High-Energy Cross Sections. I. The Size of the Nucleus*

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An analysis is made of the recently available measurements of the absorption (reaction) cross sections of nuclei for 1.4-Bev neutrons. The nature of the approximations involved in interpreting such data is discussed, and it is pointed out that these measurements are more directly related to the nuclear density distribution than are the lower-energy measurements. If it is assumed that the density drops off smoothly to zero at the edge of the nucleus, in accord with electron scattering results, it is found that the size of the nucleus determined from these nuclear experiments is in good agreement with the size determined from electromagnetic experiments. This fact indicates that the spatial distribution of the protons is probably equal to that of the neutrons, and cannot be smaller by as much as 1×10^{-13} cm, even for heavy nuclei.

A simple formula is given for the nuclear density distribution; the radius of the *uniform* (square-well) density distribution which yields the same $\langle r^2\rangle_{\text{Av}}$ is $r_0=1.19A^{\frac{1}{3}}\times10^{-13}$ cm; the corresponding square-well radius for nuclear interactions is larger, and depends on the type and energy of observation. Some results are presented of a method for treating the effect of the finite range of interaction of the neutrons.

INTRODUCTION

RECENT results of electromagnetic measurements of nuclear size[1] make it evident that nuclear matter is considerably more dense than neutron and other purely nuclear measurements[2,3] had indicated. This has been suggested[1,4,5] to be a consequence of the fact that the nuclear density must fall to zero gradually rather than abruptly at the edge of the nucleus, and that the nuclear forces have finite range; recently it has also been suggested[6] that the proton distribution may be more highly concentrated than the neutron distribution.

Little quantitative work has followed these suggestions, perhaps because of the complexity of medium-energy nuclear physics. At very high energies the situation becomes simpler because the semiclassical picture of independent nucleons in the nucleus is more nearly valid, and because the experiments yield nonelastic cross sections directly. In the preceding paper,[7] Coor, Hill, Hornyak, Smith, and Snow show that the cross sections measured at Brookhaven in the 1.4-Bev neutron beam lead, with the standard analysis, to a nuclear radius which is smaller than the radius deduced from the lower energy nuclear measurements. In other words the Brookhaven results nearly agree with the electromagnetic radii, without taking into account the tapering of the nuclear density distribution or the finite range of interaction of the neutrons. In the present note we consider these two effects in order to relate the high-energy nonelastic cross sections measured at Brookhaven to the actual nucleon density distribution. By assuming a density distribution in accord with electron scattering results we find that the nuclear *size* as measured by high-energy neutron scattering agrees with the electromagnetic determinations.

In a subsequent note[8] we consider the implications of the cross sections which have been measured with cosmic rays at ultrahigh energies.

ANALYSIS

The Brookhaven group has described in the preceding paper their measurements of the attenuation of the 1.4-Bev neutron beam as a function of detector solid angle. These measurements enable them to ascertain the nonelastic part of the cross section as well as the total cross section. The nonelastic cross section, which we shall call the reaction cross section[9] σ_r, is simply related to the elementary nucleon-nucleon total cross section $\bar{\sigma}$, and the average density of nucleons at radius r, $\rho(r)$, under the conditions that: (a) the mass number of the nucleus be not too small (we formulate this more quantitatively below); and (b) the nucleons scatter or absorb the neutron wave independently of each other. For the independence condition to hold strictly it is necessary that both the neutron wavelength $\lambda\!\!\!^{-}$ and the range of nucleon-nucleon interactions be smaller than the average internucleon spacing.

If the latter condition were fulfilled, one would feel more confident in treating each nucleon as uninfluenced by the others, but in fact the range of nuclear forces is about equal to the internucleon spacing in the central region of the nucleus. However, the average internucleon distance increases near the edge of the nucleus, and it turns out that this is the important region for

* This work was supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] F. Bitter and H. Feshbach, Phys. Rev. **92**, 837 (1953); V. L. Fitch and J. Rainwater, Phys. Rev. **92**, 789 (1953); Hofstadter, Hahn, Knudsen, and McIntyre, Phys. Rev. **95**, 500 (1954).
[2] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 15.
[3] T. B. Taylor, Phys. Rev. **92**, 831 (1953).
[4] R. R. Wilson, Phys. Rev. **88**, 350 (1952).
[5] L. N. Cooper and E. M. Henley, Phys. Rev. **92**, 801 (1953).
[6] M. H. Johnson and E. Teller, Phys. Rev. **93**, 357 (1954).
[7] Coor, Hill, Hornyak, Smith, and Snow, preceding paper [Phys. Rev. **98**, 1369 (1955)].
[8] R. W. Williams, following paper [Phys. Rev. **98**, 1393 (1955)].
[9] This follows the usage of Blatt and Weisskopf, reference 2. In the Brookhaven paper and in reference 10 it is called the absorption cross section, and in cosmic-ray usage either the inelastic or collision cross section.

