discussed in the introduction, is satisfied in the 400-Mev region.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor L. Wolfenstein for numerous valuable discussions on many aspects of this problem. The help of Dr. S. A. Friedberg and other members of the Low Temperature Laboratory in providing liquid hydrogen is appreciated. The cooperation of the personnel of the Carnegie Institute of Technology Nuclear Research Center was an important contribution to the successful completion of this work.

PHYSICAL REVIEW VOLUME 97, NUMBER 3 FEBRUARY 1, 1955

Scattering of 151-Mev Positive Pions by Protons*

RAYMOND A. GRANDEY† AND ARNOLD F. CLARK‡
Carnegie Institute of Technology, Pittsburgh, Pennsylvania

(Received October 4, 1954)

Nuclear emulsions have been exposed to the external positive pion beam of the Carnegie synchrocyclotron and have been scanned for elastic π-p scatterings. Seventy events have been found while scanning plates exposed to a final energy of 151 ± 7 Mev, which, combined with results obtained at Columbia and the recent accurate determinations of the total cross section in this region by the Carnegie Group, yield a value for the differential cross section of $d\sigma/d\Omega=(7.7\pm2.1)-(3.1\pm2.4)\cos\theta+(17.3\pm7.2)\cos^2\theta$ mb/sterad. The best Fermi type phase shifts calculated from the above are: $\alpha_3=-26°$, $\alpha_{33}=50°$, and $\alpha_{31}=0°$.

Recent data on pion-proton scattering at various energies are discussed, and it is found that the present data can be fitted with just three phase shifts which can be treated on semi-empirical models.

I. INTRODUCTION

THE elastic scattering of both positive and negative pions from hydrogen has been studied over a considerable range of energies. Counter techniques have been applied at Chicago, Columbia, and Rochester to obtain angular distributions for negative pion-proton scattering from 40 to 217 Mev.[1–6]

Unfortunately, the very low positive-pion fluxes obtained externally from the present synchrocyclotrons have made it impractical to extend counter techniques in studying angular distributions for positive pion-proton scattering much above the 135-Mev point obtained by the Chicago group.[1] Nuclear emulsion techniques can be well applied in this high-energy positive-pion region. By using the hydrogen content of the emulsion as a scatterer, data can be collected with a minimum of cyclotron running time, and since the entire solid angle is available, this leads to the additional advantage of sampling scattering into all angles.

II. EXPERIMENTAL PROCEDURE

A magnetically analyzed 160-Mev external positive-pion beam is available from the Carnegie cyclotron with a flux at the focus of the analyzing magnet of from 2 to 7 mesons per cm^2 per sec.[7] This flux is so small that it causes serious experimental difficulties in a counter angular distribution experiment. The small flux requires exposures for plates on the order of one hour for intensities in the emulsion of 10^4 mesons/cm^2.

The beam is formed by allowing the main internal proton beam to strike a Cu target placed at a cyclotron radius just inside the $n=0.2$ resonance. Positive pions produced in the backward direction are deflected out of the cyclotron by the fringing field and are focused at the same time.

The beam passes through a channel cut in the 8-ft thick magnetite concrete shield wall separating the cyclotron room from the experimental area and is further analyzed upon emergence by passing through a double-focusing 45° sector magnet. The energy and muon contamination of the beam were measured by taking range curves in Cu with counters. Three scintillation counters were placed before the absorber and one large counter behind. The ratio of quadruple to triple coincidences was counted as a function of absorber thickness. The equipment and geometry of the counter group[7] was used. Figure 1 shows the range curve.

The relatively long exposure required together with

---

* This paper is a condensation of a thesis submitted by R. Grandey in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics at Carnegie Institute of Technology. The research was partially supported by the U. S. Atomic Energy Commission. Preliminary results were reported in Phys. Rev. **94**, 766 (1954).

† U. S. Atomic Energy Commission Predoctoral Fellow.

‡ Now at the Radiation Laboratory, University of California, Livermore, California.

[1] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).

[2] Fermi, Glicksman, Martin, and Nagle, Phys. Rev. **92**, 161 (1953).

[3] M. Glicksman, Phys. Rev. **94**, 1335 (1954).

[4] M. Glicksman, Phys. Rev. **95**, 1045 (1954).

[5] Bodansky, Sachs, and Steinberger, Phys. Rev. **93**, 1367 (1954).

[6] A. Roberts and J. Tinlot, Phys. Rev. **94**, 766 (1954).

[7] Ashkin, Blaser, Feiner, Gorman, and Stern, Phys. Rev. **96**, (1954).

