approximately three to four units. Thus, again, it would seem possible that a complementary shift of approximately two charges in the positions of the peaks of the light and heavy fragments would bring the calculations into line with experiment. It is possible, since the light fragment tends to have an excess of neutrons in the division, that some allowance for neutron-proton interactions might be able to account for this shift.

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Angular Momentum Coupling in Deuteron Reactions*

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Information about nuclear coupling schemes which can be derived from relative cross sections in deuteron stripping and pickup reactions in light nuclei is considered. In a few cases, experimental results are applied to give a determination of the intermediate-coupling parameter.

1. INTRODUCTION

In recent years, the intermediate-coupling shell model has received the attention of a number of authors.[1] Energy levels of p shell nuclei in particular have been studied in intermediate coupling because they are not consistent with either of the extreme coupling schemes. The most striking cases are given in the very complete and useful survey paper of Inglis[1] where approximate level schemes are given as a function of the intermediate-coupling parameter which measures the relative effective strengths of the spin-orbit and central parts of the interaction.

Our present purpose is to consider certain deuteron stripping and pickup reactions using intermediate coupling wave functions. By considering the ratio of the cross sections of a d-p reaction leading to different states of the same final nucleus one can also deduce the value of the intermediate-coupling parameter and thus obtain an independent check on the validity of the model.

We should remark that the present work was stimulated by an investigation of Christy[2] on angular momentum coupling in a certain class of resonant nuclear reactions. Christy's work confined itself to the two extreme coupling schemes and did not require the construction of explicit nuclear wave functions. These become essential, however, if we wish to depart from the extreme cases. When one does this one encounters, among others, the difficulty that the compound state is almost always quite highly excited and thus may not be a good candidate for description by a shell model. Deuteron reactions do not have this difficulty, for we may restrict ourselves to low-lying states. There is also the important fact that the essential features of these reactions are well understood and adequately described by Butler's theory.[3]

An analysis similar to ours has already been published by Lane.[1] Lane has in fact emphasized the desirability of using intermediate-coupling wave functions for many purposes beyond the elementary one of calculating level schemes. In his first-cited paper, magnetic dipole moments and transition strengths and reduced widths for nucleon emission are considered and applied with success to the low-lying states of C^{13} and N^{13}. In the present work (which was done independently of Lane) we consider stripping and pick-up reactions involving nuclei with $A=6$, 7, 13, 14.

2. DEUTERON REACTION CROSS SECTIONS

We first write the d-p cross section using the "Born approximation" theory[4] which is equivalent to, but simpler in formulation, than the original theory of Butler. Corrections such as the Coulomb effect and the interaction between proton and nucleus are neglected because of the large uncertainties in the shell model treatment of nuclear spectroscopy. The details of the Born approximation are well-known and we do not give them; we consider in detail only the overlap integral between the initial and final systems since it is from this (or rather a ratio of two of them) that we shall gain information concerning the intermediate coupling parameter. This integral can be related to a

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[1] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953); Phys. Rev. **87**, 915 (1952); N. Zeldes, Phys. Rev. **90**, 416 (1953); G. E. Tauber and T. Y. Wu, Phys. Rev. **93**, 295 (1954); **94**, 1307 (1954); A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953); Phys. Rev. **92**, 839 (1953); R. Schulten, Z. Naturforsch. **8**, 759 (1953); R. Schulten and R. A. Ferrell, Phys. Rev. **94**, 739 (1954).
[2] R. F. Christy, Phys. Rev. **89**, 839 (1953).
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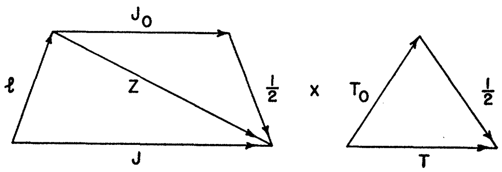

FIG. 1. Vector-coupling diagram for $\psi(zJ_0JT_0T)$.

"reduced width" of the final state (see Lane[1]) but we prefer not to adopt this point of view.

