allowed transition in Kr^{85} has been ascribed[20] to a similar rearrangement. Thus the assignment of $d_{5/2}$ to the ground state of Sb^{117} makes plausible the lack of observed transitions both to the ground state of Sn^{117} and also to the 0.726-Mev level. This is actually the assignment most strongly suggested by the shell model and is consistent with the assignments, from measured spins and moments, of $d_{5/2}$ and $g_{7/2}$ to the ground states of stable Sb^{121} and Sb^{123} respectively.

The proposed decay scheme for In^{117} and Sb^{117} is given in Fig. 5.

[20] Sunyar, Mihelich, Scharff-Goldhaber, Goldhaber, Wall, and Deutsch, Phys. Rev. **86**, 1023 (1952).

V. ACKNOWLEDGMENT

I wish to express my appreciation to R. W. Hayward for his generosity in placing the facilities of his laboratory at my disposal, to D. Hoppes for his cooperation in the experiments, to D. Cowie, N. Heydenburg, and S. Buynitzky of the Department of Terrestrial Magnetism for the cyclotron bombardments, to G. Scharff-Goldhaber, and A. W. Sunyar for their cooperation in making it possible for me to do part of the experimental work at Brookhaven, and to K. Way and R. W. King for their helpful discussions throughout the course of this work.

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Decay Scheme of the Mirror Nucleus P^{29} and Related Results*†

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The decay of P^{29} has been investigated with scintillation spectrometers, single and in coincidence. Positron emission (3.945±0.005 Mev end-point energy for the most energetic spectrum) occurs with a half-life of 4.45±0.05 sec to the ground state and to excited levels of Si^{29} at 1.28, 2.43, and possibly 2.03 Mev. Branching ratios (in percent) of 98.8±0.4 (ground state), 0.8±0.2 (1.28-Mev level), <0.15 (2.03-Mev level), and $0.24_{-0.08}^{+0.26}$ (2.43-Mev level) were measured. A study of γ rays from the decay of Al^{29} to Si^{29} led to branching ratios (in percent) of 15±9 (1.28-Mev level), <4 (2.03-Mev level), and 85±9 (2.43-Mev level). The intensity of a 1.15-Mev γ ray (cascading from the 2.43-Mev level) has been set at <11 percent of the total Al^{29} decays. Spin and parity assignments of the Si^{29} levels are discussed and compared with the results of other experiments. The ft values of the P^{29} decay are shown to agree with values calculated from the coupling constants of β decay if the theories of Feenberg and Bohr are used.

I. INTRODUCTION

PHOSPHORUS-29 had previously been found to decay to Si^{29} with a half-life of 4.6±0.2 sec by emitting positrons of 3.67±0.07 Mev.[1] The positron transition was presumed to be an image transition. The present work shows that the disintegration of P^{29} proceeds by two, or possibly three, alternative positron transitions to the low excited states of Si^{29} in competition with the image transition.[2]

II. METHOD

The method used to study the weak modes of β^+ decay in competition with the intense β^+ transition to the ground state was to search in the radioactivity for γ rays from the excited states to which the lower energy positrons decayed. Such a method is suitable even if the branching ratio to the excited state is quite small, because any low-intensity γ rays produced are of discrete energy. Hence the gamma rays can be detected relatively free of background, and can be identified.[3] The success of this method depends entirely on the use of NaI scintillation counters with their high sensitivity to and selectivity of γ rays. The apparatus used in this work for selecting, analyzing and presenting the pulse energy distribution produced in the NaI scintillation counter has been described in the literature.[4]

III. PRODUCTION AND HALF-LIFE OF P^{29}

P^{29} was produced by the $Si^{28}(d,n)$ reaction by bombarding Si crystals with 2.8-Mev deuterons from the Stanford cyclotron. The half-life of P^{29} was determined by measuring the activity of annihilation radiation (detected by the photoelectric peak in a NaI scintillation spectrometer) as a function of time. A scaler

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‡ Fulbright Travel Fellow. Present address: Physics Department, University of Oslo, Blindern, Norway.

[1] White, Creutz, Delsasso, and Wilson, Phys. Rev. **59**, 63 (1941).

[2] A preliminary report of this work has been given earlier: Roderick, Lönsjö, and Meyerhof, Phys. Rev. **90**, 371(A) (1953).

