

## Nuclear Spin Relaxation in Alkali Metals\*

D. F. HOLCOMB† AND R. E. NORBERG‡

*Department of Physics, University of Illinois, Urbana, Illinois*

(Received September 23, 1954)

Nuclear magnetic resonance measurements have been made in metallic lithium, sodium, and rubidium, using pulsed radio-frequency power. The experimental data are values of  $T_1$ , the spin-lattice relaxation time, and of  $T_2$ , the inverse line width or spin-spin relaxation time. Measurements were made at several Larmor frequencies and at temperatures between  $-65^\circ\text{C}$  and  $250^\circ\text{C}$ . Over a considerable temperature range,  $T_1$  is found to be primarily determined by interaction with the conduction electrons. The magnitudes of  $T_1$  agree fairly well with the Korringa theory in all three metals. The lithium data in particular indicate that  $P_f/P_a \cong 0.6$ . In lithium and sodium a dependence of  $T_1$  on the resonance frequency is observed, which cannot be explained on the basis of the Korringa theory. Information about the atomic self-diffusion process is also obtained. Portions of the  $T_1$  and  $T_2$

data are interpreted using the theory of Bloembergen, Purcell, and Pound. In addition, some of the  $T_1$  data are interpreted in terms of the lattice diffusion theory of Torrey. These analyses yield values for  $D$ , the coefficient of self-diffusion, of  $0.24_{-0.10}^{+0.17} \times \exp(-13\,200 \pm 400/RT)$   $\text{cm}^2/\text{sec}$  in lithium and  $0.20_{-0.15}^{+0.56} \times \exp(-10\,000 \pm 600/RT)$   $\text{cm}^2/\text{sec}$  in sodium. Although there is some ambiguity in the interpretation of the rubidium data, they indicate  $D \cong 0.23 \exp(-9400/RT)$   $\text{cm}^2/\text{sec}$ . Unusual broadenings of the resonance lines are observed at the melting points in all three metals. These broadenings are not presently understood, but some features of the data can be correlated with a mechanism involving magnetic local fields which arise from lattice imperfections.

### I. INTRODUCTION

THIS paper reports the results of a series of nuclear magnetic resonance experiments performed on three of the alkali metals, lithium, sodium, and rubidium, using the spin echo and induction decay techniques of Hahn.<sup>1</sup> The quantities which we have measured are the spin-lattice relaxation time  $T_1$  and the spin phase-memory time  $T_2$ . ( $T_2 \propto 1/\delta H$ , where  $\delta H$  is the resonance line width.) These time constants have been observed over a range of temperatures from  $-65^\circ\text{C}$  to  $250^\circ\text{C}$ , which includes the melting points of all three metals. Measurements were made at several values of the Larmor frequency in each metal.

Nuclear magnetic resonance (NMR) and electron spin resonance (ESR) measurements in metals have already yielded results of significance for the description of metallic properties. NMR measurements of the line shape, line width,<sup>2-5</sup> and paramagnetic shift of the resonance frequency<sup>3-6</sup> in metals and alloys have given information about the magnetic dipolar interactions of the metal nuclei among themselves and with the conduction electrons. In noncubic lattices, effects have been observed<sup>5</sup> which arise from interaction of the nuclear quadrupole moments with electric field gradi-

ents existing at the lattice sites. From these measurements and others, information can be deduced about the coefficients of self-diffusion, the phase changes, and the electronic distributions in some metals and alloys. Recently, a combination of NMR and ESR measurements<sup>7</sup> in lithium has resulted in the first direct measurement of the electron spin contribution to the bulk susceptibility of a metal.

However, few measurements of  $T_1$  have been reported. The only literature data seem to be some low-temperature measurements on aluminum,<sup>8</sup> copper,<sup>9</sup> and lithium<sup>10</sup> and room temperature data on copper.<sup>11</sup> Without  $T_1$  data, the shift and line width measurements cannot be exploited so as to yield their full information content about the metallic lattice and the conduction electrons. The  $T_1$  data have been lacking primarily because the usual values of  $T_1$  in metals are inconveniently short for measurement by the "steady state" techniques.

In addition, line-width data have not been available for the narrow lines characteristic of cubic metals at temperatures near their melting points. These lines are typically narrower than the smallest easily achievable magnet inhomogeneities.

$T_1$  and  $T_2$  in the alkalis fall generally between a few hundred microseconds and one second and are thus in a particularly convenient range for measurement with the pulse technique. The alkali metals were chosen for this work primarily because of their theoretical simplicity insofar as electronic properties are concerned. In addition, reported experiments<sup>2,3</sup> had already shown the existence of self-diffusion induced line width transitions in these metals. The low-melting points of the

\* Supported by the Office of Naval Research. This paper is based in part upon a thesis submitted by D. F. Holcomb to the University of Illinois in partial fulfillment of the requirements for the Ph.D. degree (January, 1954).

† National Science Foundation Predoctoral Fellow, September, 1952, to August, 1953. Present address: Department of Physics, Cornell University, Ithaca, New York.

‡ Present Address: Department of Physics, Washington University, St. Louis, Missouri.

<sup>1</sup> E. L. Hahn, *Phys. Rev.* **80**, 580 (1950).

<sup>2</sup> H. S. Gutowsky, *Phys. Rev.* **83**, 1073 (1951).

<sup>3</sup> H. S. Gutowsky and B. R. McGarvey, *J. Chem. Phys.* **20**, 1472 (1952).

<sup>4</sup> H. S. Gutowsky and B. R. McGarvey, *J. Chem. Phys.* **21**, 2114 (1953).

<sup>5</sup> N. Bloembergen and T. J. Rowland, *Acta Metallurgica* **1**, 731 (1953).

<sup>6</sup> Townes, Herring, and Knight, *Phys. Rev.* **77**, 852 (1950).

<sup>7</sup> Schumacher, Carver, and Slichter, *Phys. Rev.* **95**, 1089 (1954).

<sup>8</sup> J. Hatton and B. V. Rollin, *Proc. Roy. Soc. (London)* **A199**, 222 (1949).

<sup>9</sup> N. Bloembergen, *Physica* **15**, 588 (1949).

<sup>10</sup> N. J. Poulis, *Physica* **16**, 373 (1950).

<sup>11</sup> D. F. Abell and W. D. Knight, *Phys. Rev.* **93**, 940(A) (1954).

alkalis made it possible to work with the molten metals using relatively crude thermal techniques. Measurements were not made in potassium because of its very small nuclear magnetic moment, which would result in prohibitively small nuclear signals with the present experimental arrangement. Measurements in cesium are planned for the future.

Some of the data on sodium which we report here have already appeared in preliminary form.<sup>12</sup>

## II. APPARATUS AND MEASUREMENTS

A typical sample consisted of about 0.5 cc of an oil dispersion of metallic spheres sealed into a pyrex vial. The dispersions were produced by violent mechanical agitation of chunks of the metal in a high boiling point oil at a temperature slightly greater than the melting point of the metal. Average particle radii for the samples used were: lithium, 12 $\mu$ , sodium, 25 $\mu$ , and rubidium, 50 $\mu$ . Typical samples contained an actual volume of about 0.1 cc of the metal. Several of the dispersions were prepared by Dr. T. R. Carver, and the rest supplied by Professor H. S. Gutowsky and his group in the chemistry department of the university.

Relaxation measurements were made using radiofrequencies between 3 and 15 Mc/sec. Over this range all but the rubidium samples have an average particle size smaller than the rf skin depth. It proved possible, using any of the samples, to achieve an rf pulse power setting such that there was no induction decay following the rf pulse. We may therefore infer that the rf attenuation arising from skin effects was quite small, since it was possible to produce rf magnetic fields of amplitude  $H_1$  and pulse length  $t_w$  for which the magnetic moment nutational angle  $\theta_1 = \gamma H_1 t_w$  was equal to  $\pi$  uniformly over substantially the entire sample volume. ( $\gamma$  is the nuclear gyromagnetic ratio,  $\mu$ , the magnetic moment, is equal to  $\gamma \hbar I$ , where  $I$  is the nuclear spin.) Moreover, it can be shown that there will be no spurious effects on  $T_1$  or  $T_2$  of the type suggested by Bloembergen<sup>13</sup> even if the particles are large compared to the skin depth. These effects, for pulsed experiments, depend on the existence of appreciable diffusion of nuclei through the skin depth in times comparable to  $T_1$  or  $T_2$ . The time to traverse one skin depth is about  $\delta^2/D$ , where  $\delta$  is the skin depth and  $D$  the diffusion constant. The largest values of  $D$  in the alkalis in the temperature

TABLE I. Spectrochemical analyses of the stock materials.

Sample	Percentages for all metallic impurities $\geq 0.01\%$
Lithium A	Na 0.3; Cu, Si 0.05
Lithium B	Na 4; Sb, Pb 0.05
Lithium (Li <sup>6</sup> -enriched)	Ca, Fe, Ni 0.01
Sodium A	Li 0.1; Ca, Mg, Si, Sr 0.05
Sodium B	Li 0.4; Ca, Sr 0.1
Rubidium	(Not analyzed)

<sup>12</sup> R. E. Norberg and C. P. Slichter, Phys. Rev. **83**, 1074 (1951).

<sup>13</sup> N. Bloembergen, J. Appl. Phys. **23**, 1383 (1952).

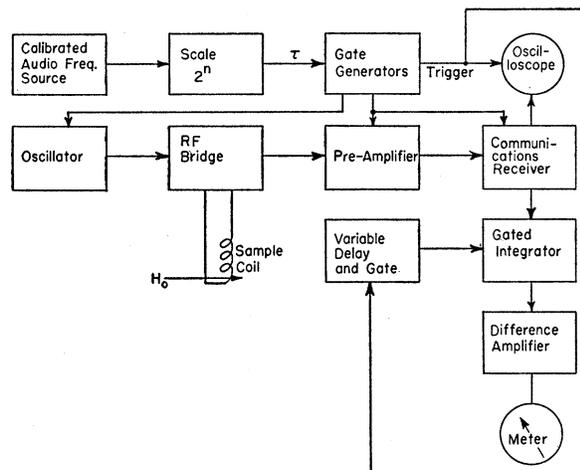


FIG. 1. Block diagram of the apparatus.

range involved here are perhaps  $10^{-5}$  cm<sup>2</sup>/sec (in the liquid phase). At temperatures corresponding to such a value of  $D$ ,  $\delta$  is in the vicinity of 50 $\mu$ . Thus  $\delta^2/D = 25 \times 10^{-6}/10^{-5} = 2.5$  sec. Since the largest values of  $T_1$  or  $T_2$  in this temperature region of large  $D$  are less than 0.1 sec, skin effects should be negligible.

The purity of the samples used was not especially good. Table I shows the results of spectrochemical analyses of the stock materials. Most of the lithium and sodium data were obtained with the A samples.

A copper-constantan thermocouple was laid against the outside wall of the sample vial and the rf coil then close wound directly onto the vial. For runs above room temperature, a constantan heater was wound noninductively on a card which was then wrapped around the sample. Temperatures below room temperature were attained by immersing the sample coil in a copper trough containing turpentine (whose dielectric constant is relatively insensitive to temperature over the temperature range involved). A coolant mixture of dry ice and acetone flowed through copper tubing soldered to the end walls and bottom of the trough. Temperatures between  $-65^\circ$  and  $300^\circ\text{C}$  could be maintained and measured to  $\pm 1^\circ\text{C}$  with these crude arrangements. The melting points of the lithium and sodium samples were checked calorimetrically by measuring time rate curves of heating and cooling, run with the samples situated in place in the magnet gap. The melting points observed were  $97 \pm 1^\circ\text{C}$  for sodium (literature value  $97.5^\circ\text{C}$ <sup>14</sup>) and  $182 \pm 1^\circ\text{C}$  for lithium (literature value  $180^\circ\text{C}$ <sup>14</sup>). The measured values varied by less than  $2^\circ\text{C}$  between rapid-heating and rapid-cooling runs. During most of the  $T_1$  and  $T_2$  measurements, the thermal gradients over the samples were less than  $1^\circ\text{C}$ .

The  $T_1$  measurements were made by the application of two rf pulses. As the spacing between these pulses is

<sup>14</sup> C. J. Smithells, *Metals Reference Book* (Butterworth's Scientific Publications, London, 1949).

