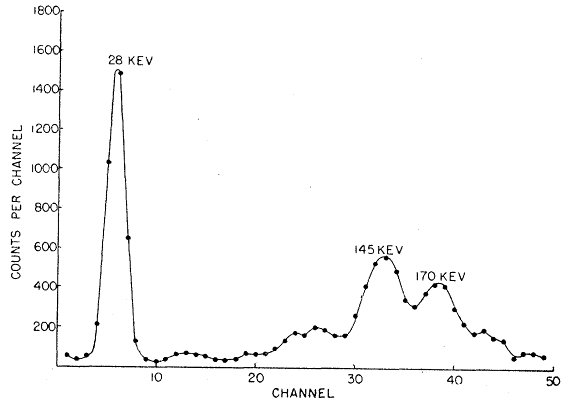


FIG. 2. Gamma spectrum of $Xe^{127}$ showing the gamma rays in coincidence with the 56-kev gamma ray.

gate crystal was changed from 200 to 170 kev. The 368-kev gamma ray was in coincidence with the 28-kev iodine x-rays and with no other gamma rays. Since most of the coincidence measurements were straightforward the curves are not presented. In Fig. 2 the gamma spectrum in coincidence with the 56-kev gamma ray is shown because it indicates that the 145-kev gamma ray is clearly evident. This gamma ray was not resolved in the gamma curves of Fig. 1. All coincidence results are incorporated in the decay scheme of Fig. 3 which agrees with that proposed by Bergström.[2]

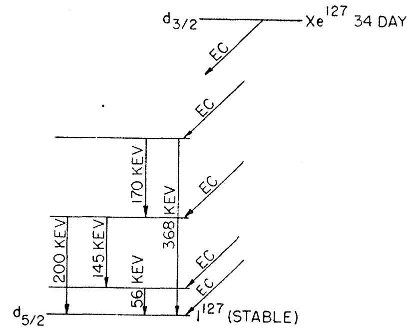


FIG. 3. Proposed decay scheme of $Xe^{127}$.

### ACKNOWLEDGMENTS

I wish to thank Dr. E. K. Hyde for helpful discussion and interest in this work. The assistance of Mr. W. W. Olson, Mr. R. D. Watt, and the linear accelerator crew for the proton bombardment is acknowledged. Thanks are also due to Dr. F. Asaro and Mr. F. S. Stephens for help in the gamma-gamma coincidence studies.

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# Internal and External Bremsstrahlung in Connection with the Beta Decay of $S^{35}$

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The *internal* bremsstrahlung, emitted in the $\beta$ decay of $S^{35}$, was investigated with a NaI scintillation spectrometer. Neither the shape of the spectral distribution nor the absolute yield was found to agree with theory. At 50 kev the experimental photon yield was 65 percent greater than the theoretical. At 100 kev it was even 180 percent greater. The total photon energy yield per $\beta$ decay, $2.23\times10^{-5}\,mc^2$, was 35 percent greater than the calculated value.

The *external* bremsstrahlung, emitted when the $\beta$ particles from $S^{35}$ were stopped in matter, was also investigated with the same apparatus. With elements of low atomic number the experimental values agreed with those calculated. In experiments using elements of higher atomic numbers, however, the experimental values differed widely from those calculated, the difference increasing with the atomic number. Thus, for lead at 50 kev the experimental value was 60 percent greater than the theoretical, and at 100 kev it was 170 percent greater.

## I. INTRODUCTION

STUDIES are available of the continuous electromagnetic radiation accompanying $\beta$ decay, i.e., the internal bremsstrahlung (IB), and of the radiation emitted when $\beta$ particles are stopped in matter, i.e., the external bremsstrahlung (EB), of $\beta$ emitters with disintegration energies above 1 Mev. Thus at this laboratory[1] IB and EB were investigated in experiments using $P^{32}$ ($E_\beta^{max}$ 1.70 Mev). The scintillation spectrometer was used because of its good $\gamma$-detection efficiency. (Readers interested in a fairly complete list of references of investigations bearing on the problems under consideration are referred to Lidén and Starfelt.[1])

The present paper is concerned with an investigation of a $\beta$ emitter with low disintegration energy. The investigation, which was carried out with $S^{35}$ ($E_\beta^{max}=168$ kev) by means of a scintillation spectrometer, included the determination of the spectral distribution and the total radiation energy yield of both the IB and the EB

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* Present address: National Bureau of Standards, Washington, D. C.