[3] The intense β^+ ray transition to the ground state, of course, produced no discrete γ ray, except annihilation quanta, although, as shown below, bremsstrahlung and annihilation in flight gave disturbing background effects.

[4] H. I. West, Jr., and L. G. Mann, Rev. Sci. Instr. **25**, 129 (1954).

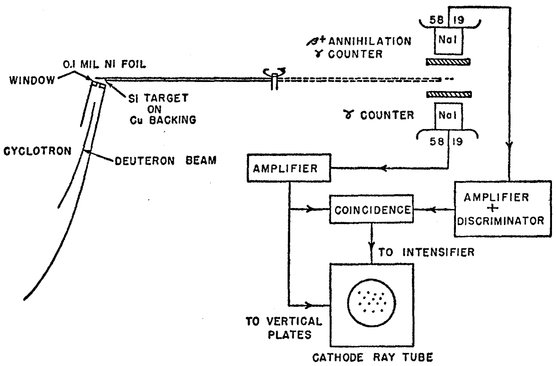

FIG. 1. Coincidence arrangement used for measurement of branching ratios of P^{29}.

and a clock together were photographed with a movie camera. Only a very small background activity (0.6 percent of the initial total activity) was obtained. The half-life of P^{29} was found to be 4.45 ± 0.05 sec.

IV. γ RAYS

Excited states of Si^{29} which could be reached by decay from P^{29} occur at 1.28, 2.03, 2.43, 3.07, 3.62, and 4.08 Mev.[5] In order to assure that γ rays from these states could be properly observed and in order to search for possible γ-ray transitions between excited

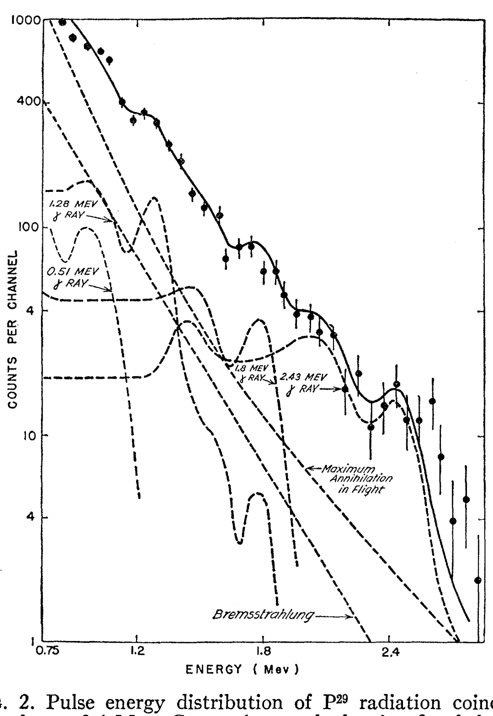

FIG. 2. Pulse energy distribution of P^{29} radiation coincident with pulses >0.4 Mev. Composite graph showing fit of the sum of 1.28-, 1.8-, and 2.43-Mev γ rays plus maximum continuous background.

[5] P. M. Endt and J. C. Kluyver, Revs. Modern Phys. **26**, 95 (1954).

states, the *prompt* γ-ray spectrum produced in the $Si^{29}(d,p)$ reaction was measured. A NaI scintillation spectrometer was used and the pulse height distribution was photographed. The results are shown in Table I. It is clear that only direct transitions from each excited state of Si^{29} to the ground state were observed.§

Next, using the same apparatus, the *radioactivity* of P^{29} was observed, and γ rays of 2.43, 1.28, and possibly 2.03 Mev were identified by their photopeak pulses. These γ-ray energies correspond to transitions between the three lowest excited states[5] and the ground state of Si^{29}. No higher-energy gamma rays were detected. The half-life of each of these γ rays was measured by observing each photopeak pulse activity as a function of time and found to be the same as that of P^{29}.

V. BRANCHING RATIOS

Assuming electron capture to be negligible,[6] the branching ratios of the various β^+ spectra to the excited states of Si^{29} are given directly by the intensities of

TABLE I. *Prompt* γ rays from the $Si^{28}(d,p)Si^{29*}$ reaction.