Consider

$$A[J_0,T_0,M_{T_0}]+\text{nucleon}[l,m]\rightarrow(A+1)[J,T,M_T], \quad (1)$$

which symbolically describes a stripping reaction. A and $A+1$ are atomic numbers, J_0 and J are nuclear angular momenta, T_0, T, M_{T_0}, and M_T refer to nuclear isotopic spins, and l, m refer to the orbital angular momentum and isotopic spin of the captured nucleon. By vector-coupling the initial nuclear wave function to that for the nucleon, we may form the wave function $\psi(zJ_0JT_0T)$ which has specified channel spin z. To simplify the notation, the dependence of ψ on space and spin coordinates will not be shown explicitly except where needed. The angular momenta, spins, and isotopic spins in $\psi(zJ_0JT_0T)$ are coupled according to the vector diagram in Fig. 1.

We now find in the cross section the factor $\sum_z|\langle\psi(JT)\psi(zJ_0JT_0T)\rangle|^2$. If we describe the nuclear wave functions in terms of the configurations l^{n-1}, l^n the integral occurring here may be factored into two parts. If we take only the first part involving the radial integrations (which is independent of z) we find (except of course for the isotopic spin vector coupling factor $[C^{T_0\frac{1}{2}T}(M_{T_0}m)]^2$) simply the single-particle model stripping cross section which of course is entirely independent of the nuclear structure. Consideration of the second factor multiplies the cross section by $n\sum_z\beta_z^2$, where we have

$$\beta_z=\int\psi(JT;\xi)\psi(zJ_0JT_0T;\xi)d\xi. \quad (2)$$

In (2), we understand by ξ the angular spin and isotopic spin coordinates only.

It is clear that β_z will in general depend on the details of the shell-model wave functions and in particular on the intermediate-coupling parameter $\zeta=$Inglis' a/K.[1] Moreover, if we consider the ratio $d\sigma/d\sigma^*$ for a deuteron reaction leading to two different states of the final nucleus (with preferably the same l value) then the kinematic factors entering in the cross section may be largely eliminated and the cross-section ratio expressed in terms of the "stripping ratio" $R(\zeta)=(\sum_z\beta_z^2)/(\sum_z\beta_z^{*2})$. The details of this are given in the Appendix. Broadly speaking, we expect this to be a reasonable procedure if the angular distributions are well described by Butler's theory[3] and if the outgoing velocities for the two levels are not too different. (Both of these criteria may be satisfied if the incident beam energy is high enough.)

3. NUCLEAR WAVE FUNCTIONS

To find the nuclear wave functions appearing in (2), we perform a standard intermediate-coupling calculation using LS wave functions as a basis. For the two-particle interaction we take Rosenfeld's saturation potential,[5]

$$H_1=\sum_{i<j}V(r_{ij})(0.1+0.23\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j. \quad (3)$$

A general expression, in terms of the L and K of Feenberg and Phillips,[6] for the matrix elements of H_1 for p-shell nuclei (and in this paper we consider only p-shell cases) has been first given by Racah.[7] In order to have consistency in phase relations with other results which we need, we use the equivalent result as given by Elliott, Hope, and Jahn.[8] For numerical work we take $L/K=6$ as suggested by Hummel and Inglis.[9]

The spin-orbit dependence is accounted for by a single-particle spin-orbit force:

$$H_2=\sum_i a(r_i)\mathbf{s}_i\cdot\mathbf{l}_i. \quad (4)$$

The matrix elements of H_2 may be readily derived in terms of p-shell fractional parentage coefficients which have been completely tabulated by Jahn and Van Wieringen.[10] Except for a sign the result is as given in Eq. (32) of the first cited paper by Lane.[1]

The procedure now is to evaluate and diagonalize the Hamiltonian matrix H for a specified J and T. We do this as a function of the coupling parameter $\zeta=a/K$, and then find for the eigenfunction

$$\psi(JT)=\sum_{\alpha LS}K(\alpha LSJT;\zeta)\psi(\alpha LSJT), \quad (5)$$

where $\alpha\equiv$partition describing the space symmetry of the corresponding basis vector and the constants K emerge from the diagonalization. In terms of these constants, we have

$$\beta_z(\zeta)=\sum_{\substack{\alpha_0L_0S_0\\ \alpha LS}}(-1)^{1+L_0+L}K(\alpha LSJT;\zeta)\times K(\alpha_0L_0S_0J_0T_0;\zeta)U(L_0S_0z\tfrac{1}{2};J_0S)\times U(L_0S|J;zL)\langle\alpha LST|\alpha_0L_0S_0T_0\rangle, \quad (6)$$

where U is a normalized Racah coefficient[11] and $\langle\alpha LST|\alpha_0L_0S_0T_0\rangle$ a coefficient of fractional parentage.