[1] K. Lidén and N. Starfelt, Phys. Rev. **97**, 419 (1955).

emitted on total absorption of the $\beta$ particles in C, Al, Cu, Ag, and Pb. A preliminary note on the initial stage of this investigation has been published.[2]

## II. EXPERIMENTAL PROCEDURE

### A. Arrangements

The scintillation spectrometer used consisted of a NaI(Tl) crystal of diameter 1.8 cm and thickness 0.9 cm, in conjunction with an EMI 5359 photomultiplier, a linear pulse amplifier, and a single-channel pulse-analyzer.[3] The crystal was covered with a thin, 2.3 $mg/cm^2$, aluminum reflector. The air in the photomultiplier housing was dried with phosphorus pentoxide to prevent the crystal from being affected by moisture.

Backscattering of photons from that part of the photomultiplier surface not covered by the crystal was prevented by the use of diaphragm $D_2$ (Fig. 1).

By calculation the radiation scattered in the hole of the diaphragms was found to be negligible.

In order to avoid characteristic x-rays in the energy range under consideration, the lead diaphragms were lined with tin and copper sheeting. The detector was shielded by 8 cm lead.

The $\beta$ particles from the source were absorbed in a beryllium disk, resting on the diaphragm $D_1$ and located halfway between the source and the crystal. The diameter of the hole in this diaphragm was the smallest permitting "visibility" of the entire crystal from any point in the source. This kept the EB disturbance below 2 percent of the IB intensity for all energies.

The source was housed in a Perspex container, in which the pressure was kept below 1 mm Hg during measurement. At this pressure the EB disturbance from the air was negligible. At normal pressure and with the geometry used this disturbance exceeded 25 percent of the IB for low energies. Within the entire energy range the calculated EB from the Perspex container was less than 1 percent.

The experimental arrangements for the study of EB were the same as those used for measurement of IB.

### B. Sources

Owing to the low intensity of the IB, a strong source would have been preferable, so that the counting rate would far exceed the background counting rate. This was, however, not possible because in thick sources EB is created in the source itself: only when the source is extremely thin and uniform will this disturbance be small in proportion to the IB. The source should be as free as possible from impurities, especially from impurities of high atomic number. The backing film should also be thin and made of material of low atomic number.

In an investigation of the $\beta$ spectrum of $S^{35}$, Albert and Wu[4] found the source itself to exert a disturbing effect at low energies when the source thickness exceeded 5 $\mu g/cm^2$, but not otherwise. Albert and Wu mounted their sources on backing films of collodion about 3 $\mu g/cm^2$ thick. In the measurement of the IB a slightly thicker source can be tolerated.

In the present investigation collodion films with a thickness of about 2 $\mu g/cm^2$ were used. These films were fastened to a Perspex ring (Fig. 1). The diameter of the ring was sufficient to prevent EB from the ring from reaching the crystal. The films were made from a solution of roughly 3 percent collodion in amyl acetate. A drop of the solution was allowed to spread over the surface of distilled water containing wetting agent. After evaporation of the solvent, the Perspex ring was placed on the film thus formed. Superfluous parts of the film were removed and the ring with the film attached was taken from the surface of the water. The thickness of the film was assessed both by weighing and by studying the interference colors of superimposed films.

The sources were made from carrier-free $S^{35}$ solution, obtained by neutron irradiation of KCl at AERE, Harwell. However, owing to the presence of sulfate in the potassium chloride, the $S^{35}$ solution used, contained 10–20 $\mu g$ $SO_4$ ions per millicurie. Other impurities were mainly B, Na, Cl, and K, together $<25$ $\mu g/mC$. The absolute activity of the $S^{35}$ solution was determined with an accuracy of about $\pm 5$ percent at Harwell by means of a $4\pi$ G-M counter.