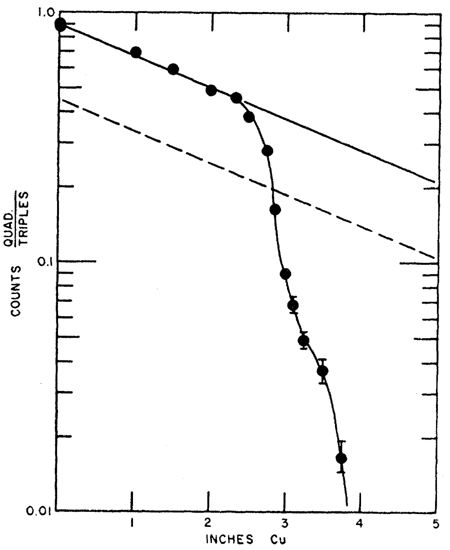

FIG. 1. Range curve for 151-Mev positive pions range = 2.83 inches with 1 inch plexiglass in front of the first counter. Contamination = (8±3) percent.

the high general radiation background level caused by neutrons escaping through the shield wall made it necessary to shield the plates during exposure. This was effected by building an 8 ft ×8 ft blockhouse of magnetite concrete about the plates with an inner core of two cubic feet of Pb. In addition, a large mass of Pb was placed on the coil can of the cyclotron in the median plane in an effort to reduce the general background in the experimental area. Ilford G-5 600 3 in. ×3 in. plates were used with the beam incident perpendicular to the 3-in. edge but in the plane of the emulsion.

III. SCANNING

The low intensity of the meson beam together with the high background made it impossible to reduce the background-to-meson ratio in the plates sufficiently to permit area scanning. Reliable results in area scanning are obtained only if the plates are very clean. It was therefore necessary to scan along the tracks. The procedure is essentially that described by Homa, Goldhaber, and Lederman,[8] hereinafter referred to as HGL. Their plates were obtained by internal exposures and had a great deal of background. Our plates are much cleaner, and we believe that we are operating close to 100 percent efficiency in the angular region under consideration. Our total cross section agrees within statistical errors with the counter transmission cross section. We have normalized our angular distribution to the counter total cross section which is statistically much better, so that the efficiency is important only if it varies appreciably with the type of elastic event. The efficiency at small and large meson angles is discussed in Sec. V. It is possible that efficiency is lower for large azimuthal angles, the azimuthal angle being the angle made by the plane of the event with the plane of the emulsion, since events are often difficult to measure in these regions. The azimuthal distribution of our 70 events is plotted in Fig. 2. The concentration of events with azimuthal angles less than 15° is regarded as a purely statistical fluctuation, since we are quite sure that our efficiency could not fall off until well over 45°. Such a possible inefficiency should not be a function of the meson angle.

IV. IDENTIFICATION OF EVENTS

Elastic pion-proton scattering events were identified by applying the usual criteria of coplanarity and angular correlation.[8] When the scattered proton stopped in the emulsion, its range was measured and energy determined as an additional check. In general, the outgoing meson was not grain-counted if the density looked normal unless there was some additional reason to suspect the event, for instance a possible third outgoing particle or a borderline satisfaction of the two criteria. One event which was quite coplanar and within one-half degree of angular correlation was rejected because of obviously high-grain density. This event was already suspect because of the presence of an electron track originating with the event. This additional criterion of "cleanliness" appears to be very good. All quasi-elastic pion-proton scatterings had either an electron track or a very short (less than 5μ) stub originating with the event. All elastic scatterings appeared clean at the origin except in a few cases where an electron could be discerned. In all of these later cases, the electron does not appear to originate with the event but is probably a chance coincidence with one of the numerous background electrons.

$\Theta/\Delta\theta$, the deviation from angular correlation measured in units of the rms errors in measurement,

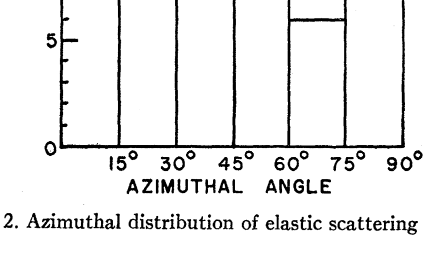

FIG. 2. Azimuthal distribution of elastic scattering events.

---

[8] Homa, Goldhaber, and Lederman, Phys. Rev. 93, 554 (1954), henceforth referred to as HGL.