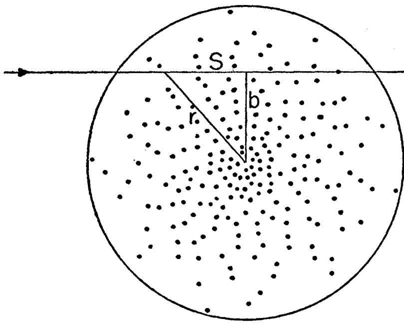

FIG. 1. The path of a neutron through the nucleus is designated by the coordinate s; the impact parameter is b. The dots are intended to illustrate the density of nuclear matter.

the calculation, at least for medium and heavy nuclei. We therefore assume that any nonadditive effects due to the failure of this condition will be small.

Following Fernbach, Serber, and Taylor,[10] we calculate σ_r by a semiclassical impact parameter method. As the neutron wave passes through the nucleus it will be attenuated exponentially[11] with an absorption coefficient $K(r)=\rho(r)\bar{\sigma}$. The size and shape of the nucleus are expressed by the density distribution $\rho(r)$, which approaches zero rapidly (usually exponentially) for large r. The total attenuation is obtained from an integral over s, the coordinate along the neutron trajectory (Fig. 1), and the cross section is then the probability of interaction integrated over the impact parameter b. Noting that $r^2=b^2+s^2$, we have:

$$\sigma_r=\int_0^\infty \left\{1-\exp\left[-\int_{-\infty}^{\infty} K((b^2+s^2)^{\frac{1}{2}})ds\right]\right\}2\pi b db. \quad (1)$$

The Brookhaven measurements have several advantages for a determination of nuclear size. They yield σ_r directly, whereas at lower energies one must rely mainly on *total* cross sections, which involve a large amount of diffraction scattering and therefore require the full apparatus of the optical model[12] and the introduction of another unknown parameter, the index of refraction.[13] At 1.4 Bev, $\bar{\lambda}=0.09\times10^{-13}$ cm, so the condition that $\bar{\lambda}$ be small is well satisfied (the mean nucleon-nucleon separation is $\sim1.8\times10^{-13}$ cm). Finally, the effective cross section $\bar{\sigma}$ for elementary collisions inside the nucleus should approach more closely to the average free nucleon-nucleon total cross section σ_{nucleon}. We are therefore able to fix $\bar{\sigma}$ rather than leaving it a free parameter to be determined from the fit of the data, as is done in the lower-energy analysis.[3] If the nucleons were truly independent we would have $\bar{\sigma}=\sigma_{\text{nucleon}}$, but as Serber has pointed out[14] the binding of the nucleons will inhibit many small-momentum-transfer collisions, so that in fact $\bar{\sigma}<\sigma_{\text{nucleon}}$. The only calculations of $\bar{\sigma}$ known to the writer are those of Goldberger[15] for 90-Mev neutrons and Bernardini, Booth, and Lindenbaum,[16] for 400-Mev protons. In both cases an approximate treatment gave $\bar{\sigma}\sim\frac{2}{3}\sigma_{\text{nucleon}}$. Despite this discouragingly large effect we shall assume $\bar{\sigma}=\sigma_{np}$,[17] measured as 43 mb by the Brookhaven group. This may be partially justified by the consideration that at this energy meson production is likely to account for over half the cross section,[18] and that the average energy transfer in the remaining "elastic" collisions probably increases. One must expect, however, that the elastic part of the nucleon-nucleon cross section becomes more forward at energies where meson production is copious; preliminary results on (pp) scattering at 1 Bev confirm this.[19] The sensitivity of our results to error in $\bar{\sigma}$ is discussed below.

