_2.

Present knowledge of the nature of nucleons does not lead to any firm expectation for the behavior of the cross section at very high energies. One would be surprised if it greatly exceeded $\pi\times$(low-energy range of nuclear forces)^2, which is 200 mb for a range of 2.5×10^{-13} cm; this is in the same region as the figure obtained by attributing a "size" of $\hbar/\mu c$ (where μ is the pion mass) to each nucleon, $\pi(2\hbar/\mu c)^2=250$ mb. The figure we obtain, 120 mb, is still well within the reasonable region, although there is some suggestion (Table II) that the cross section may still be rising at ~50 Bev.

Fermi, in formulating his statistical theory of high-energy processes,[15] chose $R=\hbar/\mu c=1.4\times10^{-13}$ cm for the radius of the excited volume, and thus $\pi R^2=63$ mb for the cross section for meson production.[16] The angular

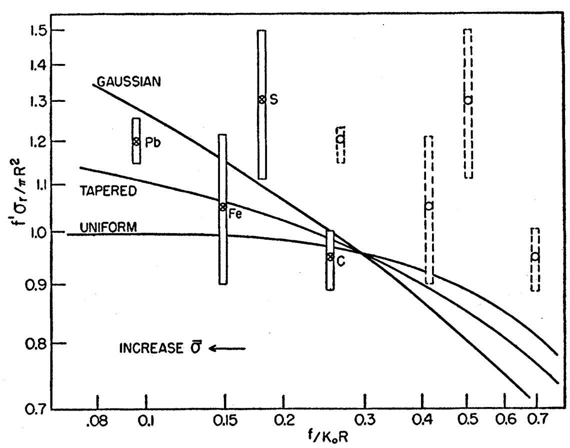

FIG. 1. Opacity, $\sigma_r/\pi R^2$, as a function of $(K_0R)^{-1}$, the mean free path in nuclear matter divided by R, for the uniform density model, on a log-log plot. The corresponding abscissas and ordinates for the tapered and Gaussian models (for which "R" has a different meaning) are multiplied by the appropriate scale factors f and f' so that all three curves refer to the same set of experimental points. The cosmic-ray data of Table I are plotted for $\bar{\sigma}=120$ mb. For comparison, they are also dashed in at the position they would occupy if $\bar{\sigma}=43$ mb, the (np) cross section at 1.4 Bev.

[13] The data on carbon are not included in this analysis because the opacity curves are probably incorrect for such a light nucleus.

[14] R. H. Rediker, Phys. Rev. **95**, 526 (1954); Froman, Kenny, and Regener, Phys. Rev. **91**, 707 (1953); E. Todd, thesis, University of Colorado, 1954 (unpublished).

[15] E. Fermi, Phys. Rev. **81**, 683 (1951).

[16] The question of whether the true total cross section should be larger than this because of possible diffraction scattering is irrelevant, since low-momentum-transfer events do not contribute to the cross section measured by the cosmic-ray techniques. The wave-diffraction picture is not very meaningful, anyway, since at these energies the nucleon must be considered to be a complex particle. It carries a meson cloud whose dimensions are much greater than the wavelength ƛ of its center of mass. Any diffraction-like process would involve a momentum transfer of the same order of magnitude as the momentum of the virtual mesons. It is not clear without further study that this would correspond to elastic scattering of the particle as a whole.

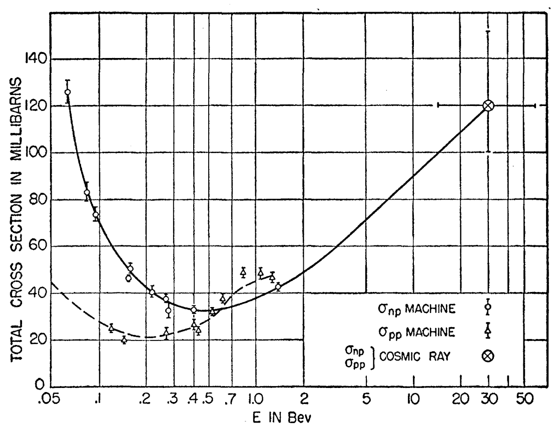

FIG. 2. Energy variation of the nucleon-nucleon total cross section. The low-energy (np) and (pp) cross sections come from various synchro-cyclotron and cosmotron experiments; the point at 30 Bev is the cosmic-ray value inferred in this paper. The flags on the high-energy point have the sense of reasonable limits rather than statistical standard deviations. The continuous curve is included only for convenience.

distribution of mesons is independent of R, in this theory, and the multiplicity varies only as $R^{\frac{3}{4}}$, so that a slow variation of R with energy would not worsen the comparison with experiment. However, it would be somewhat counter to the spirit of the model, which envisions the interaction taking place in a sharply-defined region of space.