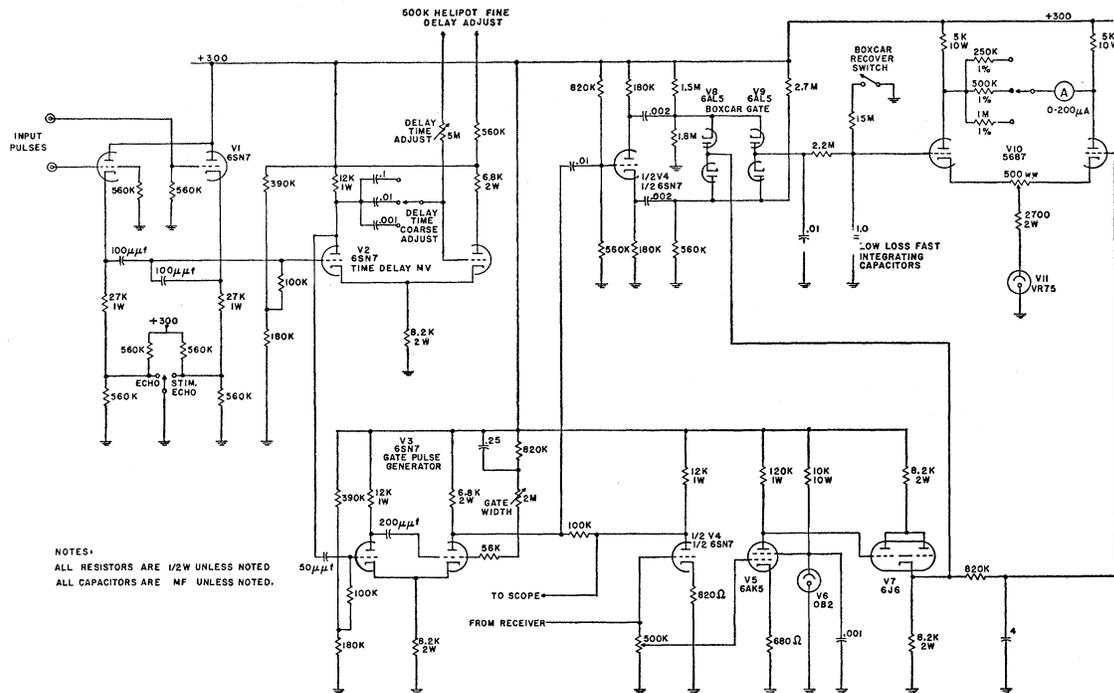


Fig. 2. Boxcar integrating circuit used to improve the signal-to-noise ratio. The necessary gating circuits are also shown in the left part of the figure.

increased, it is found that the initial amplitude of the free induction decay following the second pulse rises along an exponential curve with time constant  $T_1$ . The amplitude of the second decay reflects the degree of recovery of the nuclear spin system toward thermal equilibrium in the time interval between the first and second pulses. Most of the  $T_2$  measurements were also made using two rf pulses, following the spin echo technique of Hahn.<sup>1</sup> The amplitude of the echo is observed as a function of the pulse spacing, the resulting exponential decay having time constant  $T_2$ . Some  $T_2$  measurements were made using the technique of Carr and Purcell,<sup>15</sup> which is a modification of the Hahn method and involves the application of a series of pulses. The envelope of the echo decay may be observed by the application of a single Carr-Purcell pulse train.

The electromagnet and some of the radio-frequency apparatus have been outlined previously.<sup>16</sup> A block diagram of the experimental arrangement is shown in Fig. 1. A frequency-calibrated audio oscillator provides a basic timing wave which drives a scale of  $2^n$  (where  $n=1$  to 5). The interval output of the scaler is used as  $\tau$ , the separation of the two rf pulses in the Hahn spin echo experiment. The scaler output initiates gate circuits which govern the rf pulses. The variable-frequency pulsed oscillator delivers rf pulses (typically 200 volts, 10 to 50  $\mu$ sec duration) to a symmetrical twin- $T$  rf bridge, one arm of which contains the sample

coil. The output from the bridge is amplified in a preamplifier having about a 100-kc bandpass and a voltage gain of 100. The preamplifier is a modification of the Wallman cascode design.<sup>17</sup> The amplified signal is detected in a communications receiver which has been broad-banded to be 3 db down at  $\pm 15$  kc. The preamplifier  $B+$  voltage and the suppressor voltage on the first rf amplifier of the receiver are gated downwards during the rf pulse. This switching technique is necessary in order to avoid excessive overdriving of the rf amplifier stages.

The output of the second detector of the receiver is displayed on an oscilloscope and also fed into the electronic measuring circuit shown in Fig. 2. This measuring device is a "boxcar integrator"<sup>18</sup> and is, to a large extent, the work of L. S. Kypta and H. W. Knoebel. The essential operation is that of a cathode follower ( $V7$ ) driving two channels; a reference channel containing a simple  $RC$  integrator, and a signal channel containing an electronic switch ( $V8, 9$ ) in the form of a gated diode clamp. This gated diode clamp stage is followed by an  $RC$  integrator which utilizes very high leakage impedance condensers (polystyrene insulated). The integrated outputs of the two channels are placed on the input grids of a difference amplifier ( $V10$ ) whose unbalance output is indicated on a meter. At each

<sup>17</sup> Wallman, Macnee, and Gadsden, Proc. Inst. Radio Engrs. 36, 700 (1948).

<sup>18</sup> J. L. Lawson and G. E. Uhlenbeck, *Threshold Signals*, Massachusetts Institute of Technology Radiation Laboratory Series (McGraw-Hill Book Company, Inc., New York, 1950), Vol. 24.

<sup>15</sup> H. Y. Carr and E. M. Purcell, Phys. Rev. 94, 630 (1954).

<sup>16</sup> R. E. Norberg, Phys. Rev. 86, 745 (1952).

repetition of the pulse array, the sampling channel places a charge on the 1.0- $\mu$ f condenser. The charge is representative of the average signal occurring at  $V_7$  during the clamp gate. An equilibrium deflection of the output meter is achieved by sampling many pulse repetition intervals. As an example of the use of the circuit consider the method of measuring  $T_2$  with the Hahn two-pulse technique. The two pulses are separated by an interval  $\tau$ , and the echo occurs at time  $2\tau$ . The sampling gate interval is placed on the echo signal at time  $2\tau$  and the equilibrium meter deflection recorded. The gate interval is then placed at an arbitrary time on the noise baseline. The difference between the two readings is a measure of the echo amplitude. A plot of such amplitudes as a function of  $2\tau$  is exponential with logarithmic slope equal to  $T_2$  if the spin system obeys the Bloch phenomenological equations.<sup>1</sup> Similarly,  $T_1$  is measured by observation of the recovery of the free induction decay following the second rf pulse towards the equilibrium amplitude which corresponds to infinite pulse separation. The relative amplitudes of the decays are measured by automatically keeping the sampling gate at some time  $\tau + \epsilon$  on the decay as the time of the second pulse,  $\tau$ , is varied.

A comparison of the lithium and sodium data presented in Figs. 3 and 6 with those reported earlier<sup>12</sup> from photographic measurements made on similar samples shows the striking improvement of experimental accuracy provided by the gated integrator circuit. The device plays a narrow-banding third detector role for the measurement of transient nuclear

resonance signals analogous to that which the familiar "lock-in"<sup>19</sup> plays for the signals of steady state experiments. Full exploitation of the instrument would demand the use of a stable, coherent rf oscillator followed by a gated power amplifier and the use of a phase-sensitive first detector. The difference amplifier could then deliver a center-null output. Our experiments involved amplitude detection of nuclear signals arising from the action of an incoherent pulsed oscillator. The absence of an rf carrier at the time of measurement of the nuclear signals meant that the detector characteristic was non-linear for small nuclear signals. It was necessary to prepare a calibration plot which provided detector corrections to be applied to each measurement.

The  $T_2$  measurements using the technique of Carr and Purcell<sup>15</sup> were made as a check on the possible existence of diffusion-field inhomogeneity effects (Sec. III). For these measurements, R. L. Trogdon designed a pulser which provided the required arrays of pulses with  $\theta_1 = \pi/2$  and  $\pi$ . The Carr-Purcell  $T_2$  runs were measured by either using the boxcar integrator or by photographing the oscilloscope display.

### III. SUMMARY OF THE EXPERIMENTAL RESULTS

A brief outline of some of the experimental results obtained for  $\text{Li}^6$ ,  $\text{Li}^7$ ,  $\text{Na}^{23}$ ,  $\text{Rb}^{85}$ , and  $\text{Rb}^{87}$  is given at this point. Detailed analyses of the complete data for each isotope will be made in the subsequent sections. This initial summary is intended to emphasize the major similarities and differences among the data in the various metals.

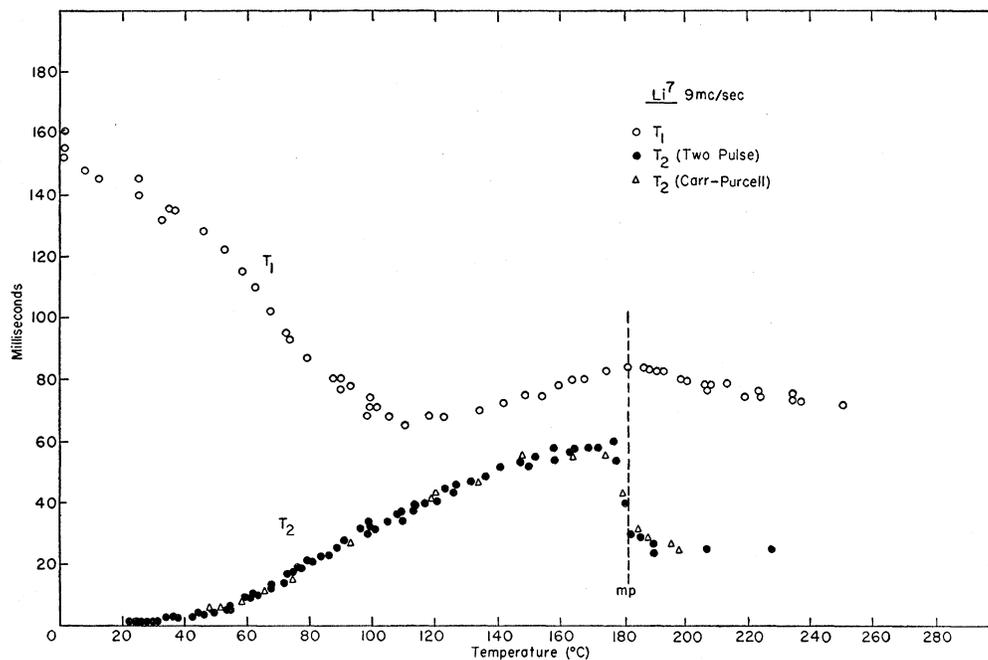


FIG. 3.  $T_1$  and  $T_2$  data for  $\text{Li}^7$ , measured in a sample of natural abundance, at a Larmor frequency of 9.0 Mc/sec.

<sup>19</sup> Bloembergen, Purcell, and Pound, Phys. Rev. **73**, 679 (1948).

Figure 3 shows  $T_1$  and  $T_2$  obtained between 0°C and 250°C for the  $\text{Li}^7$  resonance at 9.0 Mc/sec in a sample of natural isotopic abundance (92.1 percent  $\text{Li}^7$ , 7.9 percent  $\text{Li}^6$ ).  $T_1$  is seen to decrease as the temperature is increased until about 110°C, where a minimum in  $T_1$  occurs. Thereafter  $T_1$  increases until 180°C, then decreases again for higher temperatures. This  $T_1$  behavior will be shown in the next section to result from the superposition of two contributions to the nuclear relaxation. The first relaxation contribution arises from the interaction with the conduction electrons, is designated  $(T_1)_e$ , and should be inversely proportional to the absolute temperature. The second is a contribution from the direct nuclear dipole-dipole interaction,  $(T_1)_d$ .  $(T_1)_d$  goes through a minimum at the temperature at which the inverse of the mean time which a diffusing nucleus spends at a given lattice site is of the order of  $\omega_0$ , the Larmor angular frequency. The initial increase of  $T_2$  with increasing temperature is a continuation of the line narrowing observed by Gutowsky and McGarvey<sup>3</sup> to begin near 230°K. The narrowing line width in this region appears to arise entirely from the diffusion-limited nuclear dipolar interaction.  $T_2$  increases until the  $T_1$  contribution to  $T_2$  becomes important [see Eq. (3), Sec. IV]. At the melting point a sharp reduction of  $T_2$  occurs. This corresponds to a surprising broadening of the resonance line upon melting. The broadening is doubly unusual in that it is not accompanied by any associated  $T_1$  decrease. The presence of the  $T_1$  minimum at 110°C indicates that the correlation frequency for the self-diffusive motion has become much greater than the Larmor frequency by the time the melting point is reached, and therefore one would expect  $T_1 \cong T_2$ ,<sup>20</sup> if the high-temperature interaction is able to produce nuclear relaxation. We return to this point in Sec. IV, C.

The complete  $T_1$  and  $T_2$  curves of Fig. 3 were reproducible for both samples *A* and *B* with no irreversible effects observed over the indicated temperature range. One possible source of a spurious broadening at a melting point is a suddenly increased rate of diffusion

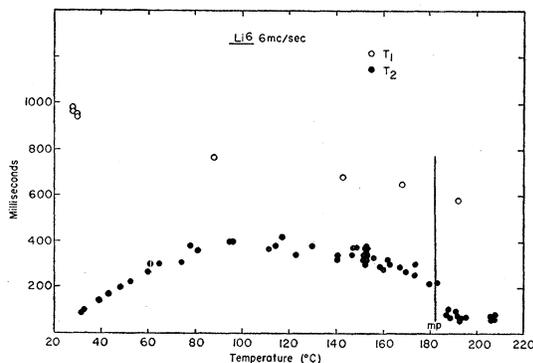


FIG. 4.  $T_1$  and  $T_2$  data for  $\text{Li}^6$ , measured in a sample enriched to 94 percent  $\text{Li}^6$ , at 6.0 Mc/sec.

<sup>20</sup> R. K. Wangsness and F. Bloch, Phys. Rev. **89**, 728 (1953).

of the nuclei through the static inhomogeneous magnetic field distribution impressed over the sample by the magnet. The effect of such a process can be shown<sup>1,16,21</sup> to be an echo envelope decaying as  $\exp(-t/T_2 - kt^3)$ , where  $k$  is a constant involving the magnetic field gradient and the diffusion coefficient. The additional  $kt^3$  term, if undetected, can result in misinterpretation of the data so as to yield  $T_2$ 's substantially lower than the true values. Since there was evidence of non-exponential echo envelopes for lithium at high temperatures, check runs of the  $T_2$  data were made by using the method of Carr and Purcell.<sup>15</sup> Their technique removes, in first order, the diffusion-inhomogeneity  $kt^3$  term. The experimental results obtained with the Carr-Purcell technique are plotted as triangles in Fig. 3. The line-broadening at the melting point is shown to be real.