γ-ray energy[a] (Mev)	Estimated relative intensity
6.38	4
4.93	10
3.07	2
2.43	1
2.03	4
1.28	6

[a] Known excited states of Si^{29} which can be reached by the (d,p) reaction with 2.8-Mev deuterons are at 6.38, 6.10, 5.95, 4.93, 4.90, 4.84 Mev, in addition to those mentioned in the text.

the γ rays per positron decay. Hence coincidence measurements were made with two NaI scintillation spectrometers between the γ rays and the annihilation quanta of the positrons from P^{29}. The coincidence arrangement is shown in Fig. 1. Absorbers were used to prevent the positrons from entering the counters. The counter labelled β^+ *annihilation* γ *ray* was set to observe mainly annihilation quanta (i.e., pulses >0.4 Mev) while the counter labelled γ *ray* was set to observe only γ rays (pulses >0.74 Mev). Coincident pulses from the two counters caused the γ-*ray* counter pulse to be recorded. Figure 2 shows this coincident pulse energy distribution. It was fitted by discrete γ rays of 2.43, 1.8, and 1.28-Mev energy after subtraction of the continuous background.

An appreciable continuous background was found to be present which was caused by bremsstrahlung[7]

§ *Note added in proof.*—Some of the γ rays observed here have also been observed by L. C. Thompson, Phys. Rev. **96**, 369 (1954).

[6] Theoretically electron capture can be shown to contribute at most one percent to the branching ratios in the present case. See E. Feenberg and G. Trigg, Revs. Modern Phys. **22**, 402 (1950).

[7] Bremsstrahlung from radioactive sources has been observed and compared with theory by Goodrich, Levinger, and Payne, Phys. Rev. **91**, 1225 (1953).

and annihilation in flight[8] of the positrons. The absolute pulse height distribution of the background was calculated from the work of Heitler.[9] The number of coincident annihilation-in-flight pulses depends sensitively on the place of annihilation because of the angular correlation between the two quanta.[10] Because the exact region in which all the positrons were annihilated was not known, only upper and lower limits could be set to the annihilation-in-flight background. The maximum annihilation-in-flight background is shown in Fig. 2, the minimum in Fig. 3.

Because of the strong γ-ray transition from the 2.03-Mev level to the ground state found in the prompt γ-ray spectra from the $Si^{28}(d,p)$ reaction, the possible presence of a 2.03-Mev γ ray in the coincident distribution could not be ignored. Only when the minimum annihilation in flight background was subtracted, could a 2.03-Mev γ ray be fitted into the observed distribution as shown in Fig. 3. The intensities of the 2.43, 1.8, and 1.28-Mev γ rays are not appreciably changed by this fit.

The coincident 1.8-Mev γ ray was assigned to (138 sec)Al^{28}, produced in the $Si^{30}(d,\alpha)$ reaction.[11] The intensity of the 1.8-Mev γ ray was in approximate agreement with expectations based on calculation of chance coincidences and bremsstrahlung—1.8-Mev γ-ray coincidences.[12]

The peak labelled *0.51-Mev γ ray* in Figs. 2 and 3 was produced by the chance pile-up of annihilation quanta in the *γ-ray* counter while one of the quanta was in true coincidence with its twin in the *β^+ annihilation γ-ray* counter. Measurements made with a pure positron source (N^{13}) verified this hypothesis and calculations based on this measurement yielded an intensity for the chance peak in reasonable agreement with observation.

The half-life of the coincident pulse distribution was also measured and found to be the same as that of P^{29}, when the 1.8-Mev γ-ray pulses were assigned to Al^{28}.

In order to be able to determine branching ratios from the data of Figs. 2 and 3, the coincidence system was calibrated for γ-ray intensity per positron with the known[13] positron radioactivity Na^{22} in the identical geometry. The branching ratios of the decay of P^{29} to Si^{29} are listed in Table II.

[8] Annihilation in flight has been observed and compared with theory by Gerhart, Carlson, and Sherr, Phys. Rev. **94**, 917 (1954). We are very much indebted to Dr. Sherr for sending us a manuscript of this paper prior to publication and for enlightening discussions.

[9] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, pp. 151–177, 204–209, 230–231.

[10] H. Roderick, Ph.D. thesis, Stanford University, 1954 (unpublished), Appendix I.

[11] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

[12] The relative yield of Al^{28} to P^{29} was determined from half-life measurements of pulses >1.4 Mev.

[13] R. Sherr and R. H. Miller, Phys. Rev. **93**, 1076 (1954).