As the Brookhaven group points out,[7] the neutron cross sections above are not adequate to determine the *form* of the nuclear density distribution $\rho(r)$; the data are compatible with any reasonable form. We therefore adopt a *shape* of the density distribution which conforms well with the information from electromagnetic experiments, leaving the scale factor to be determined by the neutron cross sections. The relevant facts are: (1) the high-energy electron scattering data are most compatible with a proton density in gold and lead which is uniform in the central region and drops off smoothly at the edge,[20] the distance for falling from 90 percent to 10 percent being $\Delta r_{10-90}\approx(2.0\text{–}2.4)\times10^{-13}$ cm; (2) several different experiments define a "size" of the proton distribution, the exact meaning of which depends, for each experiment, on the distribution shape. Following Ford and Hill,[21] we give for each of these the equivalent square-well radius—the radius obtained from analysis of the experiment when a uniform density distribution is assumed. (Note that if the true distribution is nonuniform, not all experiments will give the same equivalent square-well radius.) Writing $r_0A^{\frac{1}{3}}$ for the square-well radius, we find (in units of 10^{-13} cm)

[10] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949). For short-range, inelastic interactions our condition on $\bar{\lambda}$ serves to justify the use of an impact parameter. More stringent conditions are required for elastic scattering [N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **18**, No. 8 (1948); E. J. Williams, Revs. Modern Phys. **17**, 217 (1945)].

[11] See, for example, L. L. Foldy, Phys. Rev. **67**, 107 (1945); M. Lax, Revs. Modern Phys. **23**, 287 (1951). This absorption coefficient is proportional to the imaginary part of the optical model potential.

[12] K. M. Watson, Phys. Rev. **89**, 575 (1953); N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953); N. C. Francis and K. M. Watson, Am. J. Phys. **21**, 659 (1953).

[13] Actually the complete optical model has been used by Coor *et al.*, in determining corrections to the measured σ_r, and determining the mean energy of the neutron beam.

[14] R. Serber, Phys. Rev. **72**, 1114 (1947).

[15] M. L. Goldberger, Phys. Rev. **74**, 1268 (1948).

[16] Bernardini, Booth, and Lindenbaum, Phys. Rev. **88**, 1017 (1952).

[17] σ_{pp} seems to be higher, about 48 mb (Shapiro, Leavitt, and Chen, Phys. Rev. **95**, 663(A) (1954).

[18] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **95**, 1026 (1954).

[19] Smith, McReynolds, and Snow, Phys. Rev. **97**, 1378 (1955).

[20] R. Hofstadter (private communication); D. G. Ravenhall and D. R. Yennie, Phys. Rev. **96**, 239 (1954).

[21] K. W. Ford and D. L. Hill, Phys. Rev. **94**, 1630 (1954); D. L. Hill and K. W. Ford, Phys. Rev. **94**, 1617 (1954).

$r_0=1.17$ from μ-mesonic x-ray experiments,[22] 1.20 from high-energy electron scattering,[20] and 1.22 from the semiempirical mass equation.[23] The first two experiments also indicate that r_0 is quite constant as one proceeds toward lighter elements, at least down to $Z=22$ and probably much lower.[24]

The constancy of r_0 permits us to assume (though it does not prove) that all nuclei have a similar form and the same central density ρ_0; that is, for any nucleus that both Δr_{10-90} and the appropriate radius parameter R are proportional to $A^{\frac{1}{3}}$. While this assumption is surely unrealistic for the lightest nuclei, it may be true for the heavier nuclei.[25] (In what follows we shall put most weight on the data for heavy nuclei, which in any case yield the most reliable neutron cross sections.) Guided by the high-energy electron scattering results, we now choose a specific form for the nuclear density distribution, determine the radius parameter R from

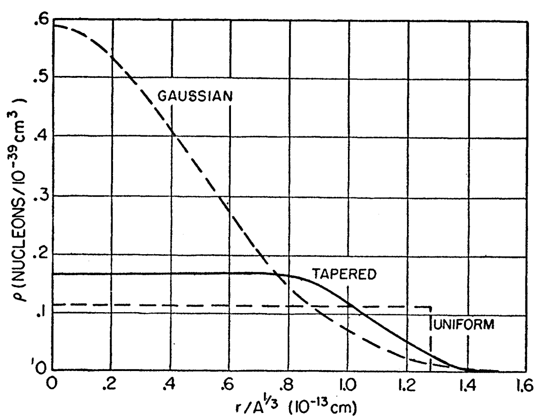

FIG. 2. Tapered nuclear density distribution function plotted against a universal radius $r/A^{\frac{1}{3}}$. Gaussian and uniform distributions, fitted to the same high-energy neutron data, are shown for comparison.