Figure 4 shows the results for  $\text{Li}^6$  obtained at 6.0 Mc/sec in a sample enriched to 93.8 percent  $\text{Li}^6$ , 6.2 percent  $\text{Li}^7$ .<sup>22</sup> The accuracy of the data obtained from this sample does not approach that for  $\text{Li}^7$  in natural abundance, but it is clear that  $T_1$  decreases steadily as the temperature is increased, with no evidence of a minimum. As in Fig. 3,  $T_2$  increases as the nuclear dipolar interaction averages out with increasing diffusion rate, and then levels off rather sooner, with respect to  $T_1$ , than in the  $\text{Li}^7$  data above. At the melting

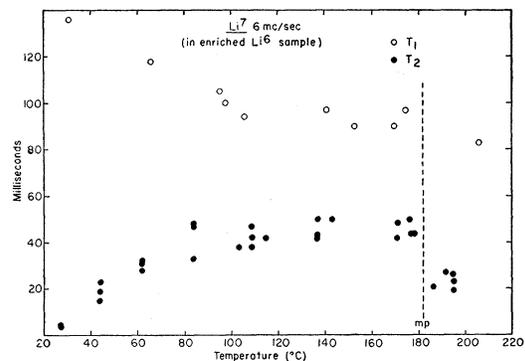


FIG. 5.  $T_1$  and  $T_2$  data for  $\text{Li}^7$  in the enriched  $\text{Li}^6$  sample referred to in Fig. 4, at 6.0 Mc/sec.

point  $T_2$  for  $\text{Li}^6$  decreases suddenly, although somewhat less markedly than for  $\text{Li}^7$ .

Figure 5 shows data taken on the  $\text{Li}^7$  resonance in this same  $\text{Li}^6$ -enriched sample. The accuracy is very poor because of the small concentration of  $\text{Li}^7$  nuclei in this sample, but it is seen that the general features of the data are similar to the data in Fig. 3, with two exceptions. The nuclear dipolar  $T_1$  minimum is very much weaker because of the different nuclear environment. As in the case of the  $\text{Li}^6$  data in Fig. 4,  $T_2$  in the

<sup>21</sup> T. P. Das and A. K. Saha, Phys. Rev. **93**, 749 (1954).

<sup>22</sup> We are indebted to the Stable Isotope Division of the Oak Ridge National Laboratory (U. S. Atomic Energy Commission) for supplying the enriched metal.

region just below the melting point is appreciably less than  $T_1$ .

Figure 6 shows 9.0-Mc/sec data for  $\text{Na}^{23}$ . Here again,  $T_1$  is in general inversely proportional to the absolute temperature, with a slight decrease occurring at the melting point. This reduction will be interpreted as reflecting the volume dependence of  $(T_1)_e$ . Most of the low temperature  $T_2$  data in Fig. 6 have been published previously.<sup>12</sup> They extend to higher temperature the line-width transition reported by Gutowsky.<sup>2</sup>  $T_2$  appears to become limited by  $T_1$  near room temperature, and at the melting point displays a broadening similar to that observed in lithium. A few  $T_2$  measurements were made in sodium using the Carr-Purcell technique and verified the  $T_2$  behavior shown in Fig. 6.

Figure 7 shows data obtained for the  $\text{Rb}^{85}$  and  $\text{Rb}^{87}$  resonances in a sample of natural abundance (73 percent  $\text{Rb}^{85}$  and 27 percent  $\text{Rb}^{87}$ ). Few  $T_2$  points were measured because of experimental limitations at the short  $T_2$  values in rubidium and because of rather poor

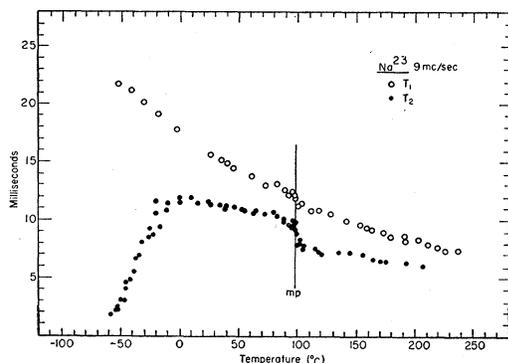


FIG. 6.  $T_1$  and  $T_2$  data for  $\text{Na}^{23}$  at 9.0 Mc/sec.

signal-to-noise ratios. The data in the two isotopes were taken at about the same magnetic field (7500 oe), corresponding to Larmor frequencies of 3.0 Mc/sec for  $\text{Rb}^{85}$  and 10.5 Mc/sec for  $\text{Rb}^{87}$ . The  $\text{Rb}^{87}$  data show a  $T_1$  minimum near 0°C. The dashed curve indicates  $T_2$  corresponding to the  $\text{Rb}^{87}$  line width data of Gutowsky and McGarvey<sup>4</sup> and the assumption of a Lorentz line shape, that is, the absorption as a function of frequency is assumed proportional to  $1/[1+T_2^2(\omega-\omega_0)^2]$ , where  $\omega_0$  is the Larmor angular frequency. Above the melting point  $T_1$  is inversely proportional to the absolute temperature within the large experimental scatter. In the liquid,  $T_2$  is equal to  $T_1$  within the error of the observations.

In  $\text{Rb}^{85}$  the low-temperature  $T_1$  data are qualitatively similar to those in  $\text{Rb}^{87}$ , with the minimum shifted (as it should be for lower Larmor frequency) towards lower temperature, and lying beyond the range of measurements.  $T_2$  presumably equals  $T_1$  below the melting point, but there are no reliable  $T_2$  data. Again,  $T_1$  decreases with increasing temperature above the melting point, and appears to be entirely  $(T_1)_e$ . In  $\text{Rb}^{85}$ ,

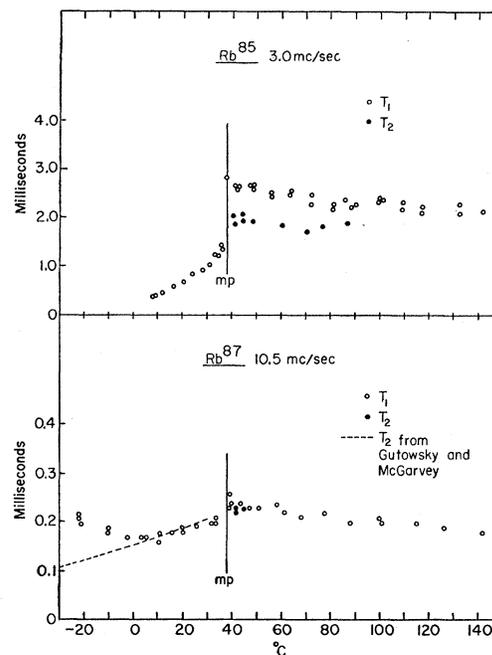


FIG. 7.  $T_1$  and  $T_2$  for a rubidium sample of natural isotopic abundance, 73 percent  $\text{Rb}^{85}$  and 27 percent  $\text{Rb}^{87}$ . The  $\text{Rb}^{85}$  measurements made at 3.0 Mc/sec and the  $\text{Rb}^{87}$  measurements made at 10.5 Mc/sec correspond to roughly the same value of  $H_0$ , the applied static magnetic field.

$T_2$  in the liquid metal is significantly less than  $T_1$ , which indicates the presence of a line broadening similar to that observed in molten lithium and sodium.

Some significant common features of the data in all of the alkalis investigated are a behavior of  $T_2$  near room temperature which reflects a diffusion-induced transition of the dipolar line width,  $T_1$  generally proportional to  $1/T$  with several instances of the occurrence of  $T_1$  minima, and an unexpected high-temperature line-width term, very pronounced in lithium, less so in sodium, and visible in rubidium only as an extra broadening in one isotope.

#### IV. INTERPRETATION OF THE EXPERIMENTAL RESULTS

In the subsequent sections we shall attempt to separate and examine the interactions arising from several sources. In general, we shall assume that an observed spin-lattice relaxation time is the resultant of contributions from a number of lifetime-limiting processes whose effects add as transition probabilities, i.e.

$$1/T_1 = \sum_i (1/T_1)_i \quad (1)$$

If all the interactions which contribute significant line broadenings correspond to the same line shape, then the line widths add and

$$1/T_2 = \sum_i (1/T_2)_i \quad (2)$$

Each line-width term may be further reduced to the

sum of a static broadening ( $1/T_2'$ ), which arises from the essentially constant local fields, and a quantum indeterminacy broadening associated with the spin state lifetime  $T_1$ ,<sup>19</sup> that is,

$$(1/T_2)_i = (1/T_2')_i + (1/2T_1)_i. \quad (3)$$

The combination of (2) and (3) permits the separation of the individual static line width terms,

$$\left(\frac{1}{T_2'}\right)_i = \frac{1}{T_2} - \left(\frac{1}{2T_1}\right)_i - \sum_{j \neq i} \left(\frac{1}{T_2}\right)_j. \quad (4)$$

### A. Interaction with the Conduction Electrons

We begin the detailed examination of the data by considering the interaction of the nuclei with the conduction electrons. Heitler and Teller<sup>23</sup> pointed out in 1936 that there should be a very potent relaxation process for the nuclear magnetic moments in a metal via a spin-spin interaction with the conduction electrons.

The Hamiltonian describing the magnetic interaction of the nucleus with the conduction electrons is discussed by Bloembergen and Rowland.<sup>5</sup> There are three main terms in the interaction, the  $\mathbf{I} \cdot \mathbf{S}$  hyperfine nuclear spin-electron spin term which occurs because of the finite value of the electron wave function at the position of the nucleus, the "outside" nuclear spin-electron spin interaction, and a nuclear spin-electron orbit term. The first term is generally by far the largest, and gives rise to a strong relaxation process for the nuclei. Another observable effect of the hyperfine interaction upon the nuclear magnetic resonance in a metal is a paramagnetic shift of the resonance frequency for a nucleus in the metal with respect to the resonance frequency for the same nuclear species in an isolated atom (usually approximated experimentally by a salt).<sup>24</sup> Townes, Herring, and Knight<sup>6</sup> have shown the shift to be a measure of the electron spin contribution to the magnetic susceptibility of the metal, that is,

$$\frac{\Delta H}{H_0} = \frac{8\pi}{3} \chi_a P_f = \frac{I}{2I+1} \frac{\Delta E}{\mu_e \mu_n} \frac{P_f}{P_a} \chi_a \quad (5)$$

in which  $\Delta H$  is the shift of the resonance frequency and  $H_0$  is the applied static magnetic field.  $\chi_a$  is the electron spin contribution to the susceptibility per atom,  $\Delta E$  is the hyperfine splitting of the ground state of the free atom,  $P_f$  is the probability density evaluated at the nucleus for an electron in the Fermi surface, and  $P_a$  is the probability density at the nucleus for the valence electron in the free atom.

In 1950, Korringa<sup>25</sup> and, independently, Slichter<sup>26</sup> examined the relation between the shift  $\Delta H/H_0$  and

<sup>23</sup> W. Heitler and E. Teller, Proc. Roy. Soc. (London) **A155**, 629 (1936).

<sup>24</sup> W. D. Knight, Phys. Rev. **76**, 1259 (1949).

<sup>25</sup> J. Korringa, Physica **16**, 601 (1950).

<sup>26</sup> C. P. Slichter (unpublished).

$(T_1)_e$ , the electron contribution to the nuclear spin-lattice relaxation time. Korringa performed a first-order perturbation calculation, using Bloch wave functions for the electrons. The diagonal matrix elements of the Hamiltonian are responsible for the paramagnetic or "Knight" shift, and the off-diagonal terms give the transition probabilities between adjacent  $m$ -levels of the nucleus and thus determine  $(T_1)_e$ .

The nuclear spin-electron orbit term in the Hamiltonian vanishes to first order because of the local electric fields which quench the orbital angular momentum of the electrons in the metal. The quenching should be complete except for a small effect arising from the electron spin-orbit interaction. The spin-spin terms outside the nucleus should be small compared to the hyperfine term, especially so in the alkalis where  $P_f$  is rather large. Korringa's result for  $(T_1)_e$ , considering only the hyperfine interaction, may be written

$$\frac{1}{(T_1)_e} = \frac{4\pi kT}{\hbar} v_0^2 \rho^+(\epsilon_0) \rho^-(\epsilon_0) \left( \frac{P_f}{P_a} \frac{\Delta E}{2I+1} \right)^2, \quad (6)$$

where  $\rho^\pm(\epsilon_0)$  are the densities of electron states per unit volume per unit energy for spins up and down respectively, evaluated at the Fermi surface, and  $v_0$  is the atomic volume. Korringa also considered the outside spin-spin and the nuclear spin-electron orbit contributions, which give rise to small terms in addition to (6).  $(T_1)_e$  is predicted by (6) to be inversely proportional to the temperature and to be substantially independent of the applied static magnetic field  $H_0$ . These features are not changed by the addition of the aforementioned extra terms to (6). In a magnetic field the Fermi surfaces for electrons of spin up and spin down are separated by an energy  $2\gamma \hbar H_0$ , so there is a small dependence of  $\rho^+(\epsilon_0)$  and  $\rho^-(\epsilon_0)$  on  $H_0$ . However,  $\gamma_e \hbar H_0$  is only about 0.01 percent of the Fermi energy  $\epsilon_0$ , and the effect on  $(T_1)_e$  should be very small indeed.