If the $S^{35}$ solution is placed directly on the film, the latter will be destroyed. The solution was therefore first

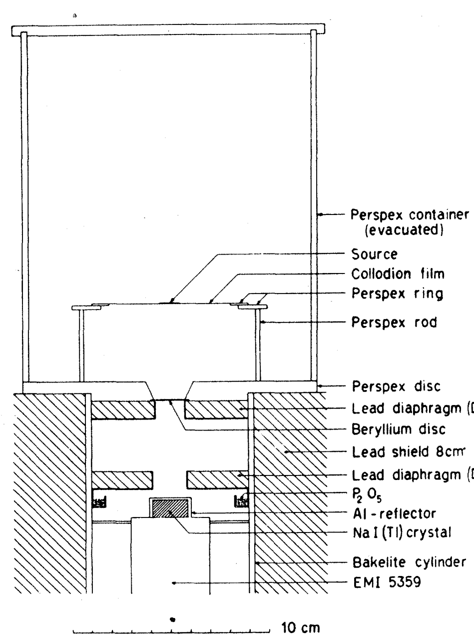


FIG. 1. Experimental arrangement used for the measurement of internal and external bremsstrahlung.

[2] K. Lidén and N. Starfelt, Arkiv Fysik **7**, 83 (1954).

[3] W. C. Elmore and M. Sands, *Electronics Experimental Techniques* (McGraw-Hill Book Company, Inc., New York, 1949), first edition, p. 228.

[4] R. D. Albert and C. S. Wu, Phys. Rev. **74**, 847 (1948).

evaporated to dryness and the residue then redissolved in distilled water. A suitable amount of this solution was afterwards evaporated with the aid of an infrared lamp on that side of the film which had been in contact with the water. The rest of the film was screened off by means of an asbestos diaphragm. In this way sources thinner than 35 $\mu g/cm^2$ were obtained.

On account of the difficulty to handle the film the absolute activity of an IB source was not determined by weighing but as follows. The IB source was sandwiched between two aluminum radiators. Then the intensity of the IB plus EB from this source was compared with the intensity of the radiation from a similar source, prepared by evaporating a weighed amount of the $S^{35}$ solution directly on one of the two aluminum radiators. The average atomic number of the sources was close to that of aluminum, and therefore any slight variation in the thickness of the source would not appreciably influence the result of this comparison.

Autoradiographs were taken to check any irregularity in thickness of the source. The source was placed on the photographic emulsion, from which it was separated by the backing film only. The variation in activity between different parts of the source with an average thickness less than 7 $\mu g/cm^2$, was found to be less than sevenfold, as measured with a microphotometer. For the thickest source (average thickness $<35$ $\mu g/cm^2$) used in the measurements of the IB the maximum variation was found to be about tenfold. This infers that the thickest parts of the thin source were less than 35 $\mu g/cm^2$. Calculation of the intensity of the EB emitted when the $\beta$ particles of $S^{35}$ passed through a layer of this thickness, showed this disturbance to be less than 5 percent of the IB at all energies.

The influence of the EB from the source itself was also investigated by another method. In experiments using sources of different thicknesses, measurements were made of the bremsstrahlung $(N_i+N_e)dE$ in the energy interval between $E$ and $E+dE$ from the source alone, and then of the bremsstrahlung $N_r dE$ from the source sandwiched between two aluminum radiators. Here $N_i dE$ is the number of IB photons from the source and $N_e dE$ the EB contribution from the source itself. $N_e$ increases with increasing source thickness, but as the atomic numbers of the impurities and of the radiators are about equal, $N_r$ varies only slightly with the source thickness. The ratio $B=(N_i+N_e)/N_r$ may then be taken as a measure of the uniformity of the source. Of all the sources used for the measurement of IB, the ratio $B$ for those thinner than 35 $\mu g/cm^2$ was found to vary only slightly (Fig. 2). The source with a maximum average thickness of about 17 $\mu g/cm^2$ and $B=0.32$ was distinctly nonuniform.

Figure 2 shows that an increase of the maximum average thickness of the source from 4 to 35 $\mu g/cm^2$ did not cause any demonstrable definite increase of the EB. Judging by Fig. 2, this disturbance was less than 5 percent for all the sources used. This figure was thus in fairly good agreement with that found by the first method.