the Brookhaven neutron data, and show that the resulting nuclear size and shape agree with the electromagnetic data. The form chosen[26] (which we shall call the tapered density distribution) has a constant central region attached to a smoothly-dropping polynomial:

$$\begin{aligned} &\rho(r)=\rho_0 && r\leqslant R,\\ &\rho(r)=\rho_0(2r^3/R^3-9r^2/R^2+12r/R-4) && R\leqslant r\leqslant 2R,\\ &\rho(r)=0 && r\geqslant 2R. \end{aligned} \tag{2}$$

It is illustrated in Fig. 2 where the uniform and Gaussian forms are also shown. The "radius" as defined by Ravenhall and Yennie,[20] $\int_0^\infty \rho(r)dr/\rho(0)\equiv c$, is $c=1.5R$, and $\Delta r_{10-90}=0.6R$.

The density distribution is thus of the form:

$$\rho(r)=\rho_0 f(r/R). \tag{3}$$

The radius parameter R for any nucleus is given by $R=aA^{\frac{1}{3}}$, where a is a constant to be determined, and the central density ρ_0 is given by $\rho_0=1/(a^3v)$, where v is the volume-normalization constant characteristic of f; in particular, $v\equiv\int_0^\infty f(x)4\pi x^2dx=24\pi/5$ for the tapered distribution. For a specified $f(r/R)$, Eq. (1) now reduces to the form $\sigma_r=\pi R^2O(l_0/R)$, where the opacity, O, is a function only of the mean free path at the center of the nucleus, $l_0\equiv K_0^{-1}=(\rho_0\bar{\sigma})^{-1}$, divided by R.

Figure 3 shows on a log-log plot the results of a numerical calculation of O *versus* $(K_0R)^{-1}$ for the tapered model. Since $(K_0R)^{-1}=va^2/\bar{\sigma}A^{\frac{1}{3}}$, and $O=\sigma_r/\pi a^2A^{\frac{2}{3}}$, for a given model the measurements of σ_r should determine a, the only unknown quantity, from the best fit to the opacity curve. The Brookhaven data[7] are fitted to the curve of Fig. 3 with most weight being assigned to the heavy elements; the value of the size parameter which results is $a=0.736\times10^{-13}$ cm. A similar analysis for the uniform-density model yields $a=1.28\times10^{-13}$ cm $(=r_0)$, and for the Gaussian $(\rho=\rho_0\exp(-r^2/R^2))$, $a=0.675\times10^{-13}$ cm.

The nuclear size determined in this way agrees with the electromagnetic experiments if one adopts the tapered model, but does not agree for the uniform or Gaussian models. To see this we can calculate the

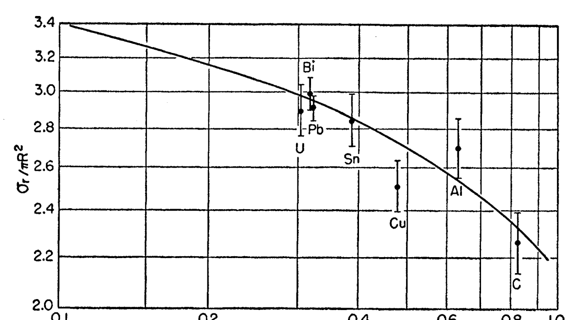

FIG. 3. Opacity, $\sigma_r/\pi R^2=\sigma_r/\pi a^2A^{\frac{2}{3}}$ as a function of $(K_0R)^{-1}=4.8\pi a^2/\bar{\sigma}A^{\frac{1}{3}}$, the mean free path at the center of the nucleus divided by R, for the tapered model. Experimental points are the Brookhaven measurements plotted with $\sigma=43$ mb, $a=0.736\times10^{-13}$ cm.

[22] V. L. Fitch (private communication). Two small corrections have been applied to the published results (reference 1), reflecting the effects of the new meson mass and of vacuum polarization, but these essentially cancel each other.

[23] A. E. S. Green, Phys. Rev. **95**, 1006 (1954).

[24] Our selection of data has excluded medium-energy electron scattering which has been quoted (reference 1) as indicating an r_0 of 1.1. A new analysis indicates that it should be nearer 1.2 [A. E. Glassgold (private communication)]. We have also omitted the evidence from energy differences of pairs of mirror nuclei, which indicates an r_0 somewhat larger than 1.20 for light nuclei [B. Jancovici, Phys. Rev. **95**, 389 (1954); B. C. Carlson and I. Talmi, Phys. Rev. **96**, 436 (1954); see, however, D. C. Peaslee, Phys. Rev. **94**, 717 (1954)]. The density distribution in light nuclei has also been considered by Gatha, Shah, and Patel, Proc. Phys. Soc. (London) **A67**, 773 (1954).