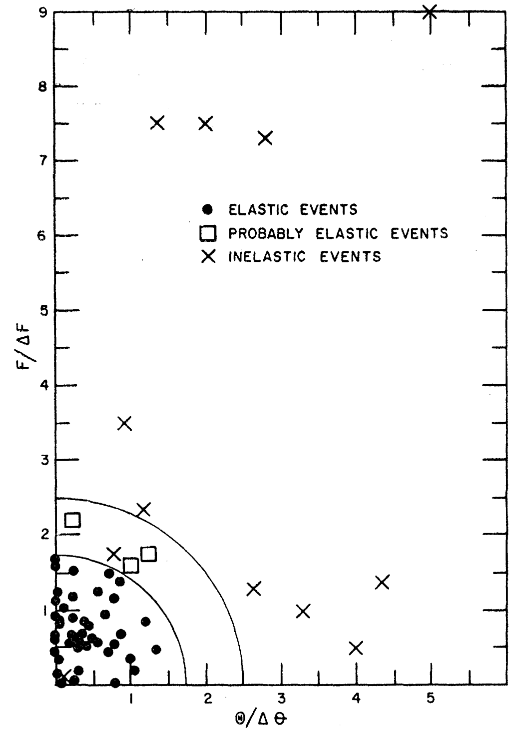

FIG. 3. Coplanarity—Angular correlation plot.

is plotted against $F/\Delta F$, the corresponding measure of deviation from coplanarity in Fig. 3 for all events which were at all close to satisfying the two criteria. F is defined by HGL. It is the volume of the pyramid formed by going out unit distance along each of the three tracks and connecting the end points. It equals zero for coplanarity. The elastic events cluster about the origin. We have applied the same criterion for acceptance as HGL. We find, as they did, that, assuming a fairly broad distribution of quasi-elastic events, statistically one event might be expected to fall in the region of acceptance. As noted, one such event was found and ruled out because of grain density and an electron track. We have made no further correction for this effect.

V. ANALYSIS OF DATA

One of the energies investigated by HGL was 151±7 Mev. This energy is fortuitously the same energy that we have in our plates after correcting the energy obtained from the range curve for the energy loss in the plates and in the absorber used to eliminate protons passing through the analyzing magnet. This made it feasible to get significant data by pooling the results. HGL had no good method of determining directly muon-electron contamination in their beam. They estimated it as (10+10) percent. From this estimate they calculated their track length scanned as 8500±850 cm at this energy. In the process of scanning, we found 453 stars in 14430±450 cm of pion track. This gives us a mean free path for star formation of 31.8±1.9 cm. This value is sufficiently accurate to permit a better determination of the HGL track length from the 260 stars observed by them. Their track length is 8220±700 cm HGL have observed 41 events and we have found 63 which can be counted in the total cross section. These are events found while scanning along track which enters in the total track length. A few additional events are available from a small amount of area scanning done by us, a few found beyond the limits of track measurement and some found while area scanning to relocate events. These additional events are included in the angular distribution. Care was taken to accept only events which would not have been missed regardless of the difficulty of seeing an event in order to prevent biasing the angular distribution.

Since the range of the recoil proton vanishes at a 0° scattering angle for the meson and is less than 5μ at 5°, a lower angle cutoff was taken at this point. Also small angle scatterings from hydrogen can be easily confused with elastic scattering from heavier nuclei if they are noticed at all. A large angle cutoff was taken at 175°, since events beyond this angle could be confused with a chance crossing of tracks, especially if they occurred near another intersection of tracks. These two cutoffs include 0.8 percent of the solid angle. In addition, HGL assume (94±6) percent scanning efficiency. These two corrections amount to 4 percent. The mean free path for elastic scattering of 151-Mev positive pions by protons is thus 0.96(22650±800 cm)/(104±10 events) = 209±22 cm. The total cross section is 151±19 millibarns.

The counter value for the total cross section at this energy is 168±4 mb.[7] This value is far beyond our accuracy. Our result is in agreement, although on the low side.

Angular Distribution

There are 56 events from HGL and 70 from our group available for investigation of the angular distribution. The center-of-mass (c.m.) angle has been calculated for each event. The data have been analyzed

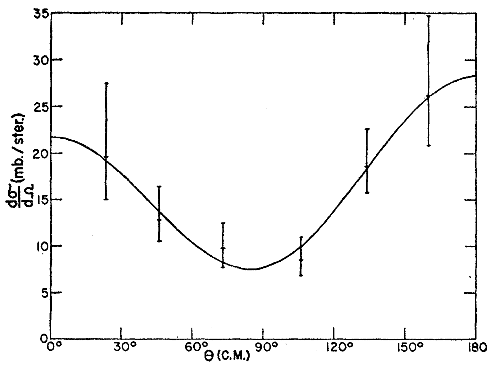

FIG. 4. Angular distribution (c.m.) for 151-Mev positive pions.