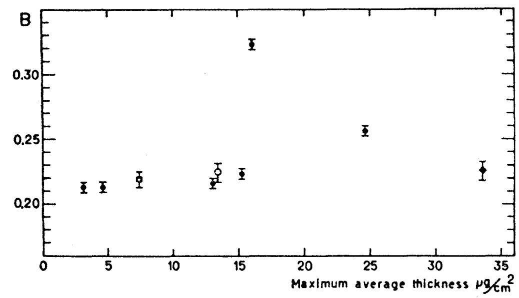


FIG. 2. The influence of the source thickness on the measurement of internal bremsstrahlung as represented by $B$, the ratio between the counting rate from the source alone and from the source plus aluminum radiators for energies between 20 and 40 kev. Squares and open circles represent the sources used in the measurements of internal bremsstrahlung.

In the measurement of the EB of different elements the radiators used consisted of circular disks (diameter $=13$ mm) thick enough (about 50 $mg/cm^2$) to stop all the $\beta$ particles. A weighed amount of the $S^{35}$ solution was evaporated with the aid of an infrared lamp on such a disk and was afterwards covered by another radiator of the same kind. $4\pi$ geometry was thereby obtained. Sources of different thicknesses were compared to ensure that no disturbance from the source itself influenced the results.

### C. Measurements

The weak sources and the low relative intensity of the radiation, especially at high photon energies, resulted in long observation times. Particular care was therefore necessary to obtain the required accuracy of the energy calibration and of the channel-width. Thus, the electronic apparatus was warmed up for 2–3 hours before any measurements were made, and the temperature of the room was kept constant. Changes in the surface layer of the crystal with consequent variation in energy calibration was minimized by keeping the air in the photomultiplier housing dry. The energy calibration was performed with $K$ x-ray lines[5] from Sn, Pm, Yb, and Pb as well as with the $Ce^{141}$ $\gamma$-ray line at 145 kev. The variation of the energy calibration during measurement of a spectrum was less than $\pm 2$ percent. The proportionality between photon energy and pulse height was good within the energy range studied. The channel-width, about 8 kev, was measured with an accuracy of $\pm 3$ percent with a pulse generator.

The IB was measured with three different sources. The EB from aluminum was studied not only with the use of these sources, but also with a source evaporated directly on an aluminum radiator. The spectra of the other elements were investigated with the use of 2–4 different sources in each case.

[5] K. Lidén and N. Starfelt, Arkiv Fysik 7, 193 (1954).

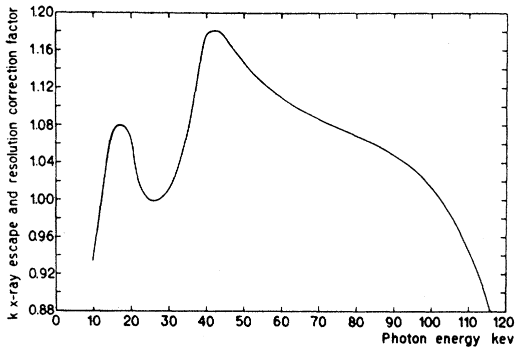


FIG. 3. Correction factor of the pulse-height distribution of the internal bremsstrahlung from $S^{35}$ for $K$ x-ray escape and resolution.

The activity of the sources varied from 70 to 350 microcuries.

### D. Corrections

The spectrum of the radiation reaching the crystal was calculated mainly according to Lidén and Starfelt[6] from the measured pulse-height distribution. In the treatment of the experimental results of the EB of silver, only energies not influenced by the peak at about 23 kev were considered. This peak consists of continuous bremsstrahlung as well as of $K$ x-rays caused by bremsstrahlung photons and by $\beta$ particles. The ratio between these different contributions is known from Lidén and Starfelt.[5] However, their calculation is based on the assumption that the spectral distribution of the EB from elements of higher atomic numbers is the same as that of aluminum, and therefore their results cannot, of course, be used here. No accurate correction for the $K$ x-rays could be made for the lead EB spectrum.

The calculated influence of Compton electrons, of escape electrons as well as of photons backscattered from the photomultiplier, was negligible in the energy range under consideration.

The greatest correction of the measurements is that for the escape of $K$ x-rays from the top and cylindrical surfaces of the crystal. This effect and that of the resolving power were corrected for together.