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[10] H. A. Jahn and H. Van Wieringen, Proc. Roy. Soc. (London) **A209**, 502 (1951).
[11] L. C. Biedenharn, "Tables of the Racah Coefficients," Oak Ridge National Laboratory Report ORNL-1098, 1952 (unpublished); Simon, Van der Sluis, and Biedenharn, Oak Ridge National Laboratory Report ORNL-1679, 1954 (unpublished).

TABLE I. The low-lying p^n-states of $A=6, 7, 13, 14$ nuclei in intermediate coupling.

Nucleus	State of excitation (Mev)	Wave functions: Intermediate coupling	LS limit	jj limit
Li^6	Ground	${}^{13}S_1{}^{[2]}$ ${}^{13}D_1{}^{[2]}$ ${}^{11}P_1{}^{[11]}$	${}^{13}S_1{}^{[2]}$	$p_{3/2}{}^2$
Li^6	2.19	${}^{13}D_3{}^{[2]}$	${}^{13}D_3{}^{[2]}$	$p_{3/2}{}^2$
Li^6	3.57	${}^{31}S_0{}^{[2]}$ ${}^{33}P_0{}^{[11]}$	${}^{31}S_0{}^{[2]}$	$p_{3/2}{}^2$
Li^6	4.52	${}^{13}D_2{}^{[2]}$	${}^{13}D_2{}^{[2]}$	$p_{3/2}{}^2$
Li^7, (Be^7)	Ground	${}^{22}P_{3/2}{}^{[3]}$ ${}^{22}P_{3/2}{}^{[21]}$ ${}^{24}P_{3/2}{}^{[21]}$ ${}^{22}D_{3/2}{}^{[21]}$ ${}^{24}D_{3/2}{}^{[21]}$	${}^{22}P_{3/2}{}^{[3]}$	$p_{3/2}{}^3$
Li^7, (Be^7)	0.48	${}^{22}P_{1/2}{}^{[3]}$ ${}^{22}P_{1/2}{}^{[21]}$ ${}^{24}P_{1/2}{}^{[21]}$ ${}^{24}D_{1/2}{}^{[21]}$ ${}^{22}S_{1/2}{}^{[111]}$	${}^{22}P_{1/2}{}^{[3]}$	$p_{3/2}{}^3$
Li^7, (Be^7)	4.62	${}^{22}F_{7/2}{}^{[3]}$ ${}^{24}D_{7/2}{}^{[21]}$	${}^{22}F_{7/2}{}^{[3]}$	$p_{3/2}{}^3$
Li^7, (Be^7)	7.47	${}^{22}F_{5/2}{}^{[3]}$ ${}^{24}P_{5/2}{}^{[21]}$ ${}^{22}D_{5/2}{}^{[21]}$ ${}^{24}D_{5/2}{}^{[21]}$	${}^{22}F_{5/2}{}^{[3]}$	$p_{3/2}{}^3$
C^{13}, (N^{13})	Ground	${}^{22}P_{1/2}{}^{[441]}$ ${}^{22}P_{1/2}{}^{[432]}$ ${}^{24}P_{1/2}{}^{[432]}$ ${}^{24}D_{1/2}{}^{[432]}$ ${}^{22}S_{1/2}{}^{[333]}$	${}^{22}P_{1/2}{}^{[441]}$	$p_{3/2}{}^8\, p_{1/2}$
C^{13}, (N^{13})	3.68	${}^{22}P_{3/2}{}^{[441]}$ ${}^{22}P_{3/2}{}^{[432]}$ ${}^{24}P_{3/2}{}^{[432]}$ ${}^{22}D_{3/2}{}^{[432]}$ ${}^{24}D_{3/2}{}^{[432]}$	${}^{22}P_{3/2}{}^{[441]}$	$p_{3/2}{}^7\, p_{1/2}{}^2$
C^{13}, (N^{13})	?	${}^{22}F_{5/2}{}^{[441]}$ ${}^{24}P_{5/2}{}^{[432]}$ ${}^{22}D_{5/2}{}^{[432]}$ ${}^{24}D_{5/2}{}^{[432]}$	${}^{22}F_{5/2}{}^{[441]}$	$p_{3/2}{}^7\, p_{1/2}{}^2$
C^{14}	Ground	${}^{31}S_0{}^{[442]}$ ${}^{33}P_0{}^{[433]}$	${}^{31}S_0{}^{[442]}$	$p_{3/2}{}^8\, p_{1/2}{}^2$
C^{14}	6	${}^{31}D_2{}^{[442]}$ ${}^{33}P_2{}^{[433]}$	${}^{31}D_2{}^{[442]}$	$p_{3/2}{}^7\, p_{1/2}{}^3$
N^{14}	Ground	${}^{13}S_1{}^{[442]}$ ${}^{13}D_1{}^{[442]}$ ${}^{11}P_1{}^{[433]}$	${}^{13}S_1{}^{[442]}$	$p_{3/2}{}^8\, p_{1/2}{}^2$
N^{14}	2.31	${}^{31}S_0{}^{[442]}$ ${}^{33}P_0{}^{[433]}$	${}^{31}S_0{}^{[442]}$	$p_{3/2}{}^8\, p_{1/2}{}^2$
N^{14}	3.95	${}^{13}S_1{}^{[442]}$ ${}^{13}D_1{}^{[442]}$ ${}^{11}P_1{}^{[433]}$	${}^{13}D_1{}^{[442]}$	$p_{3/2}{}^7\, p_{1/2}{}^3$

