Measurement of the Scattering Constant in Nuclear Emulsion*

LYLE W. SMITH
Brookhaven National Laboratory, Upton, New York
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The general solution to the diffusion equation is used to show that a measurement of track-to-track scattering of identical particles yields a value of the scattering constant in an unambiguous manner.

SINCE the operation of high-energy particle accelerators has made available well-collimated monokinetic beams of charged particles, several investigators[1–5] have reported measurements on the multiple Coulomb scattering of such particles. These measurements afford a calibration of the scattering constant in nuclear emulsion and allow a comparison with the predictions of the theory[6–8] as to the magnitude of the scattering constant and its variation with energy, cell length, etc. It is the purpose of this note to point out that such calibration experiments do not necessitate the use of an elaborate ultralinear microscope stage. Exposures may be made with suitable track densities so that the scattering of one track with respect to another of the same energy instead of with respect to a straight line gives the required information in an unambiguous manner.

We consider the form of the theory as developed by Scott and Snyder.[6] They show that the 3-point distribution in scattering-induced curvature is equivalent to the distribution in lateral displacement along a track, and we take their exact solution to the diffusion equation, subject only to the small angle approximation,

$$p(c|z)=\frac{z}{4\pi}\int_{-\infty}^{\infty}du\exp\left\{i\tfrac{1}{2}zcu-\frac{zh(u)}{\lambda u}\right\},\qquad(1)$$

where $p(c|z)$ is the probability distribution of curvature c observed on a track of length z,λ is the mean free path for scattering and the exact form of the elementary scattering law is contained in $h(u)$. If we now denote by $P(c|z)$ the distribution in curvature deduced by the differences in lateral displacement between two identical tracks, then

$$P(c|z)=\int_{-\infty}^{\infty}dc_1p(c-c_1|z)p(c_1|z),\qquad(2)$$

and performing the indicated integrations, obtain

$$P(c|z)=\tfrac{1}{2}p(\tfrac{1}{2}c|2z).\qquad(3)$$

Averaging over c to determine the mean curvature, we have

$$\bar{C}(z)=2\bar{c}(2z),\qquad(4)$$

where $\bar{c}$ refers to curvature of a single track and $\bar{C}$ refers to relative curvature between two tracks. The quantity usually measured is the second difference, δ in lateral displacement y where $\delta=y_1+y_3-2y_2$ and the y_i are the lateral displacements at three equally separated points. Then $c=4\delta/X^2$, X being the separation between points or "cell length." Using this relation (with X respectively $2z$ and $4z$), we obtain

$$\bar{\Delta}(z)=\tfrac{1}{2}\bar{\delta}(2z),\qquad(5)$$

i.e., the average second difference $\bar{\Delta}$ as measured between two identical tracks is one-half the average second difference $\bar{\delta}$ of one track, twice as long, measured with respect to a straight line. It is to be noted that this result is independent of the exact form of the elementary scattering law and the initial angle between the two tracks.

Track to track scattering has been used previously for rough energy estimates[9] but the method is applicable to a good measurement of the scattering constant. In addition to the advantage of not requiring a precision microscope stage, noise due to emulsion distortion is essentially eliminated and the remaining noise due to reading errors is independent of cell length and is easily subtracted.

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9 See, e.g., Lord, Fainberg, and Schein, Phys. Rev. **80**, 970 (1950). *Note added in proof.*—M. Koshiba and M. F. Kaplon, Phys. Rev. **97**, 193 (1955), have also pointed out the advantages of track to track scattering and have used the method for energy measurements by assuming a Gaussian scattering distribution.