Korringa's result for the Knight shift, also calculated in the free electron approximation, is

$$\frac{\Delta H}{H_0} = \frac{\gamma_e \Delta E}{\gamma_n (2I+1)} v_0 \rho^+(\epsilon_0) \frac{P_f}{P_a}. \quad (7)$$

Combination of (6) and (7) leads to what is usually referred to as the "Korringa relation,"<sup>25,26</sup>

$$T_1 \left( \frac{\Delta H}{H_0} \right)^2 = \frac{\hbar \gamma_e^2}{\pi k T 4 \gamma_n^2}. \quad (8)$$

It should be noted that Overhauser<sup>27</sup> has derived an expression for  $(T_1)_e$  by a slightly different method, and his result agrees exactly with (6).

In addition to the magnetic interactions, there could be a significant coupling between the quadrupole mo-

<sup>27</sup> A. W. Overhauser, Phys. Rev. **89**, 689 (1953).

ment of a nucleus with spin  $\geq 1$  and an electric field gradient at the nucleus arising from the conduction electrons. Such a term has been discussed by Bloembergen and Rowland,<sup>5</sup> but should be negligible for the cubic alkali metals.

Figures 8 through 11 show  $T_1$  data in the various metals, plotted as a function of the reciprocal of the absolute temperature. The plotted points differ slightly from the experimental data given in Sec. III in that  $T_1$  has been corrected for the volume dependence of  $(T_1)_e$  as given by (6). The product  $v_0^2 \rho^2$  is proportional to  $v_0^{4/3}$  in the free electron approximation. Because of thermal expansion of the metal, this introduces a small temperature dependence into (6) in addition to the general  $1/T$  behavior of  $(T_1)_e$ . This small additional temperature dependence has been removed, normalizing to the experimental points at 25°C in lithium and sodium and at 40°C in rubidium. Coefficients of thermal expansion were taken from the literature.<sup>28</sup> Equation (8) predicts that the product  $T_1(\Delta H/H_0)^2$  should be proportional to  $1/T$  with no volume dependence. If the sodium  $T_1$  data of Fig. 6 are so combined with the literature shift values,<sup>4</sup> the results agree with the 9-Mc/sec data of Fig. 10.

The straight lines in the figures are drawn with the inverse temperature dependence predicted by (6), normalized to the experimental points at room temper-

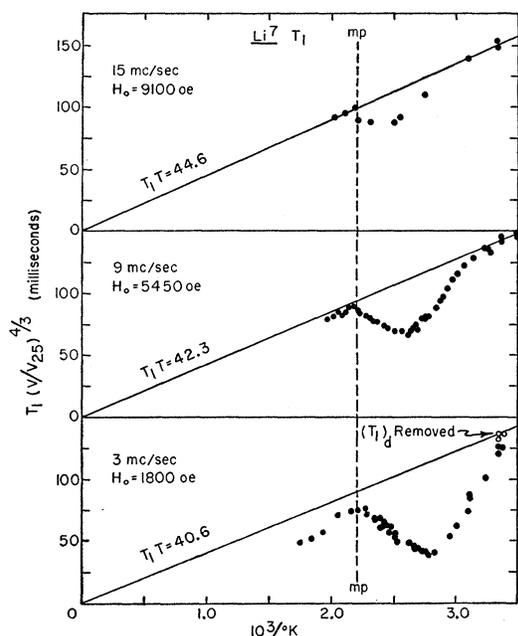


FIG. 8.  $T_1$  data for  $\text{Li}^7$ , taken at three values of the Larmor frequency and adjusted to remove the  $v_0^{-4/3}$  dependence predicted by Eq. (6). The data are plotted against the reciprocal of the absolute temperature in units of  $10^3/^\circ\text{K}$ . The straight lines are drawn with the  $1/T$  dependence predicted by Eq. (6) and normalized to the data at 300°K.

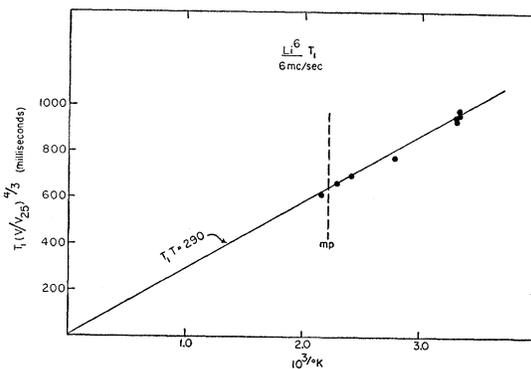


FIG. 9.  $T_1$  data for  $\text{Li}^6$ , adjusted to remove the  $v_0^{-4/3}$  dependence and plotted against reciprocal temperature. The straight line is determined as in Fig. 8.

ature in lithium and sodium, and at 40°C in rubidium. It is evident that the major  $T_1$  contributions in these metals are indeed inversely proportional to the absolute temperature. One also notices that in  $\text{Li}^7$  and  $\text{Na}^{23}$  there is a 5 to 10 percent variation of the entire  $T_1$  curve as the magnetic field  $H_0$  is changed by a factor of 3. This point will be considered later in this section.

Table II presents a correlation of the experimental  $T_1$  data with the theoretical expressions (6) and (8). Values for lithium and sodium are taken at 300°K, where the temperature dependence and other considerations indicate that the contribution to the experimental  $T_1$  from interactions other than that with the electrons is small. For the same reason, values for rubidium are taken at 40°C, just above the melting point.

$(T_1)_e$  is calculated from Eq. (6) by using the available theoretical values of  $m/m^*$  and  $P_f/P_a$ , as indicated in the table. The free electron approximation is modified only to the extent of the use of an effective mass in the evaluation of  $v_0\rho$ . [ $v_0\rho = 3/(4\epsilon_0)$  and  $\epsilon_0 = \hbar(3\pi^2/v_0)^{3/2}/2m^*$ .] The general result is that (6) is fairly successful in predicting  $T_1$  when an  $m^*$  is used. It should be noted that we have ignored possible effects of correlations among the electrons upon the density of states  $\rho$ . A recent calculation by Pines<sup>29</sup> indicates that  $\rho$  might be affected to perhaps 10 percent by electron correlation and exchange.

The results for  $\text{Li}^6$  and  $\text{Li}^7$  are mutually consistent when values at constant  $v_0$  are compared. [The  $(\Delta E/(2I+1))^2$  term in (6) means that  $(T_1)_e$  should be proportional to  $\gamma_n^{-2}$ .] The  $\text{Rb}^{85}$  and  $\text{Rb}^{87}$  results are also in good mutual agreement.

The value for  $P_f/P_a$  in lithium is from a recent recalculation by Kohn with an improved method. The previous value was 1.0.<sup>30</sup> The  $T_1$  data at 15.0 Mc/sec, if taken to represent  $(T_1)_e$ , indicate that  $P_f/P_a \cong 0.6$ . The measured  $T_1$ 's are all somewhat smaller than the values predicted by (6). The consistent discrepancy

<sup>29</sup> D. Pines (private communication).

<sup>28</sup> *International Critical Tables* (McGraw-Hill Book Company, Inc., New York, 1933).

<sup>30</sup> W. Kohn and N. Bloembergen, *Phys. Rev.* **80**, 913 (1950); *Phys. Rev.* **82**, 283 (1951).

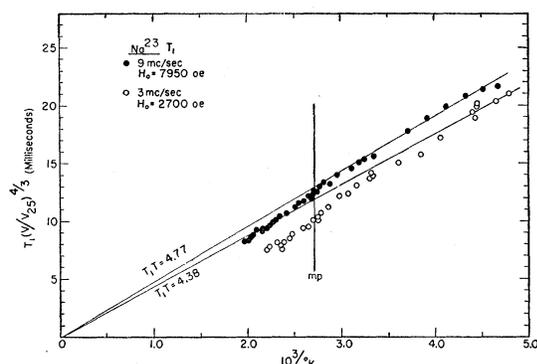


FIG. 10.  $T_1$  data for  $\text{Na}^{23}$ , taken at two values of the Larmor frequency, adjusted to remove the  $\nu_0^{-4/3}$  dependence and plotted against reciprocal temperature. The straight lines are determined as in Fig. 8.

may indicate the presence of an additional relaxation term, and is discussed at the end of this section.

Inspection of the final columns of Table II reveals that the "Korringa relation" (8) yields consistently low  $T_1$ 's. It should be noted that this discrepancy is in the opposite direction to the effect to be expected if the terms omitted from (6) actually contributed to  $(T_1)_e$ . If these terms were important, the experimental  $T_1$ 's would be shorter than those predicted by the Korringa relation. Equation (7), which is more successful, particularly in sodium, is one of the two expressions involved in (8). A check calculation of the shifts predicted by (7) shows that the discrepancy found between experiment and the results of (8) is also present in (7). The measured alkali shifts are all larger than (7) would predict. The Townes, Herring, and Knight expression (5) for the shift is in very good agreement with the measured shifts if current measured<sup>7,31</sup> or theoretical  $\chi_a$  values are used. In lithium, in particular, the recently measured<sup>7</sup>  $\chi_a$  agrees with Pines<sup>32</sup> calculated value and both coincide with  $\chi_a$  calculated from  $\Delta H/H_0$ . The failure of (7) seems to arise from the fact that a free electron calculation of the spin susceptibility, even using an effective mass, is not adequate. It should be noted that (7) becomes more successful with increasing  $Z$ .

The  $\text{Li}^7$  data, Fig. 8, are particularly interesting in that the  $T_1 T$  product of about 45 is very different from the results of Poullis<sup>10</sup> at low temperatures. He has observed a dependence on  $\nu_0$  qualitatively similar to ours, although even more pronounced. However, comparing our 15.0-Mc/sec data with Poullis's results at 15.4 Mc/sec (at which frequencies both sets of data are most nearly proportional to  $1/T$ ) one finds his  $T_1 T$  at 20°K to be about 25 sec deg as compared with our 45. Although Poullis used the "saturation method"<sup>19</sup> described by Bloembergen, in which there exists the

possibility of a calibration error, such an error seems unlikely since Poullis was able to make direct observations of recovery to equilibrium for the longer  $T_1$ 's as a check on his calibration. The other possibility is a peculiar plateau of the  $T_1$  against  $T$  plot between 20°K and 273°K, which seems even more unlikely. Unfortunately, experimental limitations on measurements at short  $T_2$ 's prevented us from measuring  $T_1$  at liquid nitrogen temperature.

It is difficult to reconcile a significant dependence of  $(T_1)_e$  upon the applied static field or the resonance frequency with Eq. (6). As previously pointed out, the only field dependence of  $(T_1)_e$  arising from the  $\mathbf{I} \cdot \mathbf{S}$  hyperfine term lies in the small effect of  $H_0$  upon the product  $\rho^+(\epsilon_0)\rho^-(\epsilon_0)$ , and this effect should be completely negligible. Even if the dipole-dipole or spin-orbit contributions to  $(T_1)_e$  were appreciable, they should be independent of  $H_0$ . It is even more difficult to account for the observed effect in terms of a dependence of  $(T_1)_e$  upon  $\nu_0$ , since the correlation frequency describing the motion of the conduction electrons is much greater than  $\nu_0$ , and changes in  $\nu_0$  should not affect the efficacy of the interaction.

There is one piece of information from the data of Table II which indicates that we are dealing with a dependence of  $T_1$  upon the resonance frequency rather than the applied field,  $H_0$ . Consider the relative magnitudes of  $T_1$  measured for  $\text{Li}^6$  and  $\text{Li}^7$ . The results at 300°K are: for  $\text{Li}^6$  at 6.0 Mc/sec and 9600 oe,  $T_1=965$  msec; for  $\text{Li}^7$  at 6.0 Mc/sec and 3600 oe,  $T_1=138$  msec; and for  $\text{Li}^7$  at 15.0 Mc/sec and 9100 oe,  $T_1=150$  msec.

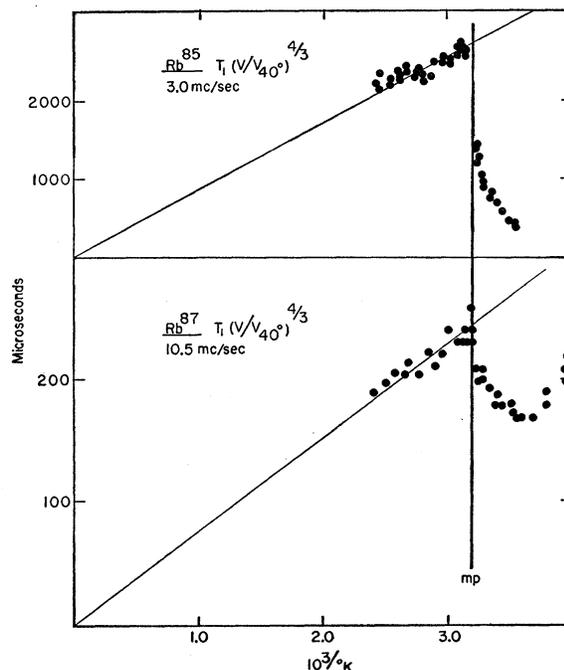


FIG. 11.  $T_1$  data for the rubidium isotopes, with the  $\nu_0^{-4/3}$  dependence removed from the data above the melting point, plotted against reciprocal temperature.