The states for which this procedure has been carried out are listed in Table I. The correlation between wave functions and levels is made on the basis of Inglis' review article[1] and the survey paper by Ajzenberg and Lauritsen.[12] The third column lists, in spectroscopic notation, ${}^{2T+1,2S+1}L^{[\alpha]}$, the LS wave functions which enter in each case. The last two columns list the states to which the intermediate coupling functions go in the two coupling schemes.

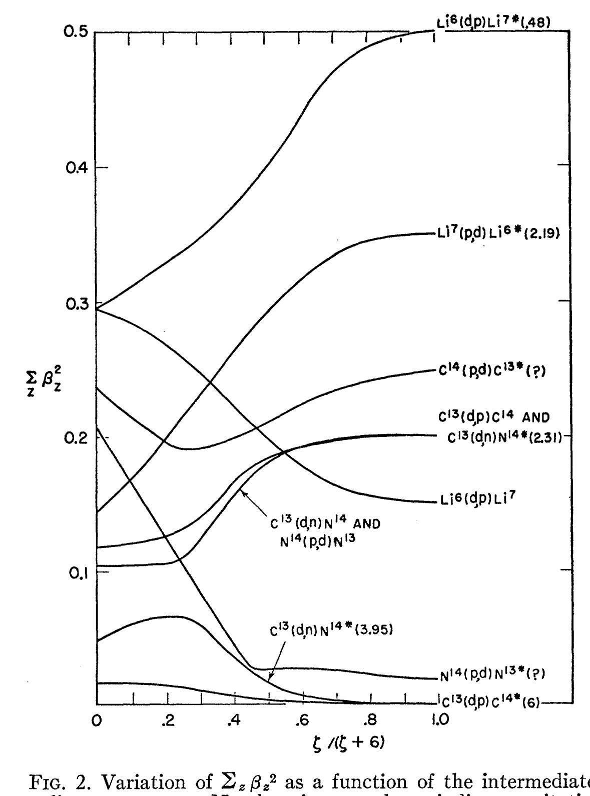

FIG. 2. Variation of $\sum_z \beta_z{}^2$ as a function of the intermediate-coupling parameter ζ. Numbers in parentheses indicate excitation energies.

[12] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **24**, 321 (1952).