TABLE I. Angular distribution of pion-proton scattering: $\pi^{+(-)}+p\rightarrow\pi^{+(-,0)}+p(p,n)$. $d\sigma/d\Omega=a+b\cos\theta+c\cos^2\theta$ mb/sterad. a_c, b_c, and c_c are calculated from treatments of α_1, α_3, and α_{33}, under the assumption that the other phase shifts are negligibly small.

E(Mev)	η	a	a_c	b	b_c	c	c_c	Proc.	Ref.
65	0.88	0.89±0.13		−1.38±0.09		0.21±0.36		−0	5
78	0.98	1.9 ±0.3	1.50	−1.7 ±0.4	−2.46	1.6 ±0.9	3.29	++	1
110	1.18	3.6 ±0.7	3.52	−4.8 ±0.8	−4.13	7.5 ±1.9	8.74	++	1
120	1.24	0.49±0.11	0.44	0.34±0.16	0.30	1.16±0.34	1.26	−−	1
120	1.24	0.6 ±0.4	1.20	−1.9 ±0.5	−1.73	3.2 ±1.7	2.53	−0	1
135	1.33	3.9 ±2.3	6.11	−7.1 ±2.8	−4.10	18.0 ±6.8	16.1	++	1
140	1.35	7.5 ±1.8	6.43	−3.7 ±2.3	−3.87	15.1 ±5.5	17.0	++	17
144	1.37	0.82±0.16	0.68	0.73±0.24	0.34	1.52±0.50	2.08	−−	1
144	1.37	1.05±0.5	1.70	−1.9 ±0.5	−1.55	3.9 ±2.0	4.17	−0	1
151	1.41	7.7 ±1.9	7.3	−3.3 ±2.4	−2.96	17.3 ±7.0	19.5	++	
169	1.51	0.64±0.19	0.83	0.47±0.29	0.24	3.04±0.62	2.43	−−	2
169	1.51	1.82±0.72	1.98	−0.61±0.63	−0.61	4.25±2.27	4.87	−0	2
187	1.60	0.81±0.13	0.87	0.35±0.20	0.53	3.08±0.38	2.62	−−	4
187	1.60	1.46±0.24	2.02	−0.16±0.30	0.27	5.63±0.88	5.24	−0	4
194	1.64	1.12±0.24	0.85	0.65±0.62	0.56	2.87±0.80	2.42	−−	2
194	1.64	1.73±0.80	2.42	−0.09±0.74	0.19	5.89±2.61	4.84	−0	2
217	1.76	0.85±0.15	0.71	0.75±0.22	0.65	2.36±0.44	2.06	−−	3
217	1.76	1.36±0.22	1.68	1.23±0.26	1.14	4.82±0.76	4.11	−0	3

by dividing the angular region into six thirty-degree regions. The average differential cross section in each region is calculated on the assumption that the total cross section is 168 mb. A least squares best fit is then made to the assumed angular dependence $d\sigma/d\Omega=(a+b\cos\theta+c\cos^2\theta)$ mb/sterad, where θ is the meson scattering angle in the c.m. system. The results are corrected for the fact that the average value of $d\sigma/d\Omega$ over an interval does not necessarily fall in the center of the interval. The distribution is: $d\sigma/d\Omega=(7.7\pm2.1)-(3.1\pm2.4)\cos\theta+(17.3\pm7.2)\cos^2\theta$ millibarns/steradian. The errors are least-squares errors. The experimental points and the above curve are plotted in Fig. 4. The errors on the experimental points are asymmetric because of the small number of events. A measure of how well the curve fits the points can be obtained by calculating $M=\Sigma_i(\Delta i/\epsilon_i)^2$, where Δi is the deviation of point i from the curve and ϵ_i is the associated standard deviation of the point. A value of $m-n$ is expected when fitting m points with n parameters, i.e., $6-3=3$. The calculated value is $M=0.98$ which is very low. It is seen from this that the curve can be changed considerably and maintain a reasonable agreement with the data. It is obvious that it is not necessary to use another form for the cross section. For example, no D-wave contributions are required.

Phase Shift Analysis

Ashkin and Vosko have devised a graphical method for the analysis of both positive and negative pion-proton scattering differential cross sections assuming only S- and P-wave scattering and charge independence.[9] We have analyzed our data using an algebraic computation derived directly from this method. If we use Fermi's notation, the least-squares solution for the Fermi-type phase shifts gives: $\alpha_3=-26°$, $\alpha_{33}=50°$, and $\alpha_{31}=0°$.