<sup>31</sup> S. R. Rao and K. Sarithri, Proc. Indian Acad. Sci. **6**, 207 (1942).

<sup>32</sup> D. Pines, Phys. Rev. **95**, 1090 (1954).

TABLE II. Comparison of experimental and theoretical values of  $(T_1)_e$ .

	Experiment				Theory				
	$\nu_0$ Mc/sec	$H_0$ oersteds	$T$ °K	$T_1$ msec	Korringa $T_1$ expression Eq. (6)		$(T_1)_e$ msec	"Korringa relation" Eq. (8)	$(T_1)_e$ msec
					$m/m^*a$	$P_f/P_a$		$\Delta H/H_0^b$	
Li <sup>6</sup>	6.0	9600	300	965±35	0.69	0.5 <sup>c</sup>	1590		(616)
Li <sup>7</sup>	15.0	9100	300	150±5	0.69	0.5 <sup>c</sup>	227	2.61×10 <sup>-4</sup>	88
	9.0	5400		141					
	6.0	3600		138					
	3.0	1800		(136)					
Na <sup>23</sup>	9.0	8000	300	15.9±0.3	1.02	0.81 <sup>d</sup>	16.8	11.2×10 <sup>-4</sup>	10.3
	3.0	2700		13.9					
Rb <sup>85</sup>	3.0	7300	313	2.75±0.2	1.12	1.0 <sup>e</sup>	3.3	65×10 <sup>-4</sup>	2.1
Rb <sup>87</sup>	10.5	7500	313	0.245±0.02	1.12	1.0 <sup>e</sup>	0.29	65×10 <sup>-4</sup>	0.18

<sup>a</sup> H. Brooks (private communication to D. Pines), reference 32.

<sup>b</sup> See reference 3.

<sup>c</sup> W. Kohn, Phys. Rev. **96**, 590 (1954). We are indebted to Dr. Kohn for communicating his results to us prior to publication.

<sup>d</sup> See reference 6.

<sup>e</sup> Estimated.

In none of these values does there appear to be a significant  $(T_1)_d$  contribution (see Sec. IV, B). From (6),  $(T_1)_e$  should be proportional to  $\gamma_n^{-2}$ .  $\gamma_r^2/\gamma_6^2=7.0$ , therefore the measured Li<sup>6</sup> value predicts a  $(T_1)_e$  of 138 msec for Li<sup>7</sup>. The  $\gamma_n^{-2}$  dependence seems to hold for constant  $\nu_0$  rather than constant  $H_0$ . The experimental value for Li<sup>7</sup> at 9100 oe, even assuming the maximum error, corresponds to a minimum value for  $(T_1)_e$  of 1020 msec, which is outside the error on the Li<sup>6</sup>  $T_1$  at 9600 oe.

One additional feature of the apparent frequency dependence should be noted. The volume-corrected  $T_1$  curves pass smoothly through the melting point in sodium, and appear to do so in lithium. If the frequency or field dependence arises from any interaction involving the lattice, one would expect a break in the  $T_1$  curve at the melting point. The conduction electrons appear to be the only likely source of an interaction which is insensitive to melting, but the frequency or field dependence probably arises from some interaction other than those which have been mentioned.

### B. Effects of Self-Diffusion: Dipolar $T_1$ and $T_2$

We now turn to the effects of lattice diffusion on  $T_1$  and  $T_2$  in the alkali metals. A general theory of the  $T_1$ 's and  $T_2$ 's which result from interactions of the nuclei with fluctuating local magnetic fields has been given by Bloembergen, Purcell, and Pound<sup>19</sup> and in Bloembergen's thesis.<sup>33</sup> (The Bloembergen, Purcell, and Pound paper will be referred to as BPP in the following.) We will be interested in the BPP theory as it relates to nuclear dipole-dipole interactions and the effects of diffusional motion of the nuclei. The BPP theory was developed in detail for the case of isotropic liquid media, but many of the results are valid for the body-centered cubic lattice of the alkalis. We will also make use of a more complete theory developed by Torrey<sup>34</sup> for  $T_1$  in the case of translational diffusion in a lattice.

<sup>33</sup> N. Bloembergen, *Nuclear Magnetic Relaxation* (Martinus Nijhoff, The Hague, 1948).

<sup>34</sup> H. C. Torrey, Phys. Rev. **92**, 962 (1953).

BPP consider the case in which the local magnetic fields at one nuclear site arise from surrounding nuclear dipoles of the same species. The BPP expression for the contribution to the line width from essentially static local fields is

$$\left(\frac{1}{T_2'}\right)_d = \frac{3}{2} \gamma^2 \hbar^2 \left( \frac{I(I+1)}{3} \frac{1}{2\pi} \int_{-2/T_2'}^{2/T_2'} J_0(\omega) d\omega \right)^{\frac{1}{2}}, \quad (9)$$

where  $J_0$  is the Fourier transform of the time spectrum of dipolar local fields. The subscript  $d$  indicates nuclear dipolar fields as the source of the interaction. For the case of interaction with only one nearest neighbor, Bloembergen<sup>33</sup> obtains

$$\left(\frac{1}{T_2'}\right)_d = \left[ \frac{6}{5} \frac{\gamma^4 \hbar^2}{b^6} \frac{I(I+1)}{2\pi} \frac{1}{\int_{-2/T_2'}^{2/T_2'} \frac{\tau_c}{1+\omega^2\tau_c^2} d\omega} \right]^{\frac{1}{2}}, \quad (10)$$

where  $\tau_c$  is a correlation time for the relative motion of the nuclei and  $b$  is the internuclear distance. The theory has been extended slightly to the case of a homogeneous distribution of nuclear dipoles,<sup>35</sup> and, in this case, one has

$$\left(\frac{1}{T_2'}\right)_d = \frac{3}{2} \gamma^2 \hbar^2 \left( \frac{I(I+1)}{3} \frac{1}{2\pi} \times \int_{-2/T_2'}^{2/T_2'} \int_{r=a}^{\infty} \frac{32\pi N}{5r^4} \frac{\tau_c}{1+\omega^2\tau_c^2} dr d\omega \right)^{\frac{1}{2}}, \quad (11)$$

where the  $r$ -integration is taken from a minimum nuclear impact parameter  $a$ . For translational diffusion, the correlation time may be written  $\tau_c = r^2/12D$ , where  $D$  is the diffusion coefficient,  $D_0 \exp(-E_D/RT)$ .  $N$  is the number of nuclei per unit volume. Only the low-frequency components of  $J_0$  enter into expression (9) for  $T_2'$ . The effective components are those for which  $\omega^2\tau_c^2 \leq (2/T_2')^2\tau_c^2 \ll 1$ . Using this fact, one obtains

<sup>35</sup> R. E. Norberg, thesis, University of Illinois, 1951 (unpublished).

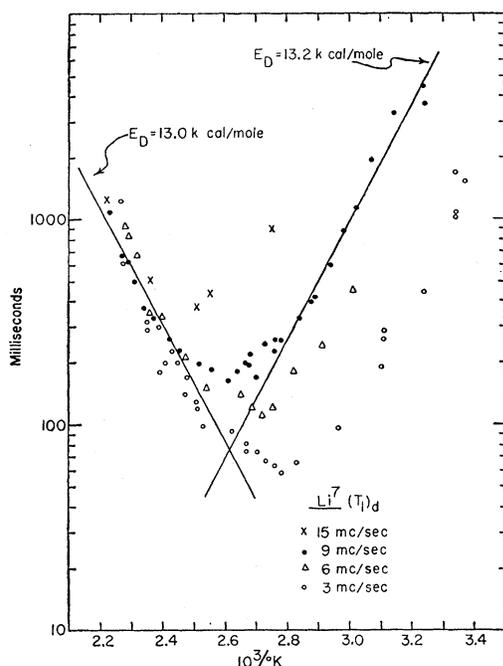


FIG. 12.  $(T_1)_d$  for  $\text{Li}^7$  at four different frequencies, plotted against reciprocal temperatures, showing the minimum in  $(T_1)_d$  and the shift of the minimum with frequency. The straight lines are least-squares fits of those 9-Mc/sec data which lie outside the region of the curved minimum.

$J_0(\omega) = 8\pi N/12Da$  and, from (9),

$$(1/T_2')_d = 4\gamma^4 \hbar^2 I(I+1)N/5Da. \quad (12)$$

In the case of the b.c.c. alkali lattices,  $a$  will be taken as the nearest neighbor distance.

The fluctuating local fields also produce spin-flipping if the characteristic frequency of the fluctuations is in the vicinity of the Larmor frequency. The flipping provides a spin-lattice relaxation process since magnetic energy is exchanged with the energy of lattice motions. By a method similar to that used in deriving Eq. (11), BPP obtain

$$\left(\frac{1}{T_1}\right)_d = \frac{5\pi}{3} N\gamma^4 \hbar^2 I(I+1) \int_a^\infty \frac{1}{r^4} \left\{ \frac{\tau_c}{1+\omega_0^2 \tau_c^2} + \frac{2\tau_c}{1+4\omega_0^2 \tau_c^2} \right\} dr. \quad (13)$$

From (13) the important features of the  $T_1$  behavior may be seen. At low temperatures we have the relation  $\omega_0^2 \tau_c^2 \gg 1$ , therefore  $T_1 \propto \tau_c$ . Again using  $\tau_c = r^2/12D$  and integrating (13), we obtain

$$(1/T_1)_d = 144\pi\gamma^4 \hbar^2 I(I+1)ND/25\omega_0^2 a^5 \quad (14)$$

for this region of low temperatures.  $(T_1)_d$  is inversely proportional to  $D$ , and therefore decreases exponentially as the temperature increases. When a temperature is reached such that  $\omega_0 \tau_c = 1/\sqrt{2}$ , Eq. (13) gives a

minimum value for  $(T_1)_d$ . At higher temperatures where  $\omega_0^2 \tau_c^2 \ll 1$ , if we again substitute for  $\tau_c$  and integrate (13), the result is

$$(1/T_1)_d = 2\pi\gamma^4 \hbar^2 I(I+1)N/5Da, \quad (15)$$

so that  $(T_1)_d$  is now proportional to  $D$  and increases exponentially as the temperature goes toward  $\infty$ .

### 1. Lithium

Let us first examine the  $\text{Li}^7$   $T_1$  minimum. To represent the combination of  $(T_1)_e$  and any frequency-dependent  $(T_1)_x$  which may exist, we select a smooth curve drawn through the  $T_1$  data at room temperature and those just above the melting point. The curve is nearly proportional to  $1/T$  and includes a small decrease at the melting point corresponding to the volume change. The  $(T_1)_d$  data resulting from the reciprocal subtraction corresponding to Eq. (1) are shown in Fig. 12. The asymptotic form of the extremes of the minima is seen to be exponential, in agreement with (14) and (15). A least-squares fit of the 9.0-Mc/sec data yields the activation energies indicated. The displacement of the temperatures  $T_m$  at which the minima occur as a function of the Larmor frequency  $\omega_0$ , is well fitted by the relation  $\ln \omega_0 \propto -E_D/RT_m$ , which arises from the condition that  $\omega_0 \tau_c$  be equal to a constant at the minimum and the assumption that  $\tau_c \propto \exp(-E_D/RT)$ . A semilog plot of the frequencies against  $1/T_m$  is exponential with a slope of about 13 kcal/mole. The minimum  $(T_1)_d$ 's are proportional to  $\omega_0$ , which is a general result for motional relaxation, independent of a detailed theory.

In order to determine a diffusion coefficient from the  $(T_1)_d$  data, we consider first the high-temperature region, where the relation  $\omega_0^2 \tau_c^2 \ll 1$  holds. Here, Eq. (15) is independent of the shape of the cutoff of the spectrum of local fields and is thus unaffected by the particular choice of correlation function. If we take  $a = 3.02$  Å (the nearest-neighbor distance in lithium),  $N = 4.37 \times 10^{22}/\text{cc}$  [from  $N(\text{b.c.c.}) = 2/(\text{lattice parameter})^3 \times \text{percent abundance of Li}^7$ ], and the measured  $(T_1)_d$  of 0.58 sec at  $10^3/^\circ\text{K} = 2.30$ , we obtain  $D = 5.0 \times 10^{-8} \text{ cm}^2/\text{sec}$ . If we accept 13.2 kcal/mole as the activation energy (see  $T_2'$  data below), this result corresponds to  $D = 0.23 \exp(-13200/RT)$ . The contribution of the 8 percent of  $\text{Li}^6$  moments to  $(T_1)_d$  and  $(T_2')_d$  can be shown to be about 0.3 percent, and it has been neglected in the above considerations. On the low-temperature side of the minimum, (14) fails to yield results consistent with the data. Using  $D = 0.23 \exp(-13200/RT)$ , Eq. (14) predicts  $(T_1)_d = 4.3$  sec at  $10^3/^\circ\text{K} = 3.00$  and 9.0 Mc/sec, whereas the observed value is 0.95 sec. It is to be noted that (14) depends upon the choice of correlation function and, because of the  $a^5$  dependence, is very sensitive to a choice of minimum impact parameter.