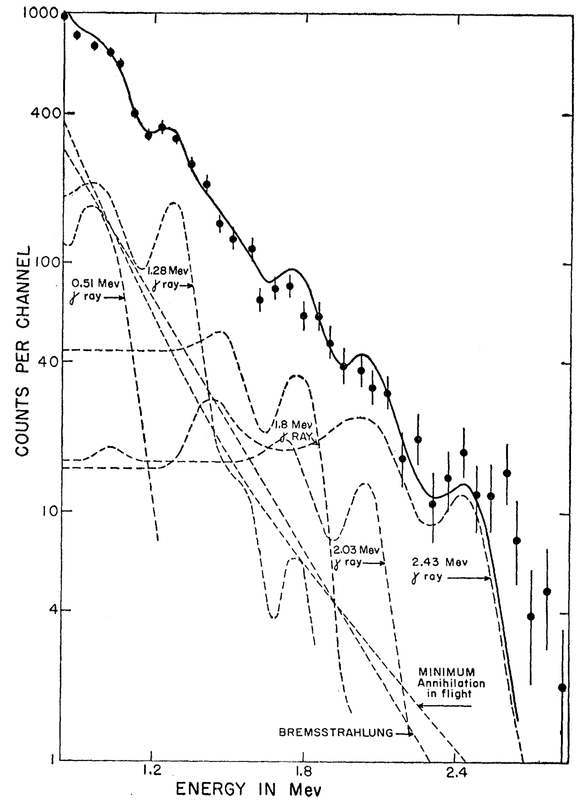

FIG. 3. Pulse energy distribution of P^{29} radiation coincident with pulses >0.4 Mev. Composite graph showing fit of the sum of 1.28-, 1.8-, 2.03-, and 2.43-Mev γ rays plus minimum continuous background.

VI. CASCADE TRANSISITONS AND Al^{29} DECAY

393-sec Al^{29} decays by β^- emission to the excited states of Si^{29}.[11] Previous investigation[14] had left some doubt as to which excited states of Si^{29} were involved in the Al^{29} decay. Therefore the γ rays from the decay of Al^{29} were investigated to clarify this matter and also to obtain further information about possible cascade transitions between the low excited states of Si^{29}.

Al^{29} was produced by an (n,p) reaction on Si. Gamma rays, from the radioactivity thus produced, yielded the pulse energy distribution shown in Fig. 4. The 1.28-Mev and 2.43-Mev γ rays are assigned to Al^{29} while the 1.78-Mev γ ray is assigned to Al^{28}.[15] Cascade γ-ray

TABLE II. Branching ratios of the P^{29} decay to Si^{29}.

Level of Si^{29}[a] (Mev)	Branching ratio (percent)
2.43	$0.24^{+0.26}_{-0.08}$
2.03	$\leqslant 0.15$
1.28	0.80 ± 0.20
0	98.8 ± 0.4

[a] An upper limit to the branching ratios to higher excited states of Si^{29} was calculated to be 0.05 percent.

[14] Seidlitz, Bleuler, and Tendam, Phys. Rev. **76**, 861 (1949).

[15] Al^{28} was produced in the $Si^{28}(n,p)$ reaction.

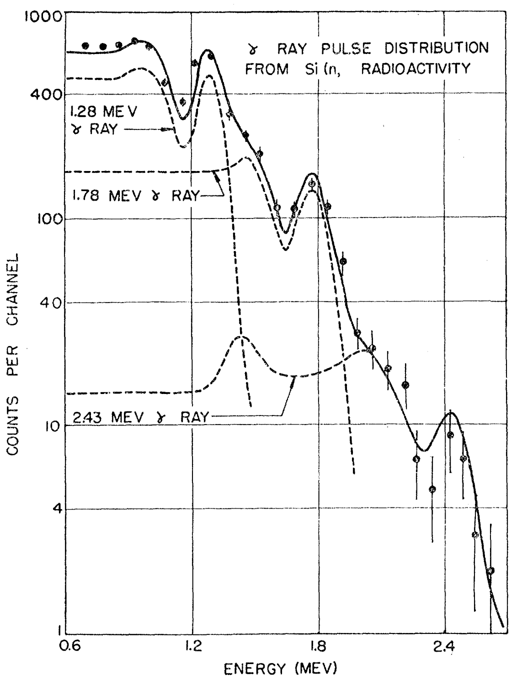

FIG. 4. γ-ray pulse energy distribution from Al^{28} and Al^{29} radioactivity produced by Si(n,p) reaction.