We omit here the numerical details of the calculation. They are contained in an unpublished report.[13]

4. COMPARISON WITH EXPERIMENT

We are now enabled to deduce the coupling parameter in two ways. The first is by comparison of the predicted level schemes with the observed ones. The procedure here is equivalent to that of Inglis,[1] minor differences being that we use a slightly different interparticle potential and derive the transition curves by exact diagonalization rather than by the approximate procedure often used by Inglis. The results are not significantly different from those of Inglis and we do not reproduce the curves here. Specifically we find as the optimum value $\zeta=1.4$, 1.1, 4.4 for $A=6$, 7, 14, respectively. For $A=13$, only one odd-parity excited state has been identified below 10 Mev and hence, following Inglis, we arbitrarily take $\zeta=5$ and with this value predict a $5/2^-$ level near 5.3 Mev.

The second determination of ζ comes from deuteron reactions. In Fig. 2 we plot against $\zeta/(\zeta+6)$ the quantity $\sum_z \beta_z{}^2$ and in Fig. 3 the "stripping ratio" $R(\zeta)$ for a variety of reactions.

We now discuss briefly a few pertinent experiments. In each case the comparison is made by first fitting Butler curves to the angular distribution and then using the best value of r_0 thus found to calculate the F_l factor of the Appendix. We are then enabled to compute $R(\zeta)$ from experiment and a comparison with the curves of Fig. 3 gives us the value of ζ. We are of course free to take different ζ values for the two nuclei entering in a reaction, but it is perhaps to be expected that the values should be about the same for neighboring nuclei and we shall in fact make this assumption.

(a) $Li^7(p,d)Li^6$, Li^{*6}

In this experiment, by Standing,[14] the proton energy is 17.5 Mev and the first two states of Li^6 are observed.

[13] T. Auerbach and J. B. French, Atomic Energy Commission Report NYO-3711, 1954 (unpublished).
[14] K. G. Standing (private communication).

The best value of r_0 is given by Standing as 5.5×10^{-13}. The peaks occur near $\theta=18°$ and the cross-section ratio at this angle is $d\sigma/d\sigma^*=2.0\pm0.2$.

We find $R(\zeta)=0.65(d\sigma/d\sigma^*)=1.3\pm0.1$.

From Fig. 3 we have $1.4\leqslant\zeta\leqslant2.1$, where of course the quoted "error" corresponds simply to the transcription of the experimental error and does not include any error assignments for parameters which have been chosen, or for more fundamental defects in the entire theory.

The value of ζ found here is in excellent agreement with the value $\zeta=1.4$ found from the level scheme.

(b) $Li^6(d,p)Li^7$, Li^{*7}

Two experiments are available: Holt and Marsham[15] use an incident energy of 8 Mev; Whaling and Bonner[16] measure the same reaction at several energies, the highest being 1.4 Mev which, from our point of view, is still quite low. In each case the first two states are observed.

The data of Holt and Marsham are fitted with $r_0=4.9\times10^{-13}$ cm. We compare at $\theta=25°$. There we find $R(\zeta)=0.52(d\sigma/d\sigma^*)=0.68\pm0.5$.

This corresponds (by Fig. 3) to $2.3\leqslant\zeta\leqslant3.5$. The data of Whaling and Bonner lead to about the same value. The value of ζ is considerably larger than that given by the level scheme. However, as discussed in detail by Inglis,[1] the picture of almost pure LS coupling in this nucleus which is implied by a very small ζ is incompatible with the relatively large observed ratio between the ^2F and ^2P splitting. We assume that the 4.62-Mev level has $J=7/2^-$. It is true that this situation is not at all improved by increasing ζ, but we may expect that it could be if we were to use a more adequate model and interaction.

The difference between the ζ values of (a) and (b) could perhaps be reconciled by taking different values for the two nuclei but at the moment this seems an unnecessary refinement.

(c) $C^{13}(d,p)C^{14}$, C^{*14}

The experiment with 4-Mev deuterons by Benenson[17] indicates that the ground-state reactions at 10° and 25° are about 3 percent and 40 percent as intense as the first excited state. This is in disagreement with the theory (assuming the excited state to be 2^+) which gives $R(\zeta)\geqslant7$, and hence $d\sigma/d\sigma^*>0.2$ and >0.5 for the two angles. It has since become clear[18] that in the region 6–7 Mev of excitation in C^{14} there are three levels. The 6.1-Mev level observed by Benenson corresponds presumably to $l=0$, thus giving a rather better fit than $l=1$ to his data and explaining the relatively large cross section. It is possible that the 6.72-Mev level is the 2^+ level predicted by the theory.

