Electric Monopole Transitions in C^{12} and O^{16}†

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The matrix elements for the electric monopole ($0^+ \rightarrow 0^+$) transitions between the ground and 7.68-Mev state in C^{12} and between the ground and 6.06-Mev state in O^{16} may be estimated from inelastic electron scattering and from the pair emission lifetime, respectively. The two are equal to each other within the rather large error of the electron scattering determination, and are given by $(\Sigma_P r_P^2)_{f0} \cong 3.8 \times 10^{-26}$ cm^2, where 0 and f represent initial and final states of the nucleus, and r_P is the radial distance of a proton from the center of the nucleus. Calculations based on the alpha-particle model and on an elastic-fluid model yield three to five times this experimental value. Therefore, a calculation was made in the case of C^{12}, based on the jj-coupling independent-particle model, according to which two nucleons undergo transitions between the $p_{\frac{3}{2}}$ shell and the $p_{\frac{1}{2}}$ shell. The matrix element vanishes if there are no internucleon forces. Pair forces are included to first order, and the sum over configurations is performed exactly by means of a Green's function. For simplicity it is assumed that the independent-particle potential is an infinitely deep square well, and that the pair interaction has zero range. Even assuming that the pair interaction has its free-space triplet magnitude, the calculated matrix element is only about one-sixth the experimental value. It is concluded, therefore, that a model that is more collective than the independent-particle model with pair interactions, and less collective than the alpha-particle or elastic-fluid models, is required to account for the experimental results.

I. EXPERIMENTAL SITUATION

THE half-life for emission of electron-positron pairs from the 6.06-Mev state in O^{16} has been measured by Devons, Goldring, and Lindsey[1] to be $(5.0 \pm 0.5) \times 10^{-11}$ sec. According to the theory of Oppenheimer and Schwinger and of Dalitz,[2] this provides a value of 3.8×10^{-26} cm^2 for the matrix element (M.E.) of the electric monopole operator $\sum_P r_P^2$ between initial and final 0^+ states, where r_P is the radial distance of a proton from the center of the nucleus.

The energy of the 7.68-Mev state in C^{12} has been measured accurately by Dunbar, Pixley, Wenzel, and Whaling.[3] Its angular momentum and parity are not as definitely known as for the 6.06-Mev state in O^{16}, although it is most probably also a 0^+ state. Harries[4] has observed pairs of approximately the correct energy. Bell and Jordan[5] have observed 7.4-Mev gamma rays, but attribute these to neutron capture in an iron shield. Beghian, Halban, Husain, and Sanders[6] have observed 3.16-Mev gamma rays that would correspond to a transition from the 7.68-Mev state to the 4.43-Mev (2^+) state, and do not find any gamma rays of about 7.5-Mev energy. These cascade gamma rays are also observed by Uebergang,[7] and an angular correlation study by Seed[8] indicates that the transitions correspond to $0^+ \rightarrow 2^+ \rightarrow 0^+$. We assume, therefore, that the 7.68-Mev state is actually a 0^+ state.

Differential cross sections for elastic and inelastic scattering of 190-Mev electrons from carbon have been measured by Fregeau and Hofstadter.[9] As indicated in an earlier paper,[10] the ratio of the 7.68-Mev excitation cross section to the elastic cross section can be used to determine the electric monopole matrix element, since the Born approximation is valid for such a light nucleus. For primary electron energies that are large in comparison with the excitation energy, the square root of this cross-section ratio is approximately equal to

$$\left| \frac{(\sum_P [j_0(qr_P) - 1])_{f0}}{(\sum_P j_0(qr_P))_{00}} \right| \underset{q \rightarrow 0}{\longrightarrow} \frac{q^2}{6Z} (\sum_P r_P^2)_{f0}, \qquad (1)$$

where 0 and f refer to the ground and excited states of the nucleus of atomic number Z, $\hbar\mathbf{q}$ is the momentum transfer from electron to nucleus, and j_0 is a spherical Bessel function. Thus a plot of the square root of the cross-section ratio against q^2 gives M.E. in terms of the slope at the origin. The experiments cover a range of q^2 from about 0.5 to 1.5 in units of 10^{26} cm^{-2}, so that the extrapolation in to $q=0$ requires some guesswork. A rough value for the initial slope can nevertheless be obtained in this way, and corresponds to about the same value M.E. $= 3.8 \times 10^{-26}$ cm^2 as is found in the case of O^{16}. A determination of M.E. can also be made from the absolute value of the inelastic cross section as a function of angle. While not as reliable, it is in agreement with the above value.