[25] For a discussion of some of the causes of nonuniformity of the nuclear density distribution, see D. L. Hill and K. W. Ford, Phys. Rev. **94**, 1617 (1954).

[26] The use of a polynomial to achieve this type of profile was suggested by Dr. W. Aron and Dr. J. MacIntosh; for this problem it is far easier to handle than a function involving exponentials. Possibly a simple linear drop-off would have served as well, but the finite-range-of-interaction transformation discussed below would change it into a curve of this form anyway.

TABLE I. Properties of the three models fitted to the high-energy neutron cross sections. Distances are in 10^{-13} cm.

	Uniform density	Gaussian	Tapered [Eq. (2)]
Radius parameter a	1.28	0.675	0.736
Central density, ρ_0, in nucleons per 10^{-39} cm^3	0.114	0.585	0.166
Mean distance between nucleons, $\rho_0^{-\frac{1}{3}}$	2.06	1.20	1.82
Mean free path in nuclear matter at center of nucleus $(\rho_0\bar{\sigma})^{-1}$	2.04	0.40	1.40
Equivalent square well, r_0, for μ-mesonic x-rays (*Observed:* 1.17), and for semi-empirical Coulomb energy (*Observed:* 1.22)	1.28	0.99	1.18
Equivalent square well, r_0, for high-energy electron scattering (*Observed:* 1.20)	1.28	1.07	1.19
Thickness of taper for 10%–90% change in density for Pb nucleus (*Observed:* from electron scattering, 2.0 to 2.4)	0	5.0	2.6

electromagnetic results to be expected from the neutron-determined nuclear size, for the three models, and compare these expected values with the observations. Ford and Hill[21] give an extensive series of calculations of the electromagnetic effects predicted by proton density distributions of various shapes, quoting their results in terms of the previously defined equivalent square-well radius, $r_0A^{\frac{1}{3}}$. For medium-energy electron scattering this is simply a measure of the second moment, $r_0A^{\frac{1}{3}}=(5/3)^{\frac{1}{2}}\langle r^2\rangle^{\frac{1}{2}}_{\mathrm{Av}}$; other experiments are related to the charge distribution in a more complicated way. In Table I we have assembled our high-energy neutron determinations of nuclear size, and compared the corresponding electromagnetic predictions with experiment. It is clear that the Gaussian nucleus so determined is too *small*, the uniform density nucleus somewhat too *large*, and the tapered nucleus [Eq. (2)] a remarkably good fit (of course the approximate fit of the tapered model to the Δr_{10-90} of the electron scattering data was forced; the values are listed for convenience).

The illustration of Fig. 2 shows the three distributions in their true proportions.

The closest fit to our tapered model among the Ford and Hill families appears to be their Family II, $n=10$;[21] among the three possibilities for Au and Pb considered by Ravenhall and Yennie,[20] the closest to our curve is their $K=1.85$, $c=6.51$.

EFFECT OF THE FINITE RANGE OF NUCLEON-NUCLEON INTERACTION

In the analysis above we have identified the effective nuclear density as seen by high-energy neutrons with the true proton density as measured electrically. This cannot be correct, since it implies that our method of calculation applies to the mean density of point nucleons, whereas in fact each nucleon's influence extends over a region of radial extent at least 1×10^{-13} cm. To see the effect of this in a simple case, consider the integral of Eq. (1) for the uniform-density model: no contribution to the cross section occurs beyond $b=R$. However, we may legitimately ask what really happens to a neutron of impact parameter $b=R+0.5\times10^{-13}$ cm, say (since $\lambdabar=0.09\times10^{-13}$ cm such a difference in trajectories is well-defined), and we see that even though the mean position of the nucleons does not extend beyond R, their influence must extend farther, and there may be an interaction. This effect is often spoken of in terms of "size of the bombarding particle," but of course it is a property of the interaction of the two nucleons. Within the present framework of nuclear theory there is no reliable way to treat such problems from a fundamental standpoint. We suggest here a simple treatment, based on physical plausibility, which yields the expected result that the effect is far less important for a tapered nucleus than for a uniform-density nucleus.