It is very difficult to determine the errors involved in the above phase shifts. An attempt has been made to obtain a least-squares type of error for the above phase shifts by writing down the equations for a, b, and c in terms of the phase shifts. Error equations are derived by expanding the equations in terms of the errors (assumed small) keeping only the first powers. This yields linear equations in the errors. The treatment led to the following errors, which are of doubtful meaning: $\delta\alpha_{33}=9°$, $\delta\alpha_{31}=9°$, $\delta\alpha_2=20°$. These errors are overestimates since the restrictions imposed by the total cross section were not utilized in determining the errors.

There are, of course, Coulomb effects in direct scatterings. Ashkin and Smith[10] and Van Hove[11] have obtained expressions for the differential cross sections including Coulomb effects. The interference term depends strongly on the nuclear phase shifts. The difference in the integrated cross section above 5.5°, the experimental cutoff point in the c.m., system between constructive and destructive interference is about 0.5 percent, which means that several hundred events would be required to detect the effect.

VI. DISCUSSION OF AVAILABLE DATA

Total cross sections for π^+-p scattering have now been measured from energies of 33 to 197 Mev[1,7,12] with accuracies of from 4 to 10 percent. Preliminary data are available up to over 700 Mev.[13,14] Angular distributions for π^+-p scattering extend from 40 Mev to the 151 Mev reported here with an additional rough point at 188 Mev[1,5,8,15–17] and for both direct and exchange

---

[9] J. Ashkin and S. H. Vosko, Phys. Rev. **91**, 1248 (1953).

[10] Ashkin and Smith, O.O.R. Technical Report No. 1, Carnegie Institute of Technology, Februrary 2, 1953 (unpublished).

[11] L. Van Hove, Phys. Rev. **88**, 1358 (1952).

[12] S. L. Leonard and D. H. Stork, Phys. Rev. **93**, 568 (1954).

[13] S. J. Lindenbaum and L. C. L. Yuan, Phys. Rev. **92**, 1578 (1953).

[14] Cool, Madansky, and Piccioni, Phys. Rev. **93**, 637 (1954).

[15] J. P. Perry and C. E. Angell, Phys. Rev. **91**, 1289 (1954).

[16] Orear, Lord, and Weaver, Phys. Rev. **93**, 575 (1954).

[17] Orear, Tsao, Lord, and Weaver, Phys. Rev. **95**, 624(A) (1954).

TABLE II. Fermi-type phase shifts taken from the references listed.

Energy (Mev)	α_3	α_{33}	α_1	α_{31}	α_{11}	α_{13}	Ref.
40	-1.75 ± 0.75	$+5\ \pm1$	$+10\pm1$	0	0	0	6,18
45	$-5.7\ \pm1.2$	$+4.4\pm1.1$		$+2.4\pm1.8$			16
65	-6.2	$+9.1$	$+10.9$	-1.9	$+0.4$	-2.6	5
78	-6	$+13$		-3			1
120	-15	$+30$	$+9$	$+4$	-3	$+2$	1
135	-14	$+38$	$+10$	$+5$	-5	$+2$	1
140	-18	$+47$		-4			17
151	-26	$+50$		0			
217	-26	$+100$	-3	0	0	0	3

π^--p scattering from 40 to 217 Mev.[1-6] Table I lists the published values of the angular distributions measured to date and analyzed on the assumption of only S- and P-wave scattering. The various total π^+-p cross sections have been correlated and plotted as a function of energy in Fig. 5.

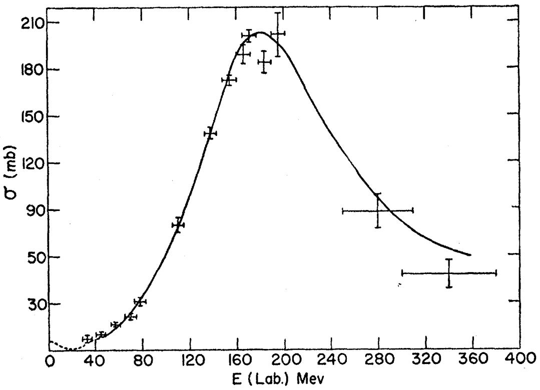

FIG. 5. Total cross section $\pi^+ + p - \pi^+ + p$.