II. COLLECTIVE MODELS[11]

In the alpha-particle model, we assume that C^{12} consists of three alpha particles with equilibrium positions at the corners of an equilateral triangle, and that O^{16} consists of four alpha particles with equilibrium

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[1] Devons, Goldring, and Lindsey, Proc. Phys. Soc. (London) **A67**, 134 (1954).
[2] J. R. Oppenheimer and J. Schwinger, Phys. Rev. **56**, 1066 (1939); R. H. Dalitz, Proc. Roy. Soc. (London) **A206**, 521 (1951).
[3] Dunbar, Pixley, Wenzel, and Whaling, Phys. Rev. **92**, 649 (1953).
[4] G. Harries, Proc. Phys. Soc. (London) **A67**, 153 (1954).
[5] P. R. Bell and W. H. Jordan, Phys. Rev. **79**, 392 (1950).
[6] Beghian, Halban, Husain, and Sanders, Phys. Rev. **90**, 1129 (1953).
[7] R. G. Uebergang, Australian J. Phys. **7**, 279 (1954).
[8] J. Seed, Phil. Mag. **46**, 100 (1955).
[9] J. H. Fregeau and R. Hofstadter, Bull. Am. Phys. Soc. **30**, No. 1, 45 (1955); and private communication.
[10] L. I. Schiff, Phys. Rev. **96**, 765 (1954).
[11] Calculations similar to those reported in this section have been made by R. H. Dalitz (private communication); see also reference 1, and Devons, Hereward, and Lindsey, Nature **164**, 586 (1949).

positions at the corners of a regular tetrahedron. Dennison[12] has achieved some success in accounting for the excited states of O^{16} with this model. In each case, the first excited 0^+ state corresponds to single excitation of the "breathing" mode, in which the alpha particles oscillate radially and in phase; the restoring force is fixed in terms of the excitation energy. The matrix element is

$$\text{M.E.}=2^{\frac{1}{2}}[ZR_\alpha\hbar/(AM\Delta)^{\frac{1}{2}}], \tag{2}$$

where M is the nucleon mass, A is the atomic weight, $\Delta\equiv E_f-E_0$ is the excitation energy, and R_α is the equilibrium distance of the center-of-mass of each alpha particle from the center of the nucleus. If we assume that R_α is smaller than the nuclear radius $R=1.3\times10^{-13}A^{\frac{1}{3}}$ cm by 1.0×10^{-13} cm, we obtain M.E.$(C^{12})=11\times10^{-26}$ cm^2, M.E.$(O^{16})=17\times10^{-16}$ cm^2. In both cases, the amplitude of oscillation is small enough for the model to be reasonable. However, as pointed out by Inglis,[13] the period of oscillation is close enough to the periods associated with the motions of the constituent nucleons in each alpha particle so that the adiabatic approximation implied in the use of the alpha-particle model is not well justified.

In the elastic-fluid model, we assume that the nucleus consists of a sphere of elastic fluid of radius R that has uniform charge and mass density. The first excited 0^+ state then corresponds to single excitation of the lowest radial mode of oscillation, in which the change in density of the fluid at radial distance r is proportional to $j_0(\pi r/R)$; again, the compressibility is fixed in terms of the excitation energy. When account is taken of the oscillation of the nuclear surface, the matrix element is found to be

$$\text{M.E.}=(3^{\frac{1}{2}}6/\pi^2)[ZR\hbar/(AM\Delta)^{\frac{1}{2}}]. \tag{3}$$

With the above expression for R, Eq. (3) gives M.E.$(C^{12})=13\times10^{-26}$ cm^2, M.E.$(O^{16})=19\times10^{-26}$ cm^2. The elastic stiffness of the fluid that is required to give agreement with the observed excitation energies is about one-seventh of that deduced from other considerations.[14]

It is not surprising that these two collective models yield such similar results. The fact that both give too large a matrix element by a factor of three to five suggests that the independent-particle model is worth investigating, since only a small number of protons are expected to take part in the transition on the basis of this model.