We have assumed that the neutron wave in homogeneous nuclear matter is attenuated according to the multiple scattering approximation, with an absorption coefficient $K=\rho\bar{\sigma}$, where ρ is the mean density of nucleons; to make a more explicit picture we ascribe to the ith nucleon, at $\mathbf{r}_i$, a contribution to K which is proportional to $\bar{\sigma}$ but depends on $|\mathbf{r}-\mathbf{r}_i|$, say $\bar{\sigma}F(|\mathbf{r}-\mathbf{r}_i|)$. Thus:

$$K(\mathbf{r})=\textstyle\sum_i \bar{\sigma}F(|\mathbf{r}-\mathbf{r}_i|). \tag{4}$$

Passing to an average over positions of the nucleons:

$$K(r)=\int \bar{\sigma}F(|\mathbf{r}-\mathbf{r}'|)\rho(r')d^3r'. \tag{5}$$

The function F must be normalized according to $\int F(x)d^3x=1$. Then we recover the simple formula for K when (a) ρ is a constant or (b) F has a delta-function form.[27] In effect the operation of Eq. (5) "folds" the density with a resolution function for the finite range of influence of a nucleon. The effective density $\rho_e\equiv K(r)/\bar{\sigma}$ retains the property $\int\rho_e d^3r=A$.

Watson's[12] formal theory of the optical model starts with an exact equation for the many-body problem in

[27] It might be expected that Eq. (1) would recover the nucleon-nucleon cross section, $\bar{\sigma}$, for $A=1$, but this proves to be true only if the nucleon's position is averaged over a large volume. Inspection of Eq. (1) shows that one would have to replace $\rho(r)$ not by a δ-function but by a smeared-out distribution which extended to a distance R such that $\pi R^2/\bar{\sigma}\gg1$. The difficulty can be traced to the essentially statistical nature of the exponential law of extinction. This law results either from a classical gas-kinetic argument or from the theory of multiple scattering of waves; in both cases the scatterers are assumed to occupy a large volume. This is the origin of the restriction on our analysis to A "not too small." In terms of the nuclear radius R and the "size of a nucleon" [the extent of the nucleon interaction function $F(r)$], say S, the condition on A can be restated as $R^2/S^2\gg1$.

terms of an assumed two-body interaction and therefore should contain the finite-range effect; however, he eventually expresses the optical model potential in terms of elementary scattering amplitudes, so that the details of the elementary interaction do not enter explicitly, and the finite-range effect is not present after some practical approximations are made. If one assumes only two-body forces, it is possible[28] to relate the optical model potential formally to a sum over the elementary two-body interaction "potentials" in the many-body problem, each potential depending on the coordinates of a nucleon. Since K is proportional to the imaginary part of the optical model potential, this relation could presumably be used as the starting point for a justification of the form of Eq. (4), but this has not been carried through.

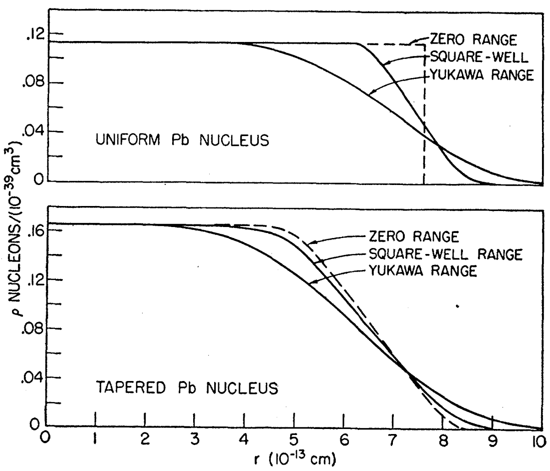

FIG. 4. Results of applying Eq. (5) for the finite-range-of-interaction correction to two nuclear density distributions: Uniform (upper curves) and tapered (lower curves). The effective density is shown for a square-well range-of-interaction of extent $\hbar/\mu c$ and a Yukawa range proportional to $r^{-1}\exp(-r\mu c/\hbar)$.