The Fermi-type phase shift analyses of the above data are summarized in Table II. These analyses have been made by the respective investigators. It is seen that three of the phase shifts, α_{11}, α_{13}, and α_{31}, are all much less than the other three at all energies. The phase shifts α_3, α_1, and α_{33} appear to be the only ones of significance in their effect on the cross sections. The S-wave phase shifts are plotted as a function of η in Fig. 6, and α_{33} is plotted in Fig. 7. η is the relative momentum in the c.m. system in units of $\mu_0 c$, where μ_0 is the reduced mass of the system. Noyes and Woodruff[18] have recently discussed the behavior of the S-wave phase shifts. The very small value of α_3 at 40 Mev and the recent results of Bernardini *et al.*[19] on photoproduction near threshold have led them to predict that α_3 passes through zero in the vicinity of 25–30 Mev and becomes positive at lower energies, a behavior which follows from a Jastrow-type potential. Noyes and Woodruff have assumed simple potential models for the S-state interactions to try to fit the data. The isotopic spin $\frac{3}{2}$ state interaction is represented by an attractive exponential with a hard repulsive core (Jastrow potential) and the $\frac{1}{2}$ state by a simple attractive exponential. Proper choice of well depths and ranges allows a reasonable fit to the observed S-wave phase shifts. The results of this phenomenological treatment by Noyes and Woodruff are plotted in Fig. 6.

Glicksman[3] has recently analyzed his 217-Mev negative-pion data under the assumption that the three small phase shifts are negligible and found that the best fit is obtained by taking α_1 slightly negative. A value of α_1 less than 4 or 5 degrees is consistent with his data. This can be done on a potential model and still retain good agreement with the low-energy data.

An effective range treatment of α_1 has been made from the available data, by assuming that three terms are required in the expansion of $k\cot\alpha_1$, where k is the meson momentum divided by $\hbar$. One may write: $k\cot\alpha_1 = 1/a + \frac{1}{2}r_0k^2 + Pr_0^3k^4$. The result of the treatment is: $\eta\cot\alpha_1 = 3.0 + 1.2\eta^2 + 1.2\eta^4$. The phase shifts are plotted as curve IV in Fig. 6. The values of the effective range, scattering length and shape dependent parameter are $r_0 = 4.2\times10^{-13}$ cm, $a = 0.47\times10^{-13}$ cm,

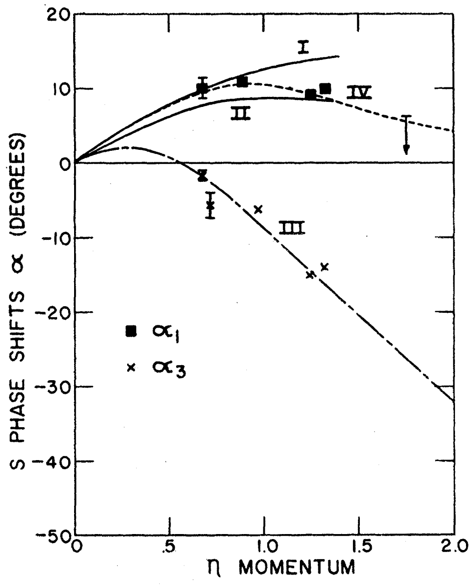

FIG. 6. Phenomenological and empirical treatment of pion-nucleon S-wave phase shifts. I and II. Noyes and Woodruff (reference 18) α_1. III. Noyes and Woodruff (reference 18) α_3. IV. Effective range fit α_1.

---

[18] H. P. Noyes and A. E. Woodruff, Phys. Rev. 94, 1401 (1954).
[19] G. Bernardini, Phys. Rev. 93, 930 (1954).

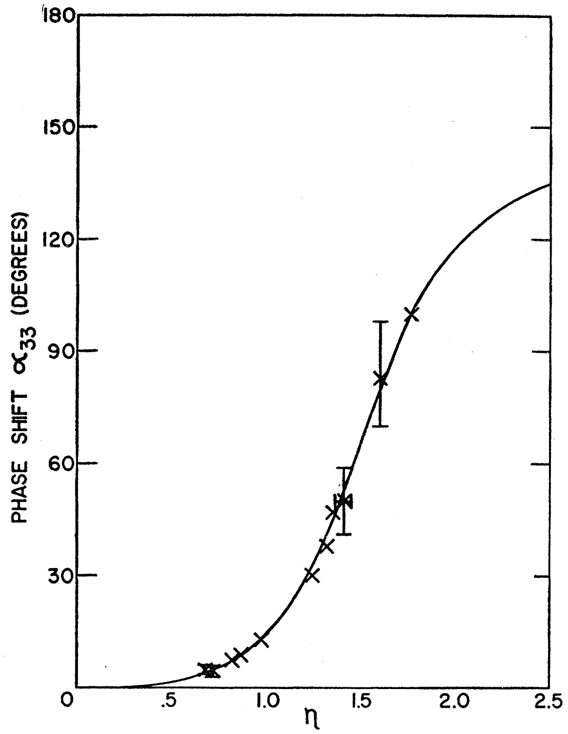

FIG. 7. Calculated values of the $\frac{3}{2}-\frac{3}{2}$ phase shift.

and $P=0.04$. The large value of r_0 and the corresponding large values of the coefficients in the expansion indicate a slow convergence of the series, i.e., the data should be fitted with several more terms. Further terms would have the effect of reducing the effective range and causing α_1 to fall off more rapidly at high energies. This should be kept in mind when better data on α_1 become available.