III. INDEPENDENT-PARTICLE MODEL FOR C^{12}

On the basis of the jj-coupling independent-particle model, the ground state of C^{12} has the $p_{\frac{3}{2}}$ shell filled with four neutrons and four protons. It is expected that the lowest excited states arise when a small number of nucleons are promoted from the $p_{\frac{3}{2}}$ shell to the $p_{\frac{1}{2}}$ shell, or possibly to the $d_{\frac{5}{2}}$ or $s_{\frac{1}{2}}$ shell. In none of these cases can an excited 0^+ state be produced by promoting a single nucleon. If however two nucleons are promoted, a 0^+ state can be obtained.[15] We assume that the 7.68-Mev state has six nucleons or two holes in the $p_{\frac{3}{2}}$ shell and two nucleons in the $p_{\frac{1}{2}}$ shell, so that it necessarily has even parity. We further assume that this state has total isotopic spin $T=0$, since the ground states of the neighboring isobars lie considerably higher. The pair of holes can be coupled together such that their total angular momentum J is 3, 2, 1, or 0; then $T=0$ for odd values of J and $T=1$ for even values of J. The pair of nucleons can similarly be coupled together such that (T,J) is equal to (1,0) or (0,1). Thus a state with $T=0$ and $J=0$ can be produced by combining the substates (1,0) and (1,0), or by combining the substates (0,1) and (0,1). According to Redlich,[16] the effect of the interaction between pairs of nucleons is generally to decrease the energy of the states of highest space symmetry with respect to the others. In the situation under consideration here, the (1,0) substate of two $p_{\frac{1}{2}}$ nucleons has a slightly higher energy than the (0,1) substate, by 0 to 0.73 Mev depending on the interaction assumed; on the other hand the (1,0) substate of two $p_{\frac{3}{2}}$ nucleons or holes has a lower energy than the (0,1) substate, by 1.19 to 1.56 Mev.[16] We therefore assume that the 7.68-Mev state results from combination of the $(T,J)=(1,0)$ substates of the two $p_{\frac{1}{2}}$ nucleons and the two $p_{\frac{3}{2}}$ holes.

The matrix element of an operator like the electric monopole operator

$$\Omega=\sum_{P=1}^{Z} r_P{}^2=\sum_{i=1}^{A}\tfrac{1}{2}(1+\tau_i)r_i{}^2=\sum_{i=1}^{A}\Omega_1(i), \tag{4}$$

which is a sum of single-particle terms, vanishes between pure independent-particle states that differ in the configurations of more than one nucleon. The effect of the interaction between pairs of nucleons is to mix in other nucleon configurations so that the initial and final perturbed independent-particle states have components that differ in the configuration of only one nucleon. The calculation is carried through only to first order in the pair interaction. On the other hand, all admixed states are included, this being accomplished by means of a Green's function. In principle, any forms can be assumed for the independent-particle potential and for the pair interaction. The complications are such, however, that for simplicity the independent-particle potential is assumed in this paper to be an infinitely deep square

[12] D. M. Dennison, Phys. Rev. **96**, 378 (1954); see also Peterson, Fowler, and Lauritsen, Phys. Rev. **96**, 1250 (1954).

[13] D. R. Inglis (private communication); see also Revs. Modern Phys. **25**, 390 (1953), Sec. 10.

[14] E. Feenberg, Revs. Modern Phys. **19**, 239 (1947).

[15] The idea that the matrix element might be small because more than one nucleon is excited to form the upper state, was suggested independently by D. H. Wilkinson (private communication); see also A. M. Lane and D. H. Wilkinson, Phys. Rev. **97**, 1199 (1955).

[16] M. G. Redlich (private communication); see also thesis, Princeton University, 1954 (unpublished); and Phys. Rev. **95**, 448 (1954).

well, and the pair interaction to be of zero range (δ function).

IV. DETAILS OF THE CALCULATION

The Hamiltonian for the nucleus is assumed to have the form

$$H=H_0+H',$$
$$H_0=\sum_{i=1}^{A} H_1(i), \quad H'=\sum_{i>j}^{A} H_p'(i,j), \tag{5}$$

where H_0 represents the independent-particle potential including the spin-orbit interaction, and H' represents the interaction between pairs of nucleons. The unperturbed energy eigenfunctions u_k form a complete orthonormal set:

$$H_0u_k=E_ku_k, \quad \int \bar{u}_ku_l d\tau=\delta_{kl}. \tag{6}$$

The first-order perturbed wave functions are

$$\psi_n\cong u_n+\sum_{k\neq n}(E_n-E_k)^{-1}u_kH'_{kn},$$
$$H'_{kn}\equiv\int \bar{u}_kH'u_n d\tau. \tag{7}$$