transitions were sought, using two NaI counters in a coincidence arrangement similar to that shown in Fig. 1. All coincidences found were accounted for by other processes, that is, cosmic ray and room background, chance coincidences and γ-ray—bremsstrahlung coincidences. An upper limit to the amount of 1.15-Mev γ ray in coincidence with the 1.28-Mev γ ray was calculated to be 11 percent of the total decays. Tables III and IV lists the intensities of the γ rays found in the decay of Al^{29} and the calculated branching ratios and *ft* values to the excited levels of Si^{29}.[16]

VII. β RAYS OF P^{29}

The end-point energy of the P^{29} positron transition to the ground state of Si^{29} was accurately measured with a lens-type magnetic spectrometer and found to be equal to 3.945±0.005 Mev.[17] The *ft* values of the competing β^+ transitions from P^{29} are listed in Table V.

VIII. DECAY SCHEME OF P^{29}

The decay scheme of P^{29} is shown in Fig. 5. The 2.43-Mev and 1.28-Mev excited levels of Si^{29} are both assigned spin 3/2, even parity, since the ground state of Si^{29} and hence of P^{29} is known to have spin 1/2, even parity.[18,19] Al^{29} is assigned spin 5/2, even parity. Since only lower limits to the *ft* values to the 2.03-Mev level of Si^{29} could be set from both P^{29} and Al^{29} and these limits were such that allowed β transitions were possible, no spin or parity assignment of the 2.03-Mev level could be made from this work.[20]

[16] After completion of this work the published paper of M. Nahmias and A. Wapstra, J. phys. et radium **15**, 570 (1954) was received. They find, for the Al^{29} decay, branching ratios of 25 and 75 percent to the 2.43- and 1.28-Mev levels, respectively. They measure the end-point energy of the weaker β^- spectrum to be 1.55±0.10 Mev.

[17] This result has been previously reported: H. Roderick and C. Wong, Phys. Rev. **92**, 204 (1953). However, a numerical mistake led to an overestimation of the error reported.

IX. DISCUSSION

A. Shell Model

The above-designated parity and spin uniquely define the orbital angular momentum of the various states that could be assigned on the basis of the single-particle shell model. These assignments are in agreement with Mrs. Mayer's shell model.[21] $_{13}Al^{29}$ has 13 protons in its odd group which leaves one vacancy in the $d_{5/2}$ shell, while Si^{29} and P^{29} have 15 particles in the odd group which assigns the 15th particle to sole occupancy in the $2s_{1/2}$ shell. The assignment of $d_{3/2}$ to the 1.28-Mev level and/or the 2.43-Mev level is consistent with the statement that the low excited states should be those in which the odd nucleon is raised to another level in the same shell, which in this case is $d_{5/2}$ or $d_{3/2}$.[21]

B. Related Experiments

Agreement between the above assigned spins and parities and the results of the (d,p) stripping[19,22] and (n,γ)[23] reactions on Si^{28} is generally good. However, an

TABLE III. Intensity of γ rays from Al^{29}.

γ-ray energy (Mev)	Intensity per decay (percent)
2.43	9.4±2.1
2.03	⩽ 3.8
1.28	89 ±3.4
1.15	⩽10.8

TABLE IV. Branching ratios of Al^{29} decay to Si^{29}.[a]

Level of Si^{29} (Mev)	Branching ratio (percent)	Log(ft)[b]
2.43	15±9	5.0±0.4
2.03	⩽3.8	⩾6.0
1.28	85±9	5.3±0.2

[a] Assuming no decay to the ground state.
[b] Assuming a total decay energy of Al^{29} equal to 3.98±0.10 Mev (see reference 16).

[18] Williams, McCall, and Gutowsky, Phys. Rev. **93**, 1428 (1954).

[19] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 467 (1953).

[20] The results of the $Si^{28}(d,p)$ stripping experiment of reference 19 lead to an assignment of even parity and spin 3/2 or 5/2 to the 2.03-Mev level.

[21] M. G. Mayer, Phys. Rev. **78**, 16 (1950).

[22] C. F. Black, Phys. Rev. **90**, 381 (1953).