Brueckner has pointed out that the α_{33} phase shifts can also be subjected to an effective range treatment.[20] Such a treatment for P waves leads to coefficients which are somewhat similar to those defined for S waves. The expansion for $k^3 \cot\alpha_{33}$ correct to three terms is written as $k^3 \cot\alpha_{33}=a^{-3}-k^2/r_0+Pr_0k^4$. The effective range is again a measure of the range of the potential giving rise to the interaction. The coefficient P is not merely shape-dependent in the case of P-state interactions. It has an inherent energy dependence as a result of the rather sharp spatial dependence of a P-state wave function in the vicinity of the potential. Brueckner has written out an expression for P. The result is that P may be rather large and negative if r_0 is somewhat larger than the scattering length a, since the ratio r_0/a has a critical effect on P.

The α_{33} data presently available have been analyzed on the assumption that three terms are sufficient in the effective-range expansion. The values obtained by Orear, Lord, and Weaver[16] at 45 Mev, the results of Bodansky, Sachs, and Steinberger[5] at 65 Mev, the results of the Chicago group[1–4] from 78 to 217 Mev, and our result at 151 Mev have been subjected to a least-squares analysis. The resultant solution can be expressed as $\eta^3 \cot\alpha_{33}=5.0-0.55\eta^2-0.44\eta^4$. The phase shifts calculated from this are plotted in Fig. 7. The results of the Rochester group at 40 Mev are plotted although the analysis was made before obtaining their data. The scattering length, effective range, and correction coefficient are $a=0.82\times10^{-13}$ cm, $r_0=2.55\times10^{-13}$ cm, and $P=-0.24$. The effective range is again somewhat larger than a Compton wavelength, and the convergence is also bad. Again, more terms are probably required, which would possibly lead to a smaller effective range.

Positive-pion cross sections are now available with very good (better than 5 percent) accuracy in the energy range from 40 Mev to 197 Mev as a result of the recent work of Ashkin *et al.*[7] in the energy range above 135 Mev. The total cross section as a function of energy has been calculated from the previously discussed expressions for α_3 and α_{33}. The result is expressed by the solid curve in Fig. 5. The agreement with the cross section is seen to be excellent up to energies above 200 Mev although this is expected since the fit to the phase shifts is good. It does indicate that α_{31} does not contribute noticeably to the total cross section. At the higher energies investigated at Brookhaven,[13,14] there is a tendency for the curve to be high. This can easily be due to the omission of higher terms in the effective range treatment of α_{33} as well as errors in extrapolating the S-wave treatment of Noyes and Woodruff.

As a more severe test of the above treatments of the phase shifts as well as of the legitimacy of neglecting the three "small" phase shifts, the differential cross sections for the various processes have been calculated at all energies above 70 Mev where they have been measured. The columns in Table I to the right of the experimental values of a, b, and c; i.e., a_c, b_c, and c_c are coefficients calculated from the three treatments of the "large" phase shifts, and the assumption that the other three are zero. The data below 70 Mev were not compared in this manner because of the Coulomb effects. The above treatments are calculated from the low energy data so it is to be expected that the agreement would be good in this region. In all, 17 cross sections involving a total of $3\times17=51$ experimental points are involved. Each phase shift introduces three parameters. We are thus fitting 51 points with 9 parameters and expect an M value for the fit of $51-9=42$, where M is as defined previously in discussion of our 151-Mev data. M has been calculated and equals 43.2. It is apparent that the agreement is good.

Concluding Remarks

We arrive at the conclusion that the presently available pion-proton scattering data can be represented well by using only three of the six phase shifts.

---

[20] K. Brueckner, Phys. Rev. 87, 1026 (1952).

The energy dependence of the phase shifts can be "explained" in terms of simple static potentials. There definitely appears to be a resonance in the state of isotopic and ordinary spin both $\frac{3}{2}$, and the resonant energy occurs near $\eta=1.67$, corresponding to an energy of 200 Mev. No states of higher angular momenta are required up to and including data at 217 Mev, if the Fermi solutions are assumed to be correct.

The low-energy behavior of the S-wave phase shifts may be completely different than described. The low-energy data are still too poor to permit us to say definitely that α_3 changes sign. It is seen in Fig. 5 that the given models predict a dip in the total positive pion-proton cross section in the region from 10 to 30 Mev. It would be difficult to distinguish this from an energy dependence for α_3 in which α_3 remains negative but levels off. However, such a dependence would not give a rise below 10 Mev, and very low-energy total cross sections would be valuable in determining the low-energy behavior of α_3.