We wish to calculate the matrix element of the electric monopole operator Ω given by Eq. (4), between perturbed initial and final states ψ_0 and ψ_f given by Eq. (7). The result through first order in H' is

$$\int \bar{\psi}_f\Omega\psi_0 d\tau=\Omega_{f0}+\sum_{k\neq f}(E_f-E_k)^{-1}H'_{fk}\Omega_{k0}+\sum_{k\neq 0}(E_0-E_k)^{-1}\Omega_{fk}H'_{k0}, \tag{8}$$
$$\Omega_{kn}\equiv\int \bar{u}_k\Omega u_n d\tau.$$

Since the initial and final states differ in the configurations of two nucleons and Ω is a sum of single-nucleon terms, we have that the zero-order part of (8), Ω_{f0}, is zero. Because of this, the restrictions on the sums over k in the first-order parts of (8) can be dropped; then since H' and Ω are Hermitian, we can put

$$\int \bar{\psi}_f\Omega\psi_0 d\tau=-S_{f0}-\bar{S}_{0f},$$
$$S_{f0}\equiv\sum_k(E_k-E_f)^{-1}H'_{fk}\Omega_{k0}, \tag{9}$$
$$\bar{S}_{0f}\equiv\sum_k(E_k-E_0)^{-1}\Omega_{fk}H'_{k0}.$$

The unperturbed energy eigenfunctions are antisymmetrized products of single-particle functions, each of which is the product of a radial function and a spin-angle-isotopic spin (s-a-i) function. The operator Ω is a sum of terms $\Omega_1(i)$, each of which can alter only the radial coordinate of one particle, so that the state k must have the same distribution of s-a-i functions as the state 0. Now the state f differs from 0, and hence also from k, in the s-a-i functions of two particles. The terms in the summand of S_{f0} can then be divided into two classes, those in which the nucleon that appears as the argument of Ω_1 is one of the two which appear as arguments of H_p', and those in which it is not. In both classes of terms, it is apparent that the two nucleons in H_p' are those that change their s-a-i functions. Then in the second class, the nucleon in Ω_1 must not change its state at all (neither radial nor s-a-i part), so that the states k and 0 are identical. The second-class part of S_{f0} is thus equal to

$$(E_0-E_f)^{-1}H'_{f0}\Omega'_{00}, \tag{10}$$

where the prime on Ω'_{00} indicates that the summation in (4) is carried only over the nucleons that do not change their state. In similar fashion, the second-class part of $\bar{S}_{0f}$ is equal to

$$(E_f-E_0)^{-1}\Omega'_{ff}H'_{f0}. \tag{11}$$

Since the single-particle states involved in Ω'_{00} and Ω'_{ff} are the same, Eqs. (10) and (11) are equal in magnitude and opposite in sign, and hence cancel when added in Eq. (9). Thus only the first-class parts of S_{f0} and $\bar{S}_{0f}$ need be considered in what follows.

A typical term in the summand of the first-class part of S_{f0} is

$$\sum_\kappa\int\int \bar{v}_c(1)\bar{v}_d(2)H_p'(1,2)v_\kappa(1)v_b(2)d\tau_1d\tau_2 \times(\epsilon_\kappa+\epsilon_b-\epsilon_c-\epsilon_d)^{-1} \times\int\int \bar{v}_\kappa(1')v_b(2')\Omega_1(1')v_a(1')v_b(2')d\tau_1'd\tau_2', \tag{12}$$

where the v's and ϵ's are unperturbed single-particle eigenfunctions and eigenvalues. None of the other particles in the nucleus changes its state as the nucleus goes from 0 to k to f, so that the integrations over the coordinates of these $A-2$ nucleons give unity. The integration over $2'$ also gives unity, as does the s-a-i part of the integration over $1'$. We write each of the v's as the product of an s-a-i part I and a radial part R; for example, $v_a(1)=I_a(1)R_a(r_1)$, $v_\kappa(1)=I_a(1)R_\kappa(r_1)$. Then, Eq. (12) can be written in the form

$$\int\int \bar{v}_c(1)\bar{v}_d(2)H_p'(1,2)w_a(1)v_b(2)d\tau_1d\tau_2,$$
$$w_a(1)=\tfrac{1}{2}(1+\tau_1)I_a(1)Q_{a\epsilon}(r_1), \quad \epsilon\equiv\epsilon_c+\epsilon_d-\epsilon_b, \tag{13}$$
$$Q_{a\epsilon}(r)\equiv\sum_\kappa(\epsilon_\kappa-\epsilon)^{-1}R_\kappa(r)\int_0^\infty \bar{R}_\kappa(r')R_a(r')r'^4dr'.$$