[15] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 1032 (1953).

[16] W. Whaling and T. W. Bonner, Phys. Rev. **79**, 258 (1950).

[17] R. E. Benenson, Phys. Rev. **90**, 420 (1953).

[18] A. Sperduto, Massachusetts Institute of Technology Progress Report, May, 1954 (unpublished).

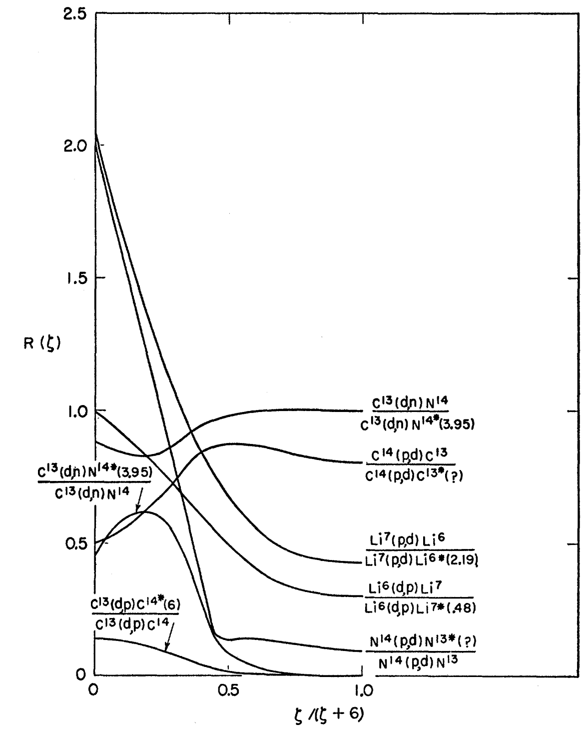

FIG. 3. Variation of the stripping ratio $R(\zeta)$ as a function of ζ. Numbers in parentheses indicate excitation energies.

(d) $C^{13}(d,n)N_{14}^*$, N_{14}^{**}

This experiment with 4-Mev deuterons is also due to Benenson.[17] The accuracy is unfortunately not very satisfactory for our purposes and at best we can determine the coupling parameter only very roughly. We do not consider the ground state reaction because it gives a quite poor fit to an $l=1$ curve. (The ground state reaction is known from the work of Bromley[19] and also of Standing[20] to have $l=1$.)

Taking $r_0=4.8\times10^{-13}$ cm and $\theta\sim20°$, we find $R(\zeta)=4.0d\sigma/d\sigma^*$; and estimating the cross-section ratio from Benenson's paper, we decide that $\zeta=3.7\pm1.5$. This range of values is consistent with the value $\zeta=4.4$ from the level scheme.

We note, incidentally, that the Q values here are quite small as is also the deuteron energy. Both these effects make the kinematical corrections quite large (the factor arising from the deuteron Fourier transform alone provides almost a factor 2 in the ratio). One may well doubt whether our understanding of the dynamics of a deuteron reaction is sufficient to make such large corrections trustworthy.

[19] D. A. Bromley, Phys. Rev. **88**, 565 (1952).

[20] K. G. Standing, Phys. Rev. **94**, 731 (1954).

5. CONCLUSION

The cases which we have examined here and the one already considered in detail by Lane[1] suggest that deuteron reactions may well be of value in learning about quite detailed features of the structure of light nuclei. To examine this, further results for several other cases involving p-shell nuclei will shortly be published. Beyond the p shell the situation is more complex; one encounters a large number of parameters and quite incomplete data concerning spins and parities. Still, information derived from deuteron reactions may be of considerable value.

We should finally point out that we have adopted a quite elementary view of the mechanism of deuteron reactions, and for that reason have been unwilling to place much reliance on the "kinematic" corrections whenever they are quite different for the two reactions compared. A more refined theory would of course extend the flexibility of the procedure. In the same way the simple treatment of the spectroscopy should eventually be replaced by a more realistic one, for example, taking into account tensor forces and inter-configuration mixing.