Torrey<sup>34</sup> has extended the BPP theory to a more precise computation of  $(T_1)_d$  for specific cases of trans-

lational lattice diffusion. Assuming that the self-diffusion process consists of a random walk with uniform flight path distance  $l$ , his result [reference 34, Eq. (86)] is

$$\left(\frac{1}{T_1}\right)_d = \frac{8\pi}{5} \gamma^4 \hbar^2 I(I+1) \frac{n}{k^2 l^3 \omega} \Psi(k, y), \quad (16)$$

where we take  $l$  to be the nearest neighbor distance, and  $\Psi(k, y)$  is a function which Torrey has tabulated for a f.c.c. lattice. Using the proper  $\Psi$  tabulation for a b.c.c. lattice<sup>36</sup> (for which  $k=0.763$ ), we have applied the theory to the  $\text{Li}^7$  data. The method of analysis was to normalize the data of Fig. 12 by forming the ratio  $T_1/(T_1)_{\text{minimum}}$  and determine the corresponding  $\omega_0\tau/2$  from the tabulated functions. A plot of  $\omega_0\tau/2$  against  $1/T$  is shown in Fig. 13, corresponding to the 3.0-Mc/sec data of Fig. 12. The analyzed data closely follow an exponential curve, with the indicated activation energy of 13.2 kcal/mole. It should be noted that Torrey uses  $\tau=r^2/6D$ , so that his  $\tau$  is equal to twice the  $\tau_c$  used here.

Torrey's expression, (16), in its low-temperature asymptotic form, provides a better agreement with the observed  $(T_1)_d$  than does Eq. (14) from the BPP theory. At  $10^3/^\circ\text{K}=3.00$ , where the 9.0-Mc/sec  $(T_1)_d$  is measured to be 0.95 sec, the Torrey expression, with  $D=0.24 \exp(-13\,200/RT)$ , predicts  $(T_1)_d=1.2$  sec. As

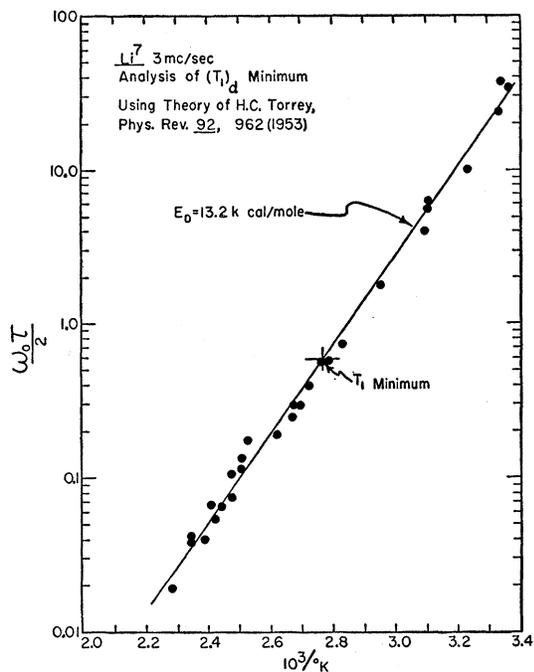


FIG. 13. Plot of  $\omega_0\tau/2$  against reciprocal temperature, obtained from analysis of  $(T_1)_d$  data for  $\text{Li}^7$  at 3 Mc/sec from Fig. 12 using the Torrey theory.

<sup>36</sup> H. C. Torrey, Phys. Rev. **96**, 690 (1954). We are indebted to Professor Torrey for providing us with the b.c.c. tabulations of his functions in advance of publication.

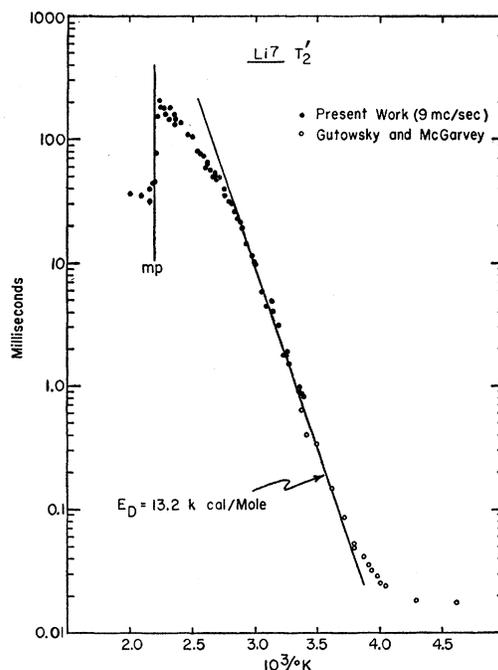


FIG. 14.  $T_2'$  for  $\text{Li}^7$ , calculated from the data of Fig. 3 using Eq. (17) and plotted against reciprocal temperature. The straight line is a best fit of the data in the region of the exponential dependence of  $T_2'$  on temperature.

stated previously, Eq. (14) gives 4.3 sec. The BPP and Torrey theories both successfully fit the moderately high temperature  $(T_1)_d$  data, for which  $\omega_0^2\tau_c^2 \ll 1$ . Here the effective local fields near  $\omega_0$  arise primarily from remote nuclei, since the neighboring nuclei contribute mainly very high frequency terms. On the other hand, the Torrey theory provides a much better fit of the low-temperature  $T_1$  data ( $\omega_0^2\tau_c^2 \gg 1$ ) than does the BPP result (14). In this region it is the near neighbors that make the main relaxation contribution, with the remote nuclei providing mainly static terms. The near neighbors are just those for which the distribution of sites deviates most from the BPP assumption of homogeneity, and for which a precise lattice description is most significant.

One advantage of Torrey's method is that it facilitates examination of the  $(T_1)_d$  data in the immediate vicinity of the minimum, where the corrections for  $(T_1)_e$  are smallest. Substitution of the maximum value of Torrey's b.c.c.  $\Psi$  function into (16) results in a prediction of  $(T_1)_d=225$  msec at the 9.0-Mc/sec  $T_1$  minimum. The observed value (Fig. 12) is about 190 msec.

In order to examine the dipolar  $T_2$  terms in the data, it is necessary to remove the electron line width contributions. Wangsness and Bloch,<sup>37</sup> in examining the range of validity of the Bloch phenomenological equations,<sup>37</sup> show that in an isotropic medium at high temperatures, where the characteristic frequency of the motion of the

<sup>37</sup> F. Bloch, Phys. Rev. **70**, 460 (1946).

TABLE III. Diffusion coefficients in lithium, resulting from interpretation of the relaxation data with an extension of the theory of BPP and with the lattice diffusion theory of H. C. Torrey.  $E_D$  is given in kcal/mole and all  $D_0$ 's are calculated for  $E_D=13.2$  and given in  $\text{cm}^2/\text{sec}$ . Indicated average result:  $D=(0.24_{-0.10}^{+0.17}) \exp[(-13\ 200 \pm 400)/RT]$   $\text{cm}^2/\text{sec}$ .

$\nu_0$	Region of the data	BPP theory		Torrey theory	
		Relation used	Result	Relation used	Result
9.0 Mc/sec	low temperature $\omega_0^2 \tau_c^2 \gg 1$	$T$ dependence of $(T_1)_d$	$E_D=13.2$	$T$ dependence of $\omega_0 \tau$	$E_D=13.2$
		Eq. (14)	$(D_0=0.05)$	Eq. (16), low $T$ asymptote	$D_0=0.30$
	high temperature $\omega_0^2 \tau_c^2 \ll 1$	$T$ dependence of $(T_1)_d$	$E_D=13.0$	Eq. (16), high $T$ asymptote	$D_0=0.20$
	$T_2'$ below 370°K	Eq. (15)	$D_0=0.23$		
		$T$ dependence of $T_2'$	$E_D=13.2 \pm 0.4$		
		Eq. (12)	$D_0=0.25$		
3.0 Mc/sec	all $(T_1)_d$ data			$T$ dependence of $\omega_0 \tau$	$E_D=13.2$
	at $T_1$ minimum			$\omega_0 \tau/2=0.592$	$D_0=0.26$
15.0, 9.0 6.0, 3.0 Mc/sec	Temperature of $T_1$ minimum as function of $\omega_0$		$E_D=13$		

magnetic moment is much greater than the Larmor frequency, one has  $T_2'=2T_1$ . In this case, Eq. (3) becomes  $T_2=T_1$ , which agrees with a classical argument that at these high frequencies for the nuclear motion the Larmor precession becomes a small effect and  $T_1$  and  $T_2$  represent the same quantity, since the uniqueness of the  $z$ -direction is lost. The motions of the conduction electrons are such that  $(\omega_c)_e \gg \omega_0$ . Therefore,  $(T_1)_e=(T_2)_e$  and Eq. (4) becomes, considering only dipolar and conduction electron terms,

$$\left(\frac{1}{T_2'}\right)_d = \frac{1}{T_2} - \left(\frac{1}{2T_1}\right)_d - \left(\frac{1}{T_1}\right)_e. \quad (17)$$

Turning to the  $T_2$  data, it was found that in  $\text{Li}^7$  near room temperature, where  $T_1$  effects are small,  $T_2$  is independent of the Larmor frequency from 3.0 to 15.0 Mc/sec, as one would expect. In order to inspect the static line width term  $(T_2')_d$ , we make the reciprocal subtraction described in Eq. (17). We have assumed that any  $(T_1)_x$  may be included with the  $(T_1)_e$  insofar as line width effects are concerned. The results for the 9.0-Mc/sec data of Fig. 3 are shown in Fig. 14. The line width data of Gutowsky and McGarvey<sup>3</sup> have been converted to  $T_2'$  under the assumption of a Lorentz line shape, which is the shape they observed at low temperatures.<sup>38</sup> For this line shape,  $T_2=2/(\sqrt{3}\gamma\delta H)$ , where  $\delta H$  is measured between the maxima of  $d\chi''/dH_0$ . The computed  $T_2$ 's increase from the rigid lattice value and join smoothly onto the low-temperature end of our observations. The data are fitted by an exponential,

corresponding to  $E_D=13.2$  kcal/mole, from the lowest temperature at which the narrowing has gotten well under way, up through nearly three decades of  $T_2'$ . Beyond 100°C, however,  $T_2'$  no longer increases at a constant rate, but rather bends down as if either the activation energy were decreasing [which the  $(T_1)_d$  data do not indicate] or another line width interaction were becoming important as the direct dipolar term averages out. The sudden broadening associated with melting is seen to correspond to an order of magnitude decrease in  $T_2'$ .

Considering the room-temperature region where  $T_2'$  seems to be entirely associated with  $(T_2')_d$  and where the  $T_1$  correction is negligible, Eq. (12) and the line drawn in Fig. 14 yield  $D=0.25 \exp(-13\ 200/RT)$   $\text{cm}^2/\text{sec}$ . As has been previously mentioned, the  $\text{Li}^6$  nuclei contribute less than 0.3 percent to  $1/T_2'$  and have been neglected here. The results of the dipolar analyses of the  $\text{Li}^7$  data are summarized in Table III. Those data which we have associated with  $(T_1)_d$  and  $(T_2')_d$  are seen to be in generally good agreement with each other and with the BPP and Torrey theories. As a result of the several independent measurements, we regard the coefficient of self-diffusion for metallic  $\text{Li}^7$  below 400°K as determined to be  $D=(0.24_{-0.10}^{+0.17}) \exp[(-13\ 200 \pm 400)/RT]$   $\text{cm}^2/\text{sec}$ . The error limits on  $D_0$  are those dictated by the assigned accuracy of  $E_D$ . The results in Table III indicate that, within the temperature range of the data,  $D$  is determined to  $\pm 15$  percent. The thermodynamical relation of Nachtrieb *et al.*<sup>39</sup> gives  $E_D=16.5L_m$ , where  $L_m$  is the latent heat of melting.

<sup>38</sup>H. S. Gutowsky and B. R. McGarvey (private communication).

<sup>39</sup>Nachtrieb, Weil, Catalano, and Lawson, *J. Chem. Phys.* **20**, 1189 (1952).

This appears to work well in many metals, but predicts  $E_D$  for lithium to be 11.34 kcal/mole. The earlier Van Liempt relation,<sup>40</sup>  $E_D = 32 \times$  melting temperature, predicts 14.5 kcal/mole.

We now consider the  $T_2$  data for both the  $\text{Li}^6$  and  $\text{Li}^7$  resonances in the enriched  $\text{Li}^6$  sample, from Figs. 4 and 5. Although the enriched sample contains 93.8 percent  $\text{Li}^6$  and only 6.2 percent  $\text{Li}^7$ , the  $\text{Li}^7$  moments still make an appreciable contribution to the  $(T_2')_a$  observed for  $\text{Li}^6$ . The expression corresponding to Eq. (12) for the case of two nuclear species in the lattice can be shown to be<sup>41</sup>

$$\left(\frac{1}{T_2'}\right)_a = \frac{4\gamma_a^4 \hbar^2 I_a(I_a+1)N_a}{5Da} + \frac{8\gamma_a^2 \gamma_b^2 \hbar^2 I_b(I_b+1)N_b}{15Da}. \quad (18)$$

Figure 15 shows the  $\text{Li}^6$   $T_2'$  data corresponding to the data of Fig. 4. The line which passes through the low-temperature points in Fig. 15 is the theoretical result of Eq. (18), if one uses  $D = 0.24 \exp(-13\,200/RT)$ . The accuracy of the data does not quite permit us to see the 8 percent difference between  $D$  for  $\text{Li}^6$  and for  $\text{Li}^7$  which should arise from the  $(m)^{-3}$  dependence of  $D_0$ . The high-temperature behavior of the  $\text{Li}^6$   $T_2'$  is quali-

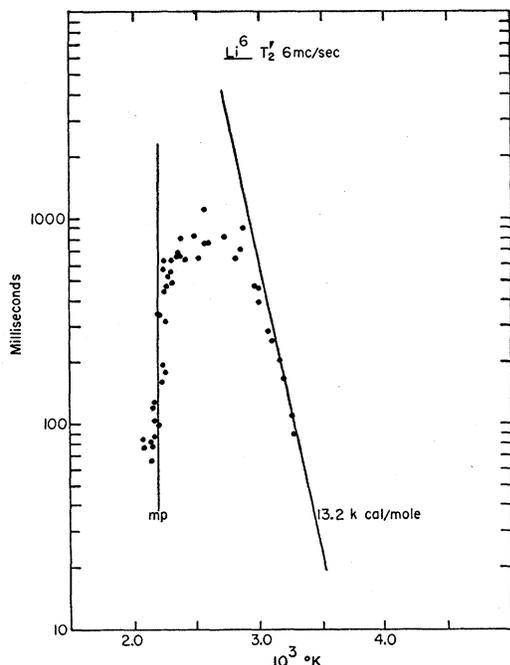


FIG. 15.  $T_2'$  for  $\text{Li}^6$ , calculated from the data of Fig. 4 and plotted against reciprocal temperature. The theoretical line is drawn with the slope determined from the data of Fig. 14 and its position determined by the prediction of Eq. (18).