The summation in the last of Eqs. (13) over all radial states κ that have the same s-a-i part I_a, can be evalu-

ated in terms of a Green's function:

$$\sum_\kappa(\epsilon_\kappa-\epsilon)^{-1}R_\kappa(r)\bar{R}_\kappa(r')=G_{a\epsilon}(r,r'),$$
$$-(\hbar^2/2M)[r^{-2}(d/dr)(r^2d/dr)-l(l+1)/r^2]G_{a\epsilon}(r,r')$$
$$+[V_a(r)-\epsilon]G_{a\epsilon}(r,r')=r^{-2}\delta(r-r'); \quad (14)$$

here, M is the nucleon mass, l is the orbital quantum number and $V_a(r)$ is the total (including spin-orbit) potential associated with the s-a-i state I_a. The calculation thus consists of (1) finding the Green's function (14) associated with the potential $V_a(r)$, (2) evaluating the radial integral in (13) for $Q_{a\epsilon}(r)$:

$$Q_{a\epsilon}(r)=\int_0^\infty G_{a\epsilon}(r,r')R_a(r')r'^4dr', \quad (15)$$

(3) evaluating the integrals in the first of Eqs. (13), and (4) combining terms (12) into S_{f0}.

In order to simplify steps (1) and (2), we assume that $V_a(r)$ is a finite constant for $0\leqq r<R$ and positively infinite for $r>R$, and in order to simplify step (3), we assume that the pair interaction has zero range:

$$H_p'(1,2)=C\delta(\mathbf{r}_1-\mathbf{r}_2). \quad (16)$$

Step (4) is independent of the assumptions made concerning the forms of $V_a(r)$ and $H_p'(1,2)$, and involves instead the combinations of single-particle s-a-i states that are used to build up the initial and final states.

The Green's function for the infinitely deep square well potential is

$$G_{a\epsilon}(r,r')=-(2M\alpha/\hbar^2)\{[j_l(\alpha r_<)n_l(\alpha r_>)]$$
$$-[j_l(\alpha r)j_l(\alpha r')n_l(\alpha R)/j_l(\alpha R)]\}, \quad (17)$$
$$\hbar^2\alpha^2/2M=\epsilon-V_a,$$

where $r_<$ and $r_>$ are the lesser and greater, respectively, of r, r', and j_l and n_l are the spherical Bessel functions that are regular and irregular, respectively, at the origin. The second square bracket term in (17) is needed to make $G_{a\epsilon}$ vanish at $r=R$, as is required by the infinite potential step. The normalized radial function for the state a is

$$R_a(r)=Bj_l(\alpha_0 r), \quad j_l(\alpha_0 R)=0,$$
$$\hbar^2\alpha_0^2/2M=\epsilon_a-V_a, \quad B^2\int_0^R j_l^2(\alpha_0 r)r^2dr=1. \quad (18)$$

Substitution of Eqs. (17) and (18) into (15) gives

$$Q_{a\epsilon}(r)=-(2M\alpha B/\hbar^2)\Big\{n_l(\alpha r)\int_0^r j_l(\alpha r')j_l(\alpha_0 r')r'^4dr'$$
$$+j_l(\alpha r)\int_r^R n_l(\alpha r')j_l(\alpha_0 r')r'^4dr'$$
$$-[j_l(\alpha r)n_l(\alpha R)/j_l(\alpha R)]$$
$$\times\int_0^R j_l(\alpha r')j_l(\alpha_0 r')r'^4dr'\Big\}. \quad (19)$$