[23] Kinsey, Bartholomew, and Walker, Phys. Rev. **83**, 519 (1951). Results revised in B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **93**, 1260 (1954). We are indebted to Professor Kinsey for sending us a preprint and for private communication of his results.

isotropic angular distribution has been found for the (d,p) reaction to the 2.43-Mev level, but there is reason to doubt this result.[19]

In the $Si^{28}(n,\gamma)$ work[23] the existence of an observable γ-ray transition between the neutron capture level and the ground state of Si^{29} is of considerable interest. This transition should be strictly forbidden if the states concerned could really be expressed by $s_{1/2}$ single particle orbitals. However, as discussed below, the ground state of Si^{29} can be considered to be a mixture of $S_{1/2}$ and $P_{1/2}$ states with even parity. In that case the transition could be considered $M1$ and the observed intensity is in reasonable agreement with that calculated from Weisskopf's formula.[24]

C. Image Transition

The ft value of the image transition P^{29} (see Table IV) is in good agreement with the results obtained using the coupling constants of β decay determined by Winther,[25] if the ground states of Si^{29} and P^{29} are assumed to be a roughly equal mixture of $S_{1/2}$ and $P_{1/2}$ states of even parity.[26] The amount of this mixture is determined by the magnetic moment of Si^{29} (0.55 nm[27]). Similarly, Bohr's theory of the nucleus[28] leads to reasonably good agreement between the observed ft value and that calculated from Winther's constants (and also to agreement with the magnetic moment of Si^{29}), if the ground state of Si^{29} is made up of the super-

TABLE V. ft values of P^{29} decay to Si^{29}.

Level of Si^{29} [a] (Mev)	End-point energy of β^+ spectrum (Mev)	Log(ft)[b]
2.426±0.007	1.519±0.009	$4.51\ {}^{+0.17}_{-0.32}$
2.027±0.007	1.918±0.009	$\geqslant 5.15$
1.278±0.007	2.667±0.009	5.03 ±0.11
0	3.945±0.005	3.723±0.007

[a] See reference 5.
[b] The ft value includes both positron emission and electron capture.

[24] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 627.
[25] A. Winther and O. Kofoed-Hansen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 14 (1953).
[26] E. Feenberg (private communication).
[27] S. S. Dharmatti and H. Weaver, Phys. Rev. **84**, 843 (1951).
[28] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).

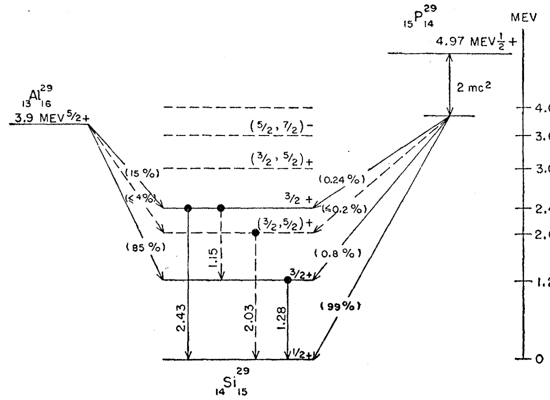

FIG. 5. Decay schemes of P^{29} and Al^{29}. Known levels of Si^{29} which presumably are not reached in either decay are shown with dotted lines. The upper limit for the intensity of the 1.15-Mev γ ray is roughly equal to the intensity of the 2.43-Mev γ ray.

position of single-particle orbital states $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ with expectation values 1/3, 1/6, and 1/2, respectively.

D. Transitions to the Excited States

Since the ground state of Si^{29}, and hence of P^{29}, can be considered as a mixture of states with different l values, it is not surprising that an allowed-unfavored β^+ transition occurs to the 1.28-Mev level ($d_{3/2}$) in spite of the l-forbiddenness rule.[29]

In fact one can calculate from the ft values given in Table IV that the 1.28- and 2.43-Mev levels of Si^{29} can be thought of as states of even parity and $D_{3/2}$ character with minimum $P_{3/2}$ admixtures of 3.6 and 12 percent, respectively.[26] There is also other evidence that the 2.43-Mev level is a many-particle state.[19] The unfavored factors[28] F of 0.096±0.025 and $0.32^{+0.34}_{-0.10}$ calculated for the transitions to the 1.28- and 2.43-Mev levels, respectively, are of the same order of magnitude as those of other allowed-unfavored transitions listed in Bohr's work.[28]

We would like to thank Dr. L. G. Mann and Mr. H. I. West, Jr. for their help with the experiments, and Dr. E. Feenberg for many stimulating discussions.

[29] Mayer, Moszkowski, and Nordheim, Revs. Modern Phys. **23**, 315 (1951).