<sup>40</sup> J. Van Liempt, Z. Phys. **96**, 534 (1935).

<sup>41</sup> D. F. Holcomb, thesis, University of Illinois, 1954 (unpublished).

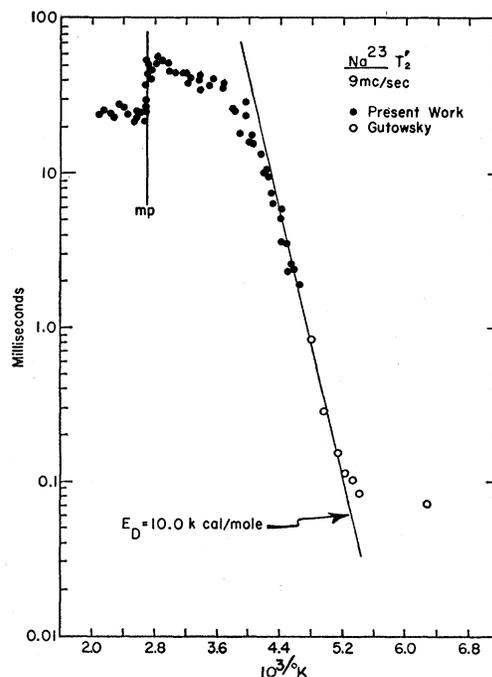


FIG. 16.  $T_2'$  for  $\text{Na}^{23}$ , calculated from the data of Fig. 6 and plotted against reciprocal temperature. The straight line is a best fit of the data in the region of the exponential dependence of  $T_2'$  on temperature.

tatively similar to that observed for  $\text{Li}^7$  (Fig. 14) and will be discussed in Sec. IV, C.

The  $\text{Li}^7$  resonance in the enriched  $\text{Li}^6$  sample gave the results which have been shown in Fig. 5. Again, as for the  $\text{Li}^6$ , the low-temperature  $T_2$ 's for the  $\text{Li}^7$  are successfully predicted by Eq. (18). There are only vestigial traces of a weak dipolar  $T_1$  minimum in the  $\text{Li}^7$  data for this sample, as should be the case because of the weak nuclear dipolar environment.

## 2. Sodium

The 9.0-Mc/sec sodium data of Fig. 6 have been converted to  $T_2'$  by using  $1/T_2' = 1/T_2 - 1/T_1$ , from Eq. (4). The results are shown in Fig. 16. We have again assumed that any  $(T_1)_x$  may be included in  $(T_1)_e$  insofar as line-width effects are concerned. As in lithium the relaxation measurements join smoothly onto the  $T_2'$  calculated from the line-width data.<sup>2</sup> The exponential section of the data has a slope of  $10.0 \pm 0.6$  kcal/mole. The error here is greater than for the lithium data of Fig. 15 because the exponential region is a decade shorter for the sodium  $T_2'$ . Equation (12) and the line drawn in Fig. 16 yield  $D = (0.20_{-0.15}^{+0.56}) \times \exp[(-10\,000 \pm 600)/RT]$ , where the limits of error on  $D_0$  correspond to those assigned to  $E_D$ . Nachtrieb and co-workers<sup>42</sup> have used the radioactive tracer technique in sodium to measure  $D = 0.242 \exp[(-10\,450$

<sup>42</sup> Nachtrieb, Catalano, and Weil, J. Chem. Phys. **20**, 1185 (1952).

$\pm 300)/RT]$  cm<sup>2</sup>/sec. Thus the nuclear magnetic resonance determination is in agreement with the results of the tracer experiments.

Equation (16), which has been previously shown to describe accurately the  $(T_1)_d$  minima observed in Li<sup>7</sup>, predicts  $(T_1)_d$  values in sodium of 0.37 and 0.12 sec respectively at the 9- and 3-Mc/sec minima. The temperature of the 3-Mc/sec minimum is calculated to be 265°K from the requirement that  $\omega_0\tau_c=0.5922$  at the minimum. The 9-Mc/sec  $(T_1)_d$  contribution is completely masked by  $(T_1)_e$  (Fig. 10) but, at 3 Mc/sec, a 10 percent dip in  $T_1$  should occur at  $10^3/^\circ\text{K}=3.77$ . The experimental 3-Mc/sec data (Fig. 10) may show this weak minimum.

As in lithium isotopes,  $T_2'$  in sodium bends away from the exponential corresponding to Eq. (12) at temperatures well below the melting point and then decreases precipitously at the melting point. Also, as in lithium, the temperature dependence of  $T_2'$  in the molten metal is poorly determined, but is certainly small. The high-temperature data will be discussed further in Sec. IV, C.

### 3. Rubidium

In the rubidium isotopes, we do not have direct  $T_2$  data below the melting point. However, the Rb<sup>87</sup> line width data reproduced in Fig. 7 and the qualitative similarities to the lithium and sodium data make it fairly certain that in Rb<sup>87</sup> we have  $T_1$  data which include a diffusion-induced  $T_1$  minimum with  $T_2 \cong T_1$  on the high-temperature side, and that in Rb<sup>85</sup> we have just the high-temperature side of a  $T_1$  minimum (and presumably  $T_2 \cong T_1$ ). The Rb<sup>87</sup> minimum at 273°K for 10.5 Mc/sec and the use of  $E_D=9$  kcal/mole indicate that the Rb<sup>85</sup> minimum should occur, for 3.0 Mc/sec, at about 250°K, well below the temperature range of our Rb<sup>85</sup> measurements. Extending a volume-corrected

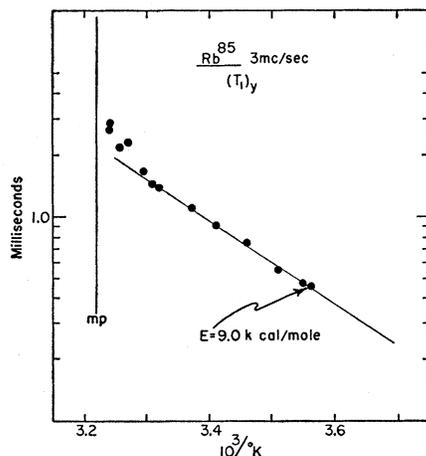


FIG. 17.  $(T_1)_y$  for Rb<sup>85</sup>, calculated from the data of Fig. 7 below the melting point by making a correction for  $(T_1)_e$ . The straight line is the best line through all data except the four points just below the melting point.

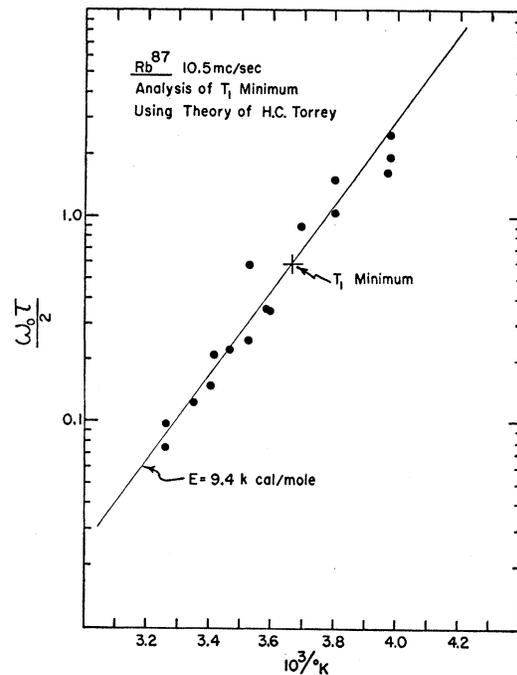


FIG. 18.  $(T_1)_y$  for Rb<sup>87</sup>, calculated from the data of Fig. 7 below the melting point by making a correction for  $(T_1)_e$ . The data have been analyzed using the Torrey theory in the same fashion as the Li<sup>7</sup> data plotted in Fig. 13.

$(T_1)_e$  from the  $T_1$  data above the melting point, we compute the nonelectron  $T_1$  contribution below the melting point with the expression  $1/(T_1)_y=1/T_1-1/(T_1)_e$ . The results for Rb<sup>85</sup> are shown in Fig. 17. Except for a few points near the melting point (rather large thermal gradients may have existed over the sample which had a volume of several cc), the  $(T_1)_y$  fall along a reasonably good exponential with activation energy 9.0 kcal/mole.

Since the Rb<sup>87</sup>  $(T_1)_y$  data correspond only to the immediate region of a  $T_1$  minimum, we have applied the method of Torrey. The results are plotted in Fig. 18 and are seen to be surprisingly exponential in view of the obvious limitation of the experimental accuracy. The slope of the Torrey plot corresponds to  $E=9.4$  kcal/mole. The previously mentioned theory of Nachtrieb predicts  $E_D=8.6$  kcal/mole for rubidium, while the Van Liempt expression predicts 10.0 kcal/mole.

From the isotopic ratio of  $(T_1)_y$ 's in the solid, however, it appears that the  $(T_1)_y$  mechanism is not entirely the nuclear dipolar interaction. The sample contains 73 percent Rb<sup>85</sup> and 27 percent Rb<sup>87</sup>. The result of the application of the coupled system  $T_1$  equation analogous to Eq. (18)<sup>41</sup> is that the ratio of  $(T_1)_d$  measured in Rb<sup>85</sup> with respect to that in Rb<sup>87</sup> is predicted to be 11.2 in the temperature region above the minimum, while the observed ratio near  $10^3/^\circ\text{K}=3.25$  is about 2. In the next section, we will interpret the  $T_2$  measurements in molten rubidium as indicating

the presence of an additional broadening interaction in Rb<sup>85</sup>. If we regard the small ratio between the  $T_1$ 's for the isotopes in the solid phase as the result of a similar reduction of  $T_1$  for Rb<sup>85</sup> in the solid, then there is the possibility that the Rb<sup>87</sup> ( $T_1$ )<sub>l</sub> minimum is entirely dipolar. If so, the Torrey plot (Fig. 18) and Torrey's relation  $\omega_0\tau_c=0.5922$  at the minimum, yield  $D_0=0.23$  cm<sup>2</sup>/sec for  $E_D=9.4$  kcal/mole. This  $D_0$  value is similar to that previously obtained for lithium and sodium.

### C. Anomalous Effects, Mainly $T_2$ at High Temperatures

In this section we consider several aspects of the data which are not clearly understood. The most striking of these effects are the decrease of  $T_2$  at the melting point in lithium and sodium and the line width contribution, in addition to  $1/(T_2)_s$  and  $1/(T_2)_l$ , which is visible in lithium and sodium over a region of about one hundred degrees centigrade below the melting points.

It was first thought that the high-temperature  $T_2$  effects were associated with an interaction of the nuclear quadrupole moment with electric field gradients caused by imperfections in the b.c.c. metallic lattices.<sup>43</sup> In order to check this hypothesis, measurements were made on the Li<sup>6</sup> and Li<sup>7</sup> resonances in the Li<sup>6</sup>-enriched metal obtained from Oak Ridge.<sup>21</sup> For a broadening arising from the postulated quadrupole interaction, there are two possible dependences on the nuclear parameters. If the interaction is subjected to averaging out because of motions in the lattice (as one would expect in the liquid) then<sup>44</sup>

$$1/T_2 \propto Q^2[4I(I+1)-3]/I^2(2I-1)^2.$$

In this case,  $(T_2)_{Li^6}/(T_2)_{Li^7}=507$ , since  $Q_6/Q_7=2.3$

TABLE IV.  $(T_2')_x$  in solid and liquid alkali metals near their melting points. The high-temperature static line widths have been calculated in the solids with  $1/(T_2')_s=1/T_2'-1/(T_2')_l$ . In the liquids,  $(T_2')_x=T_2'$ .

	$\nu_0$ (Mc/sec)	$H_0$ (oe)	Liquid $T$ (°K)	Liquid $(T_2')_x$ msec	Solid $T$ (°K)	Solid $(T_2')_x$ msec
In the Li <sup>6</sup> -enriched sample:						
Li <sup>6</sup>	6	9600	473	81±8	450	480
	3	4800		116±16		
Li <sup>7</sup>	6	3600	473	32±4	450	62
In the samples with natural isotopic abundance:						
Li <sup>7</sup>	15	9100	473	23±3	450	(250)
	9	5400		35±4		220
	6	3600		47±5		
Na <sup>23</sup>	9	8000	390	24±2	313	57
	3	2700		23±3		56
Rb <sup>85</sup>	3	7300	313	7±2		

<sup>43</sup> D. F. Holcomb and R. E. Norberg, Phys. Rev. **93**, 919(A) (1954).