In the case of C^{12} under consideration here, the radial functions for states a, b, c, d are all of the form (18) with $l=1$. The initial states a and b have $j=\frac{3}{2}$, and the final states c and d have $j=\frac{1}{2}$. Both pairs of states have the same value of α_0, since this is determined by the boundary condition at $r=R$. Thus, $\epsilon_a-V_a=\epsilon_c-V_c$; also, $\epsilon_c-\epsilon_a$ is equal to half the excitation energy $\Delta=7.68$ Mev. With our assumption that the excited state is built up as the (0,0) combination of $(T,J)=(1,0)$ substates of two $p_{\frac{3}{2}}$ nucleons and two $p_{\frac{3}{2}}$ holes, it can be shown after some calculation that steps (3) and (4) lead to the following expression for the first-class part of S_{f0}[17]:

$$-[7(\tfrac{3}{2})^{\frac{1}{2}}M\alpha CB^4/15\pi\hbar^2]\int_0^R j_1^3(\alpha r)F(r)r^2dr, \quad (20)$$

where $F(r)$ is the curly bracket of Eq. (19) with $l=1$. The first-class part of $\bar{S}_{0f}$ is given by Eq. (20) with α replaced throughout by α', where

$$\hbar^2\alpha'^2/2M=\epsilon'-V_c, \quad \epsilon'\equiv\epsilon_a+\epsilon_b-\epsilon_d. \quad (21)$$

From Eqs. (17), (18), and (21), it follows that

$$\alpha^2=\alpha_0^2+(2M\Delta/\hbar^2), \quad \alpha'^2=\alpha_0^2-(2M\Delta/\hbar^2). \quad (22)$$

In principle, all of the radial integrations implied in Eq. (20) can be performed analytically. In practice, it is desirable to calculate $F(r)$ analytically, and then to carry through the final integration over r numerically. This procedure suffers from the draw back that α, α', and α_0 are close together in magnitude: $\Delta=7.68$ Mev, and $\hbar^2\alpha_0^2/2M=46.5$ Mev for $R=3\times10^{-13}$ cm. This means that $F(r)$ is large because of the appearance of $j_1(\alpha R)$ in the denominator of the last term, since $j_1(\alpha_0 R)=0$. Moreover, since $j_1(\alpha R)$ is negative and $j_1(\alpha' R)$ is positive, the large contributions from S_{f0} and $\bar{S}_{0f}$ nearly cancel, so that the numerical work must be done with great accuracy. It is therefore desirable to expand Eq. (20) as a Taylor series in α about the point α_0, treat $\bar{S}_{0f}$ in the same way, and keep the leading term in the sum. It is easily shown that the error in this procedure is of relative order $(2M\Delta/\hbar^2\alpha_0^2)^2=0.027$, and a numerical check of the value quoted below shows that the error is actually less than one percent. The resulting expression for the sum of the first-class parts of S_{f0} and $\bar{S}_{0f}$ is the same as Eq. (20) with α replaced by α_0 and $F(r)$ replaced by

$$-(6\alpha_0^7r^2)^{-1}\{\sin x[(3\beta^2+5)-(2\beta^2+5)x^2+2x^4]$$
$$-x\cos x[(3\beta^2+5)-5x^2]\}, \quad (23)$$
$$x\equiv\alpha_0 r, \quad \beta\equiv\alpha_0 R.$$

In deriving Eq. (23), use has been made of the fact that $\tan\beta=\beta$, which is a consequence of $j_1(\beta)=0$. The matrix

[17] It is convenient to make use of some relations given by E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1935), Chap. 6, Sec. 5 and Chap. 12, Sec. 1; individual nucleon quantum numbers j, m, τ may be substituted for the individual electron quantum numbers l, m_l, m_s used by Condon and Shortley.

element of Ω is the negative of the sum of the first-class parts of S_{f0} and $\bar{S}_{0f}$, and so is equal to

$$\text{M.E.}=[7(\tfrac{3}{2})^{\frac{1}{2}}MCR(B^2R^3)^2/90\pi\hbar^2\beta^7]\times\int_0^\beta j_1{}^3(x)\{\ \}dx, \quad (24)$$

where { } denotes the curly bracket in Eq. (23).

V. NUMERICAL RESULT AND DISCUSSION

The x integral in Eq. (24) can be evaluated analytically, but is much easier to do numerically; it is equal to -1.98. From Eq. (18), the normalization constant B is given by $B^2R^3=6\beta/\sin2\beta$. Substitution of $\beta=4.493$ gives finally

$$\text{M.E.}=-1.58\times10^{22}CR\ \text{cm}^2,$$

where R is measured in cm and C in Mev-cm^3. If now we set $|\text{M.E.}|=3.8\times10^{-26}$ cm^2, as in Sec. I, and $R=3.0\times10^{-13}$ cm, we obtain $C=8.0\times10^{-36}$ Mev-cm^3. This is about six times as large as the volume integral of the triplet neutron-proton interaction, when it is assumed to be of Gaussian form.[18] Conversely, even if C were chosen to correspond to the free-space neutron-proton interaction, in which case the first-order perturbation theory used here would not be reliable, M.E. would have only about one-sixth the experimental value.