6. ACKNOWLEDGMENTS

Finally, we wish to thank Dr. D. A. Bromley and Dr. R. E. Benenson for valuable discussions concerning the experiments and Dr. K. G. Standing for information concerning his work with Li^7. We have benefited from a discussion with Dr. D. R. Inglis. S. P. Pandya, B. J. Raz, and Mrs. M. Halbert have been helpful in many ways.

APPENDIX

For the stripping reaction of Eq. (1), the "Born approximation" theory gives

$$\frac{d\sigma}{d\Omega}=C\frac{2J+1}{2J_0+1}\frac{k}{k_D}\frac{1}{(\alpha^2+\kappa^2)^2(\gamma^2+\kappa^2)^2}\times F_l^2(q,r_0,t)\{[C^{T_0\frac{1}{2}T}(M_{T_0}m)]^2\sum_z \beta_z^2\},$$

where C is an immaterial constant. We have used the Hulthén wave function for the deuteron defined by α and $\gamma\simeq 7\alpha$, $\mathbf{k}$ and $\mathbf{k}_D$ are the nucleon and deuteron momenta in the c.m. system,

$$\mathbf{q}=\mathbf{k}_D-\frac{A}{A+1}\mathbf{k},\quad \boldsymbol{\kappa}=\tfrac{1}{2}\mathbf{k}_D-\mathbf{k},\quad \hbar^2t^2=\left(\frac{2A}{A+1}\right)MB,$$

where B is the binding energy of the captured nucleon and

$$F_l=-\hbar^2\frac{A+1}{2AM}R_l(r_0)r_0^2\left\{\frac{\partial j_l(qr)}{\partial r}-\frac{j_l(qr)}{h_l^{(1)}(itr)}\frac{\partial}{\partial r}h_l^{(1)}(itr)\right\}_{r_0}.$$

Here $R_l(r_0)$ is the captured nucleon amplitude at the nuclear radius and we make the approximation that this is independent of the state of excitation.

To calculate the kinematic factors for both stripping and pickup reactions it is convenient to introduce the following quantities: Let $Q=Q$-value in c.m. system *for a stripping reaction*, $E_0=$ deuteron kinetic energy in system with A at rest, $E_1=$ nucleon kinetic energy in system with $(A+1)$ at rest, and $\theta=$ c.m. angle $(\mathbf{k}\cdot\mathbf{k}_0=kk_D\cos\theta)$. For a stripping reaction we will be given (Q,E_0), for a pickup reaction (Q,E_1). We have $(A+1)E_1-AE_0=(A+2)Q$. Then, measuring energies in Mev and wave numbers in units of 10^{13} cm^{-1}, we have

$$q=0.22\left(\frac{A}{A+2}\right)[2E_0+E_1-2(2E_0E_1)^{\frac{1}{2}}\cos\theta]^{\frac{1}{2}},$$

$$\kappa^2=0.0242Q+\frac{A+1}{2A}q^2,\quad t=0.22\left[\frac{A}{A+1}(Q+2.22)\right]^{\frac{1}{2}}.$$

We have now for the stripping reaction of Eq. (1):

$$\frac{d\sigma}{d\sigma^*}=\frac{2J+1}{2J^*+1}\left(\frac{E_1}{E_1^*}\right)^{\frac{1}{2}}\left(\frac{C^{T_0\frac{1}{2}T}(M_{T_0}m)}{C^{T_0\frac{1}{2}T^*}(M_{T_0}m^*)}\right)^2\times\left[\frac{(\alpha^2+\kappa^{*2})(\gamma^2+\kappa^{*2})}{(\alpha^2+\kappa^2)(\gamma^2+\kappa^2)}\right]^2\left(\frac{F_l}{F_l^*}\right)^2R(\zeta),$$

and for the inverse defined also by Eq. (1):

$$\frac{d\sigma}{d\sigma^*}=\left(\frac{E_0}{E_0^*}\right)^{\frac{1}{2}}\left(\frac{C^{T_0\frac{1}{2}T}(M_{T_0}m)}{C^{T_0^*\frac{1}{2}T}(M_{T_0}^*m^*)}\right)^2\times\left[\frac{(\alpha^2+\kappa^{*2})(\gamma^2+\kappa^{*2})}{(\alpha^2+\kappa^2)(\gamma^2+\kappa^2)}\right]^2\left(\frac{F_l}{F_l^*}\right)^2R(\zeta).$$