The scintillation distribution,[7] $N_3(\epsilon_\gamma)$, reaching the photocathode gives rise to a pulse-height distribution, $N_2(\epsilon)$, which differs from $N_3(\epsilon_\gamma)$. The escape of $K$ x-rays causes a discontinuity in $N_3(\epsilon_\gamma)$ at the $K$ absorption edge of iodine. The scintillation distribution, $N_4(\epsilon_\gamma)$, corrected for this effect, was deduced from the experimental pulse distribution $N_2(\epsilon)$ in the following way. $N_3(\epsilon_\gamma)$ was calculated from an estimation of $N_4(\epsilon_\gamma)$ using the $K$ x-ray escape factor $F(\epsilon_\gamma)$ of the crystal. $N_2(\epsilon)$ was then determined with Eq. (27) given in reference 6. Agreement between the calculated and the experimental values of $N_2(\epsilon)$ was obtained by trial and error. As a rule at most only three trials were necessary. At the $K$-absorption edge of lead, $N_4(\epsilon_\gamma)$ was calculated by a similar method.

Figure 3 shows the correction of the IB pulse distribution for the $K$ x-ray escape and the resolving power. This correction has two maxima, one at about 40 kev and due to the $K$ absorption edge of iodine, the other at about 15 kev and due to the absorption in the Be absorber. The resolving power was determined by measurements of the half-width of the $Ce^{141}$ $\gamma$-ray line at 145 kev. It was about 20 percent and varied by less than 5 percent of this value between the different series of observations.

Corrections were made for the $\gamma$-detection efficiency of the crystal including the effective photoelectric absorption and the effective solid angle at different energies. This correction was less than 2 percent for energies below 80 kev. The effective photoelectric absorption was calculated according to Maeder *et al.*[8]

Corrections were applied for the absorption in the aluminum reflector and the beryllium absorber. Absorption data from White[9] were used throughout the present study.

The disturbance of the measurement of the IB caused by the EB from the beryllium absorber was calculated by extrapolation of the experimental determinations of the EB for the different elements. At the low energies covered by this investigation, the EB was calculated under the assumption that it was emitted isotropically. The EB disturbance from the beryllium absorber was

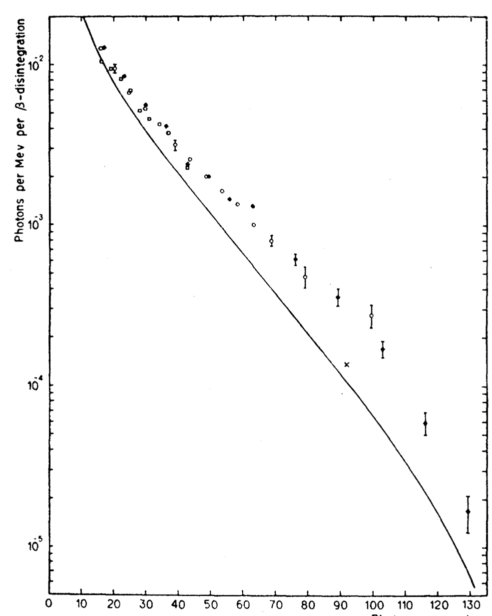


FIG. 4. Internal bremsstrahlung from $S^{35}$. The curves represent the theory as given by Knipp and Uhlenbeck. × marks the theory according to Nilsson. For explanation of symbols see Fig. 2.

[6] K. Lidén and N. Starfelt, Arkiv Fysik 7, 427 (1954).

[7] The notations are those utilized in reference 6.

[8] Maeder, Müller, and Wintersteiger, Helv. Phys. Acta 27, 3 (1954).

[9] G. R. White, National Bureau of Standards Report 1003, May 13, 1952 (unpublished).

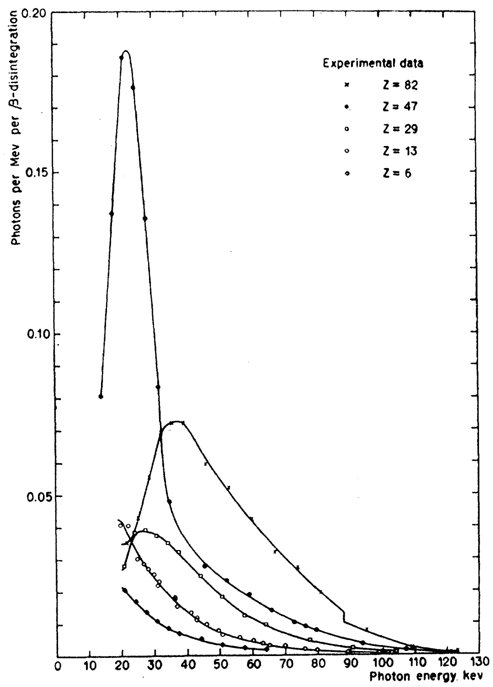


FIG. 5. Experimental total radiation, internal plus external, leaving the radiators (about 50 mg/cm²). The discontinuity at the $K$-absorption edge of lead is only a rough estimation.

found to be less than 2 percent for all energies. The results of the measurement of the IB are given in Fig. 4.