<sup>44</sup> R. K. Wangsness and F. Bloch, reference 22, Eq. (5.40).

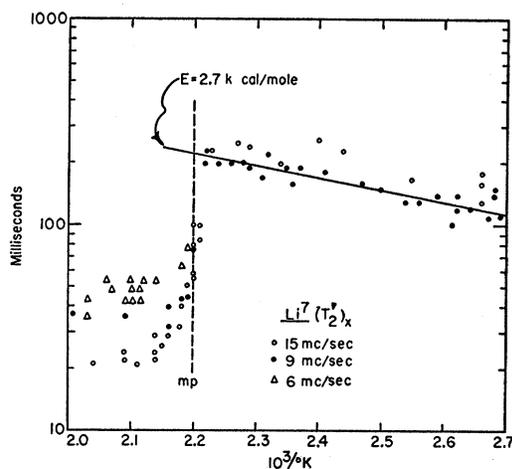


FIG. 19. The high-temperature contribution to  $T_2$  in Li<sup>7</sup> ( $T_2$ )<sub>x</sub>, which has been calculated by removing the dipolar and conduction electron contributions from the  $T_2$  data measured at several Larmor frequencies.

$\times 10^{-2}$ .<sup>45</sup> If, instead, there is some sort of static, un-averaged interaction, one expects

$$1/T_2 \propto Q[4I(I+1)-3]^2/I(2I-1),$$

in which case  $(T_2)_{Li^6}/(T_2)_{Li^7}=23$ . The magnetic moment of Li<sup>6</sup> is also smaller than that of Li<sup>7</sup>, but only by a factor of 4, so that magnetic interactions should give rise to smaller  $T_2$  ratios than those given above. As can be seen in Figs. 4 and 5,  $T_2$  above the melting point in Li<sup>6</sup> is less than a factor of 3 longer than  $T_2$  for Li<sup>7</sup> in the same sample. This ratio effectively rules out quadrupole interactions.

In order to examine the high-temperature line-width term we calculate  $(T_2')_x$  from the  $T_2'$  data of Sec. IV, B with the appropriate form of Eq. (4);  $1/(T_2')_x=1/T_2'-1/(T_2')_l$ . Table IV lists the results of this manipulation of the data for sodium, rubidium, and the two lithium isotopes. Above the melting points the  $(T_2')_x$  correction is negligible and  $(T_2')_x$  is just  $T_2'$ , that is,  $T_2$  with the  $T_1$  contribution removed. It should be noted that the high-temperature broadenings are very small compared to rigid lattice dipolar widths. The  $T_2$  of about 25 msec in molten Li<sup>7</sup> corresponds to a resonance line width of about 0.004 gauss.

The temperature dependence of the  $(T_2')_x$  data calculated from the 9.0-Mc/sec Li<sup>7</sup> data of Fig. 14 is shown in Fig. 19. Some data taken at other frequencies are also shown. The absolute values of the  $(T_2')_x$  points in the solid lithium are certainly suspect, since they are two correction processes removed from the raw data. The line drawn is a least squares fit of the 9.0-Mc/sec data, arbitrarily assuming an exponential temperature dependence. The individual  $(T_2')_x$  data in the liquid are more reliable, since the corrections to  $T_2$  are relatively much smaller than in the solid. The temperature dependence of  $(T_2')_x$  in the liquid is small. Although

<sup>45</sup> N. A. Schuster and G. E. Pake, Phys. Rev. **81**, 157 (1951).

the 15.0-Mc/sec data in the solid are in approximate agreement with the 9.0-Mc/sec results, there is a consistent inverse dependence of  $(T_2')_x$  in the liquid upon the Larmor frequency.

$(T_2')_x$  obtained from the sodium  $T_2'$  data of Fig. 16 shows a temperature dependence in the solid even smaller than that in lithium. Moreover,  $(T_2')_x$  drops at the melting point by a factor of only two. The sodium  $(T_2')_x$  curves for 9.0 Mc/sec and 3.0 Mc/sec are indistinguishable, with no frequency dependence visible in either the solid or liquid.

The exact mechanisms responsible for the unusual broadenings are not yet understood. However, many of the features of the high-temperature data are consistent with the presence of a magnetic interaction with thermally activated lattice imperfections. We shall list the experimental facts about the broadenings and discuss briefly some of their implications about the nature of the high-temperature interactions.

### 1. Decrease in $T_2$ upon Melting

The most striking feature of the data is the sudden decrease in  $T_2$  at the melting point in lithium and sodium. The decrease is unaccompanied by a drop in  $T_1$  other than the previously discussed small variation which arises from the volume dependence of  $(T_1)_e$ . An interaction is indicated which either appears only upon melting or which becomes much stronger in the liquid.

In some metals the self-diffusion coefficient increases by several orders of magnitude upon melting.<sup>46,47</sup> If this is true in the alkalis, the increased  $D$  in the liquid should effectively narrow the nuclear dipolar line width  $1/(T_2')_d$  to zero. However, if one assumes that the  $x$ -interaction arises from magnetic fields associated with the distorted electron cloud near a vacancy, then, by analogy with (12),

$$1/(T_2')_x \propto n/D_v, \quad (19)$$

where  $n = n_0 \exp(-E_F/RT)$  is the number of vacancies per unit volume.  $D_v$  is a diffusion coefficient describing the vacancy motion. If the self-diffusion mechanism is via an atom-vacancy interchange, this  $D_v$  is much larger than the atomic self-diffusion coefficient  $D$ , and therefore is the determining factor in the narrowing. The activation energy to be associated with  $D_v$  is  $E_M$ , the barrier or mobility energy. (For the atomic self-diffusion,  $E_D = E_F + E_M$ .) Thus, (19) becomes

$$(T_2')_x \propto \exp(E_F - E_M/RT). \quad (20)$$

The effect of melting upon a  $(T_2')_x$  described by (19) can be a reduction (that is, a line broadening) if the melting produces a larger increase in  $n$  than in  $D_v$ .

### 2. Results in the Solid Metals Near Their Melting Points

The  $(T_2')_x$  terms in solid lithium and sodium have several features in common. The data in Table IV

<sup>46</sup> W. Jost, *Diffusion in Solids, Liquids, Gases* (Academic Press, Inc., New York, 1952), p. 335.

<sup>47</sup> Careri, Paoletti, and Salvetti, *Nuovo cimento* **11**, 399 (1954).

indicate that  $(T_2')_x$  in solid lithium and sodium is independent of the Larmor frequency (or of  $H_0$ ).  $(T_2')_x$  first becomes appreciable in lithium and sodium about one hundred degrees centigrade below the melting points (Figs. 14 and 16). This is just the region of temperature in which the resistivities<sup>48</sup> and specific heats<sup>49</sup> become anomalous and where the gradual disintegration of the metallic lattices begins to affect x-ray data.<sup>50</sup> The anomalous resistivity and specific heat terms apparently are associated with the appearance of significant concentrations of thermally activated vacancies in the lattice. Therefore, the explanation of  $(T_2')_x$  in this temperature region in terms of a vacancy interaction seems reasonable. However, according to (20), if  $E_F > E_M$ , which is probably the case, then  $(T_2')_x$  should decrease as the temperature rises. Figure 19 shows the opposite to be true. Moreover, crude calculations indicate that a magnetic interaction with the vacancies would not be sufficiently strong to produce the observed effects upon  $T_2$ . The situation remains unresolved.

It should be noted that in a solid in which there is vacancy diffusion and in which a vacancy interaction does determine the line width, the temperature dependence indicated by Eq. (20) combined with a knowledge of  $E_D$  should permit separate determination of  $E_F$  and  $E_M$ . Bloembergen<sup>51</sup> has independently recognized this fact.

The unexplained ratios of the  $T_1$ 's measured in the solid for the two rubidium isotopes (Sec. IV, B) may be connected with the interactions which cause the  $(T_2')_x$  effects in all three alkalis, but the point is not clear.

### 3. Results in the Liquid Metals

$(T_2')_x$  in liquid lithium is independent of the isotopic species of the surrounding nuclei (Table IV). The high-temperature  $T_2$  in Fig. 3 for Li<sup>7</sup> in a sample of natural isotopic abundance is approximately the same as  $T_2$  in Fig. 5 for Li<sup>7</sup> in the 93 percent Li<sup>6</sup> sample. Since the local fields caused by the nuclear moments are very different in the two samples, the source of the local fields responsible for the reduced  $T_2$  in the liquid must be external to the nuclear spin system. The similarity of the  $T_2$ 's in the liquids is surprising in view of the nonequality of the  $T_2$ 's in the solid samples. In Sec. 4, below, we describe evidence for impurity effects on  $T_2$  in lithium which are important below the melting point but are not evident in the liquid.

$(T_2')_x$  in liquid lithium and sodium is nearly temperature independent. Bloembergen and Rowland<sup>8</sup> have observed the resonance lines in two liquid Hg-Tl alloys to be insensitive to temperature and not appreciably narrower than in the solid phases. The  $(T_2')_x$  interaction

<sup>48</sup> D. K. C. MacDonald, *J. Chem. Phys.* **21**, 177 (1953).

<sup>49</sup> L. G. Carpenter, *J. Chem. Phys.* **21**, 2244 (1953).

<sup>50</sup> C. C. Bidwell, *Phys. Rev.* **27**, 381 (1926).

<sup>51</sup> N. Bloembergen (to be published).

present in the molten alkalis may be a feature common to many liquid metals and alloys.

From the data presented here and the measurements of Bloembergen and Rowland, it appears that many liquid metals may have  $T_2$  values appreciably less than  $T_1$ . Such is not the case for those nonmetallic liquids upon which measurements have been reported. The conduction electrons appear to be the source of the magnetic local fields which determine the line widths in those liquid metals which exhibit  $T_2$  values not determined by  $T_1$ . The line widths in liquid lithium and sodium may be the result of anomalous local fields arising from distortion of the electron distribution in the vicinity of imperfections. Since no unusual behavior of  $T_1$  is observed near the melting points, the local magnetic fields in question presumably result from a polarization effect and lie only along the direction of  $H_0$ . The alternate explanation for the lack of a  $T_1$  effect would be a low-correlation frequency for the high-temperature interaction, which seems quite unlikely in the liquid metals.

#### 4. *Dependence of the Results upon the Purity of the Samples*

The indications that the high-temperature broadenings arise from interaction with magnetic field sources external to the nuclear spin system lead one to suspect impurities in the metals as a source of some of the anomalous effects. Unfortunately, the total impurity content of our samples is not well known. We do know the metallic impurities in the lithium and sodium samples from spectrographic analyses (Table I). There appears to be little dependence of the data upon these impurities. Lithium samples *A* and *B* have sodium as their primary impurity, and differ by a factor of 10 in their sodium contents. Yet the data in Fig. 3 show no difference between the  $T_2$ 's shown as dots, which were obtained using sample *A* and those indicated by triangles, which were measured in sample *B*. It appears more likely that the impurities significant for  $T_2$  might be chemical compounds on the surface or dissolved into the metal particles. Because of the high chemical reactivity of the alkali metals, it is difficult to prevent the occurrence of chemical reactions on the surfaces of the metal particles. There is some evidence that non-metallic impurities can play a role in the extra broad-

enings, at least in the region just below the melting points. A comparison of Figs. 3 and 5 shows that, as has been mentioned in Part 3, there are definitely smaller  $T_2$ 's below the melting point in Fig. 5. A similar reduction of  $T_2$  in the region just below the melting point was produced in lithium sample *A* by removing most of the oil from the sample (in air) so that there was no protective coating on the particle surfaces. Fortunately, we do have good evidence that at least the major part of the extra broadenings is independent of the surface condition. Dr. T. R. Carver was able to produce, under a helium atmosphere and in a degassed oil, a lithium dispersion in which the particles retained a bright metallic sheen. The  $T_2$  measurements given as triangles in Fig. 3 were made with this sample. They agree with the other  $T_2$  measurements in Fig. 3 which were made in a sample in which the particles were visibly darkened, probably because of the formation of lithium azide at the surfaces.

No differences in  $T_2$  among the various samples was observed above the melting point.

The sodium sample listed as *B* in Table I gave  $T_2$ 's lower than those given in Fig. 6. In *B*, lower  $T_2$ 's were also observed above the melting point. A small extra decrease of  $T_1$  was observed in the liquid, although  $T_1$  in the solid agrees with the data in Fig. 6. This is the only sample in any of the metals in which a sample dependence of  $T_1$  was observed.

Quantitative investigation of the anomalous effects considered in Sec. IV, particularly in the solid phase in the temperature region just below the melting point, evidently requires more careful control of sample purity than has been achieved in this work.

#### ACKNOWLEDGMENTS

We would like to thank Professor H. S. Gutowsky and Dr. B. R. McGarvey for communication of some of their results prior to publication. We are indebted to them and to Dr. T. R. Carver for the metallic dispersions used in the experiments. Many of the electronic circuits were designed by H. W. Knoebel and L. S. Kypta. J. Spokas assisted in some of the measurements. Discussions with Professor D. Pines and Professor D. Lazarus were very helpful. Particular thanks are due to Professor C. P. Slichter for his continuing assistance and encouragement throughout the progress of this work.