This result, together with those of Sec. II, suggests that a model that is more collective than the independent-particle model with pair interactions and less collective than the alpha-particle or elastic-fluid models, is required to account for the experimental observations. The suggestion of Christy and Fowler,[19] that low-lying excited states in the p shell nuclei arise from excitation of four nucleons, may be promising in this connection.

The writer is indebted to Dr. D. R. Yennie and Dr. M. G. Redlich for stimulating conversations, and to Professor R. Hofstadter for discussion of the experimental situation.

[18] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949); other forms for the interaction have roughly similar volume integrals.

[19] R. F. Christy and W. A. Fowler, Phys. Rev. **96**, 851(A) (1954), and private communication from W. A. Fowler. With the Hamiltonian (5) and this model for the excited state, it would be necessary to go at least to second order in H' in order for the matrix element not to vanish; it would probably then be desirable for the calculation to stress the collective rather than the perturbation aspects of the situation.

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Nuclear Moments of Nb^{93}, La^{139}, Os^{187}, and Hg^{201}

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The effective charge Z^* for a d-electron was studied by means of a hyperfine structure (hfs) investigation; the screening correction $(Z-Z^*)$ was found to range from 15 to about 19 for the charge number Z ranging from 27 to 78. Investigation of the hfs of the spectra of Nb I and La I yielded the result that $Q(\text{Nb}^{93})=(-0.2\pm0.1)\times10^{-24}$ cm^2 and $Q(\text{La}^{139})=(+0.6\pm0.2)\times10^{-24}$ cm^2, respectively, in which the polarization correction (due to Sternheimer) is taken into account. In the hfs of the spectrum of Os I the components due to the rarer odd isotope Os^{187} were detected and it was found that Os^{187} has a spin 1/2 and a magnetic moment most probably equal to $+0.12$ nm (possible range being from $+0.16$ nm to $+0.09$ nm). The quadrupole moment of Hg^{201} was calculated from the hfs of $6s6p\ {}^1P_1$ and $6s6p\ {}^3P_2$ of the spectrum of Hg I, taking the configuration interaction into account, and $Q(\text{Hg}^{201})=(+0.45\pm0.04)\times10^{-24}$ cm^2 was obtained.

I. SCREENING CORRECTION FOR A d-ELECTRON

IN the hyperfine structure (hfs) formulas for both the interval factor (A) and the quadrupole coupling constant (B) of a configuration containing d-electrons, the effective charge $Z_d{}^*$ for a d-electron frequently enters. It is usual to put $Z_d{}^*=Z-\sigma_d$, where σ_d is the screening correction for a d-electron. Casimir[1] assumed that $\sigma_d=10$, and this value has been adopted by many investigators. However, as far as the author is aware, this has no sound experimental basis. In order to fill this gap, an experimental investigation of the hfs of several atomic spectra was undertaken. A liquid-air-cooled hollow-cathode discharge tube described previously[2] was used, and a Fabry-Pérot etalon was used to resolve the hfs.

We begin with the analysis of the configuration $4d^3$ of Nb I.[3] Nb is known to consist of only one isotope Nb^{93} with spin 9/2.[4] The hfs was previously measured by Meeks and Fisher,[5] using a water-cooled hollow

[1] H. Casimir, Verhandel. Teylers Tweede Genootschap, Haarlem (1936), p. 11.

[2] K. Murakawa, J. Phys. Soc. (Japan) **9**, 391 (1954).

[3] The notation of the level symbol of the spectrum of Nb I was taken from W. F. Meggers and B. F. Scribner, J. Research Natl. Bur. Standards **14**, 629 (1935).

[4] J. E. Mack, Revs. Modern Phys. **22**, 64 (1950). P. F. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952). K. Murakawa and T. Kamei, Rept. Inst. Sci. Technol. Univ. Tokyo **7**, 219 (1953).

[5] W. W. Meeks and R. A. Fisher, Phys. Rev. **72**, 451 (1947).