At this point it is appropriate to express our indebtedness to the members of the meson scattering group for their assistance and the use of their electronics equipment.

We wish to thank Dr. Jay Orear, Dr. Lederman, Dr. Homa, and Dr. Noyes for sending reprints and for very helpful communications. Professor Gregory Breit contributed a valuable discussion of effective-range theory.

We wish to thank Mrs. R. Cutosky, Mrs. F. Feiner, Mr. J. Morrison, Mr. R. Klein, Mr. G. Stranahan, and Miss J. Kugell for their efficient scanning.

PHYSICAL REVIEW VOLUME 97, NUMBER 3 FEBRUARY 1, 1955

π^--p Interactions at 1.4 Bev*

L. M. Eisberg, W. B. Fowler, R. M. Lea,[†] W. D. Shephard,[‡] R. P. Shutt, A. M. Thorndike, and W. L. Whittemore
Brookhaven National Laboratory, Upton, New York

(Received October 6, 1954)

π^--p interactions have been observed in the hydrogen filling of diffusion cloud chambers exposed to a π^- beam of average energy 1.37 Bev at the Brookhaven Cosmotron. If 9 interactions leading to heavy unstable particles are omitted, there remain 147 interactions observed with magnetic field and 323 interactions without field. The total cross section is estimated to be 34.6 ± 2.7 millibarns. The elastic scattering cross section is 10.0 ± 0.8 millibarns. The elastic scattering angles are mostly less than 60° in the center-of-mass system and are accordingly interpreted as mainly due to diffraction scattering. The observations are consistent with diffraction by a sphere with radius $(1.18\pm0.10)\times10^{-13}$ cm and transparency 0.61 ± 0.10. However, the 20 percent of elastic scattering observed through angles $>60°$ may indicate a region of very strong interaction with radius $\sim0.5\times10^{-13}$ cm surrounded by a region of much weaker interaction. Of the inelastic events only the 95 observed in the magnet cloud chamber can be analyzed further. Of these 71 to 81 are considered to involve the production of one secondary pion and 14 to 24 two pions, depending on the assignment of unidentified inelastic cases. This indicates a slightly greater multiplicity than predicted by Fermi's statistical theory. Angle and momentum distributions of emitted pions are discussed in terms of possible pion-pion interactions and excited nucleon states, but conclusions on these questions are uncertain.

This paper reports some results concerning the nature of π^--p collisions at an energy of about 1.4 Bev. It is the second dealing with a preliminary cloud chamber survey of nucleon-nucleon and pion-nucleon interactions at the Cosmotron, the previous one having been concerned with n-p collisions at about 1.7 Bev.[1] The paper on n-p collisions will be referred to as I.

A considerable number of investigations of pion-nucleon interactions have been made at energies up to about 250 Mev,[2] but higher energy experiments have until recently been possible only with cosmic rays, and there are few cases where pions could be identified and their behavior studied.[3] A number of measurements of π-p total cross sections have been made using pion beams at the Cosmotron,[4] and π-p interactions have been studied in nuclear emulsions exposed to a 1.5-Bev π^- beam.[5]

In this experiment π-p interactions are observed in the hydrogen gas filling of diffusion cloud chambers, which are operated in a negative pion beam with average energy 1.37 Bev. (The experimental procedure is described in Sec. II.) Consequently, there is little ambiguity concerning the nature of the interacting particles. The situation is more definite than was the

---

[*] Work performed under the auspices of the U. S. Atomic Energy Commission.

[†] Now at Yale University, New Haven, Connecticut.

[‡] Now at University of Wisconsin, Madison, Wisconsin.

[1] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **95**, 1026 (1954), henceforth referred to as I.

[2] For summary see Henley, Ruderman, and Steinberger, Ann. Rev. Nuc. Sci. **3**, 1 (1953).

[3] See, for example: Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950); R. L. Cool and O. Piccioni, Phys. Rev. **87**, 531 (1952).

[4] Shapiro, Leavitt, and Chen, Phys. Rev. **92**, 1073 (1953); S. J. Lindenbaum and L. C. L. Yuan, Phys. Rev. **92**, 1578 (1953); Cool, Madansky, and Piccioni, Phys. Rev. **93**, 249 (1954); **93**, 637 (1954).

[5] Crussard, Walker, and Koshiba, Phys. Rev. **94**, 736 (1954); Walker, Crussard, and Koshiba, Phys. Rev. **95**, 852 (1954).