Heisenberg[17] has suggested a model for ultrahigh-energy collisions which envisions a nucleon as surrounded by a diffuse meson cloud whose density tapers off like a Yukawa potential. The minimum amount of "overlap" of the two nucleons which is needed to produce mesons is then expected to decrease as the energy increases, leading to a (roughly) logarithmic increase in the cross section with energy The absolute values estimated by Heisenberg are very high—over 300 mb at the energy we deal with here, rising to about a barn at 10^{12} ev—and Cocconi has pointed out[18] that the latter value probably conflicts with photoplate observations. However, the general picture of a diffuse interaction region is supported by the rise in cross section which we have observed.[19]

The character of nucleon-nucleon interactions evidently changes as the energy is raised from 1 or 2 Bev to 20 or 30 Bev. At the lower energy, the various cosmotron results[20] may be summarized by the statement that each nucleon tends to emit "its own" meson; angular effects are therefore quite pronounced.[21] At the higher energy we have seen that the cross section is larger by about a factor of three; moreover, meson production has been found to be rather isotropic[22] (in the c.m. system). Both facts suggest that the strength of the interaction between the two nucleons increases strongly as the energy increases.

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APPENDIX

The cross sections used in this note were all measured by similar methods, which we describe in general terms: the strongly-interacting high-energy component of cosmic rays in the atmosphere—at mountain altitudes about equal numbers of protons and neutrons, with a few percent of pions which we neglect—is detected by demanding an interaction in which several penetrating charged particles are produced (these prove to be mainly pions). Figure 3 shows a schematic cross section of such a detector; C and D are sets of Geiger-Mueller counters immersed in Pb and so arranged that at least two (for example) must be discharged in each tray. The detector will generally have excellent discrimination against mesons and electrons, a rather diffuse but adjustable energy threshold E_{th}, and poor directional properties. If one now defines a beam of protons by an additional set of counters such as A, and attempts to measure the attenuation of the beam as absorber Σ is inserted, he finds only about one-half the expected attenuation. The reason, of course, is that the number-energy spectrum of the protons falls so slowly—as $E^{-1.3}$ —that there are many protons of energy well above E_{th}, and these may not be removed by a collision in the absorber because the collision products can still cause the detector to register a count. This difficulty is overcome[23] by inserting a densely-packed set of counters B between the absorber and the detector, and registering a count only if exactly one counter in B is discharged. Thus high-energy interactions in the absorber will nearly always discharge more than one counter in B; they are treated as "absorptions" even though their products may cause counts in C and D. The counting-rate decrease with absorber thickness therefore yields the cross section.

If a high-energy proton transfers only a small amount of energy to a nucleus so that the secondaries do not reach tray B, the method breaks down and the measured cross section is less than the true reaction cross section. Measurements by Walker *et al.*[6] have shown that this happens only a few percent of the time if the absorber is not too thick; Froehlich and Sitte[24] have also investigated this effect, with similar conclusions.

[17] W. Heisenberg, *Kosmiche Strahlung* (Springer-Verlag, Berlin, 1953), p. 148.

[18] G. Cocconi, Phys. Rev. **93**, 1107 (1954).

[19] Evidence from the diffraction scattering of pions on protons also seems to support this picture [Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955)].

[20] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **95**, 1026 (1954).

[21] A scheme of this sort (but for high energies) is discussed by W. Kraushaar and L. Marks, Phys. Rev. **93**, 326 (1954).

[22] N. Duller and W. D. Walker, Phys. Rev. **93**, 215 (1954).

[23] A detailed discussion of this technique is given by B. Rossi, *High-Energy Particles* (Prentiss-Hall, Inc., New York, 1952), pp. 500 ff.