There is little to guide the choice of a form for F, except that it should bear some relation to the space dependence of the high-energy nucleon-nucleon interaction. We have calculated ρ_e with two simple forms for F, both expressed in terms of $\lambda\!\!\!^-_\pi = \hbar/\mu c = 1.4\times10^{-13}$ cm:

$$\text{Square well: } F = \frac{1}{(4/3)\pi\lambda\!\!\!^-_\pi{}^3} \quad r<\lambda\!\!\!^-_\pi \tag{6a}$$

$$= 0 \quad r>\lambda\!\!\!^-_\pi.$$

$$\text{Yukawa: } F = \frac{1}{4\pi\lambda\!\!\!^-_\pi{}^2 r}\exp(-r/\lambda\!\!\!^-_\pi). \tag{6b}$$

Figure 4 shows the results of evaluating Eq. (5) with the two forms of Eq. (6), for two models of the Pb nucleus: uniform density and tapered density. The uniform-density model is rounded off considerably by the square-well range-of-interaction function, and drastically by the Yukawa function; the tapered model is scarcely changed at all by the square-well function, and changed moderately by the Yukawa function. Table II gives the tapering distances Δr_{10-90}, for the various cases. (The integral of Eq. (5) is three-dimensional, and was evaluated, for the tapered model, by approximate numerical methods.)

Since the Yukawa function appreciably smears out the effective density of the model that fits the electron scattering (tapered density) we have recomputed the opacity [Eq. (2)] from the new effective density curve, and determined the nuclear size parameter from a fit to the Brookhaven data, as before. This yields a nuclear radius 15 percent smaller: $a=0.640\times10^{-13}$ cm, r_0 for electron scattering $=1.03\times10^{-13}$ cm. Thus even the heaviest nuclei come out too small if we use the Yukawa function; in lighter nuclei the decrease would be more severe.

We are therefore faced with the rather unexpected result that the nuclear radius determined by neutron cross sections is *smaller* than the electromagnetic radius, when the neutron's interaction range is assumed to be as long-tailed as a Yukawa function with characteristic length $\lambda\!\!\!^-_\pi$. The nature of the static potential corresponding to pseudoscalar meson theory is not completely known, but it is certainly more complicated than a single Yukawa function, and contains some important short-range parts.[29] At energies high enough to involve meson production nothing is known, but it seems likely that shorter distances are involved, since the nucleons are no longer slow compared to the (virtual) mesons. We therefore assume that the square-well range results are more nearly correct, and that the curves of Fig. 4 justify our neglect of the finite-range effect in using the tapered nucleus.

Rossi and Safford[30] have considered the finite-range correction from a different point of view, using a classical hard-sphere interaction in analogy with the kinetic theory of gases. Their treatment, which is not mathematically identical to ours, gives a somewhat larger effect than our square-well case.

TABLE II. Δr_{10-90} for Pb, in 10^{-13} cm. The distance Δr_{10-90} for the effective density $K/\bar{\sigma}$ to drop from 90 percent to 10 percent of its central value is given for two models of the true nuclear density in Pb, for two assumptions concerning the spatial dependence of the range of interaction of a nucleon, Eq. (6).

	Zero range	Square-well range	Yukawa range
Uniform density	0	1.7	3.8
Tapered density	2.6	3.1	4.4

[28] These remarks owe much to a very helpful discussion with Professor S. D. Drell, and a brief but enlightening conversation with Professor K. M. Watson.

[29] K. A. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953).

[30] Quoted in B. Rossi, *High-Energy Particles* (Prentice-Hall, Inc., New York, 1952), p. 359.

DISCUSSION

The least-known quantity entering into the aforementioned determination of nuclear size is the effective elementary cross section $\bar{\sigma}$, which might well be less than σ_{np}, the value we have used. Even heavy-nucleus cross sections will depend on $\bar{\sigma}$ for any tapered model—the outer edge of the nucleus is always somewhat transparent—and one must investigate the sensitivity of r_0 to the value of $\bar{\sigma}$. From the opacity curve for the tapered model it turns out (for heavy nuclei) that $d(\ln r_0)/d(\ln\bar{\sigma})=-0.1$. Since $\bar{\sigma}$ is unlikely to be decreased by more than 20 percent, the value given in Table I for r_0 is probably not more than 2 percent too low on this account.

Only two measurements are known to the writer which seem directly comparable with the Brookhaven work. Shapiro, Leavitt, and Chen[31] have measured reaction cross sections of various nuclei at Brookhaven with 860-Mev protons; their results, interpreted with a uniform-density nuclear model, yield $r_0=1.25\times10^{-13}$ cm. Eisenberg[32] determined the reaction cross section in Pb of primary cosmic-ray heavy nuclei ($Z\geqslant6$) with a median energy of two or three Bev per nucleon. Using a uniform-density model he found $r_0=1.3\times10^{-13}$ cm. Both are in good agreement with the uniform-density value $r_0=1.28\times10^{-13}$ cm from the neutron work used here.