[24] F. Froehlich and K. Sitte, Phys. Rev. **97**, 151 (1955).

This is the principal systematic error, expected to be a few percent; since it causes the measured cross section to be *less* than the true value, and our arguments are based on the fact that the measured cross section is already *larger* than the Brookhaven value, this error does not concern us further. There are, however, minor corrections in the opposite direction, which in some cases have apparently not been made by the experimenters: the thickness correction for the average path length in absorber (the incident intensity decreases as $\sim\cos^7\theta$); ionization loss in the absorber; spurious effect of knock-on electrons in the absorber (the latter two apply to protons only—modification of the apparatus to use neutrons is fairly obvious). In making the weighted averages of Table I, therefore, we have *decreased* the reported cross sections (by about 2 percent as it turns out) in those cases where we were able to estimate some needed corrections.

The neutron measurements (and some proton measurements) are done without the beam-defining tray, A (Fig. 3); since $\sim$30 percent of the beam comes in at angles $>30°$, this means the absorber must be quite wide, as shown by the dotted lines. Thus for an absorber composed of low-density materials such as paraffin or water, which have a mean free path of two feet, the added absorber will intercept many protons and neutrons which previously missed the detector; the secondaries of some of these will cause spurious counts [ray (*c*) in Fig. 3]. This may account in part for the low values obtained for σ_{np} by difference measurements. One should also remark that the mean energy used in difference experiments is deliberately chosen to be low, so that a high counting rate is obtained; the median energy of the work of reference 14 is probably only a few Bev. At such low energies, the elimination of collisions in the absorber is inefficient, and the cross section (which means only the meson-production part of the total cross section) is not large.

The energy of the protons selected is estimated from the observed counting rate, using a calculated area-solid angle-efficiency product and assuming a known proton spectrum. We have done this very roughly, assuming the efficiency rises linearly from E_{th} to a saturation value of about $\frac{1}{2}$. The proton spectrum for an atmospheric depth of 694 g cm^{-2} was compounded from several results,[25] and may be expressed, in the pertinent energy range, as $N(>E)=4\times10^{-5}(10/E)^{1.3}$ cm^{-2} sterad^{-1} sec^{-1}, E in Bev. Our results for average energies of initiating particles are in general agreement with those of Walsh and Piccioni,[26] who used a quite independent method (based on the latitude effect); their detector had a relatively low E_{th}.

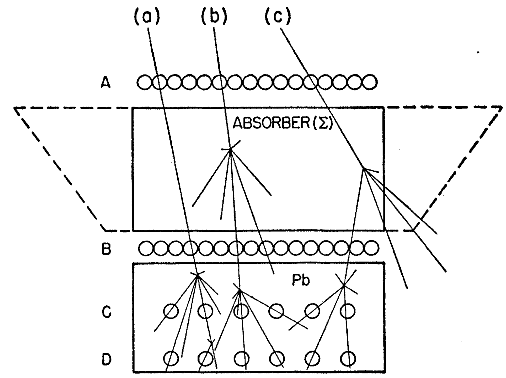

FIG. 3. Schematic diagram (sectional elevation) of the type of apparatus used to measure reaction cross sections in the cosmic-ray beam. Typically the dimensions might be 2 ft×2 ft×2 ft. A, B, C, and D are groups of Geiger-Mueller tubes. (a) A proton passes through the absorber and registers a normal count. (b) A proton interacts in the absorber; no count is registered because two counters in group B are discharged. (c) A proton which would not have registered a count with absorber absent registers a spurious count when absorber is added.

For each experiment we have made an estimate, in this way, of the median energy (which is in a sense the energy of the average event); this ranges from 15–20 Bev for Walker *et al.*[3] to 50–60 Bev for Brown[8]; we therefore use an intermediate value, 30 Bev, as "the" energy, with the remark that appreciable contributions to the flux come from a wide band centered on 30 Bev.

[25] H. S. Bridge and R. H. Rediker, Phys. Rev. **88**, 206 (1952) (contains other references); W. D. Walker (private communication).

[26] T. G. Walsh and O. Piccioni, Phys. Rev. **80**, 619 (1950).