At lower energies neutron reaction cross-section measurements present great experimental difficulties. (An analysis of recent Berkeley results will be found in the preceding paper.[7]) The reaction cross sections of *complex* charged particles are more straightforward experimentally, but difficult to interpret.[33]

As is well known, neutron *total* scattering, in the low- and medium-energy region, can be interpreted in terms of the scattering by a complex potential (the optical model), usually assumed to be a square well. For this case the radius of the *potential*, r_0', is distinctly larger than the radius we have found for the nuclear density distribution, even assuming the latter to be uniform. For example, Taylor[3] finds from an analysis of all available data from 50 Mev to 400 Mev a value of $r_0'=1.37\times10^{-13}$ for Pb, and larger values for lighter nuclei, up to 1.50×10^{-13} cm for Al. The analysis in the Mev range[34] yields $r_0'=1.45\times10^{-13}$ cm; low-energy elastic scattering of charged particles give similar results.[35]

Although no extensive calculations are yet available, it already appears unlikely that agreement will be achieved solely by the use of a realistic shape of potential well. Heckrotte[36] has analyzed the 90-Mev data with a cut-off parabolic well and finds an $\langle r^2\rangle_{\text{Av}}$ no smaller than that given by the square-well analysis. Woods and Saxon[37] have fitted the angular distribution of elastically scattered 20-Mev protons with a tapered well somewhat similar in shape to the one used in the present paper, but it turns out to be about 25 percent larger in radius.

In our view this contrast between low-energy and high-energy results suggests a real difference in the effective range of nucleon-nucleon interaction in the two cases. The high-energy experiments may be interpreted quite directly in terms of nuclear density distributions, while the low-energy interpretation must involve more detailed consideration of the range of nuclear forces.

Johnson and Teller[6] have suggested that in heavy nuclei the neutron cloud has a larger extension than the proton cloud, by perhaps 1×10^{-13} cm. We find that the Brookhaven reaction cross-section measurements yield a nuclear size in agreement with the electromagnetic determination of the *proton*-distribution size, both experiments referring primarily to heavy nuclei. A rough estimate shows that the radius determined from the Brookhaven cross sections would be essentially the neutron-cloud radius. If the effect discussed by Johnson and Teller were present, we should have found an r_0 about 15 percent larger than the one actually observed. A reasonable limit of error on the increase of r_0 would be 5 percent, indicating that any difference between proton and neutron radii[38] should be not more than one-third as great as the proposed effect.

SUMMARY

Measurement of the reaction cross section—the noncoherent part of the total cross section—of very high-energy neutrons on nuclei is shown to be the most easily interpreted nuclear method of obtaining the true size of the nucleus (the spatial extension of nuclear matter); in particular, it is more straightforward, and involves the use of fewer unknown parameters, than the coherent scattering from an equivalent complex potential. The availability of reaction cross-section measurements, from the Brookhaven Cosmotron, combined with recent evidence for the shape of the nuclear matter distribution, has allowed the determination of the true nuclear size; such a determination proves to be in excellent agreement with the results of electromagnetic experiments on the nuclear charge distribution. There is no evidence for an excess of neutrons beyond the proton distribution.

The nuclear density distribution which fits the two sets of data is $\rho=\rho_0$, $r\leqslant R$; $\rho=\rho_0(2r^3/R^3-9r^2/R^2+12r/R-4)$, $R\leqslant r\leqslant 2R$; $R=0.736A^{\frac{1}{3}}\times10^{-13}$ cm; $\rho_0=0.166\times10^{-39}$ nucleons cm^{-3}. This distribution has an "electromagnetic" radius (the size of the square well with the same $\langle r^2\rangle_{\text{Av}}$) of $1.19A^{\frac{1}{3}}\times10^{-13}$ cm.

[31] Shapiro, Leavitt, and Chen (to be published).

[32] Y. Eisenberg, Phys. Rev. **96**, 1378 (1954).

[33] Millburn, Birnbaum, Crandall, and Schecter, Phys. Rev. **95**, 1268 (1954); contains references and results concerning previous inelastic cross-section measurements.

[34] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954); R. K. Adair, Phys. Rev. **94**, 737 (1954).

[35] J. S. Blair, Phys. Rev. **95**, 1218 (1954).

[36] W. Heckrotte, Phys. Rev. **95**, 1279 (1954).

[37] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954).

[38] A decrease of proton density at the center of the nucleus—the so-called wine bottle shape—would not be noticeable in neutron cross-section experiments, which are insensitive to the central region of the nucleus.