So far the treatment of the measurements of the IB plus EB is essentially the same as that of the IB. The values of EB plus IB corrected in this way are presented in Fig. 5.

When determining the EB, the absorption in the radiator must also be taken into account. For purposes of calculation it was assumed that all radiation was emitted from the center of the source. The error introduced by this approximation was found to be less than 1 percent for all energies for C and Al, and above 40 kev for Cu, Ag, and Pb. For the smallest energies studied with the three last-mentioned elements it was found to be about 5 percent.

The calculated effect of the Compton scattering of the photons in the radiators on the radiation measured appeared to be about 1 percent.

Finally, the IB was subtracted from the measured radiation, EB plus IB. The EB values found in this way are plotted in Fig. 6.

## E. Estimation of the Errors

The errors which are of such an order of magnitude as to effect the result of the investigation of the IB are given in Table I. The errors in the correction for the EB disturbance from the Be absorber presented in Table I are mainly due to the uncertainty caused by the assumption of isotropic distribution of the EB.[10]

[10] G. Sesemann, Ann. Physik **40**, 66 (1941).

As far as EB is concerned, the error caused by the EB from the beryllium absorber must be excluded, but errors due to the absorption in the radiators and by the correction for the IB added. In other respects the errors are mainly the same as those given in Table I.

The inaccuracy in the determination of the total activity does not influence the shape of the spectral distribution, but only the total photon energy yield, and was therefore not included in the total error in Table I.

# III. RESULTS AND DISCUSSION

## A. Internal Bremsstrahlung

The theory of IB was developed in Born's approximation by Knipp and Uhlenbeck[11] and by Bloch[12] independently.

They arrived at the same expression for the probability per unit time, $S(k)dk$, for the emission of an IB photon with an energy between $k$ and $k+dk$. Knipp and Uhlenbeck rewrote the expression as

$$S(k)=\int_{mc^2+k}^{W_0}\phi(W_e,k)P(W_e,0)dW_e, \qquad (1a)$$

where $P(W_e,0)dW_e$ is the probability per unit time for the emission of a $\beta$ particle of energy between $W_e$ and $W_e+dW_e$ assuming $Z=0$, and $\phi(W_e,k)dk$ is the conditional probability that an electron emerging from the nucleus with energy $W_e$ will emit a photon of energy between $k$ and $k+dk$. $W_0$ is the maximum energy of the $\beta$ spectrum.

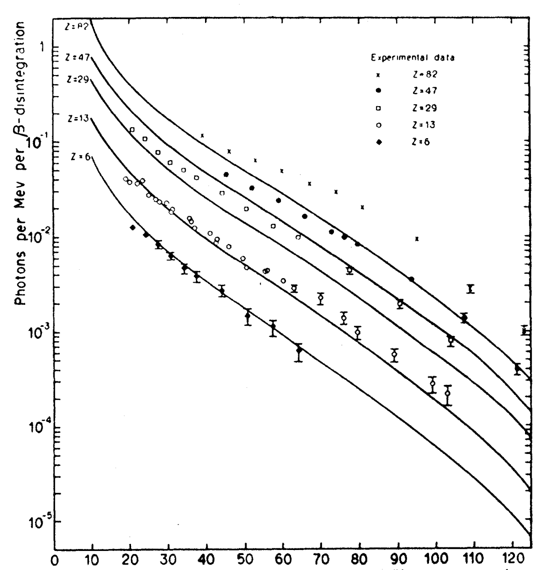


FIG. 6. External bremsstrahlung emitted when $S^{35}$ betas are completely stopped in C, Al, Cu, Ag, and Pb. The curves represent the Sommerfeld-Elwert theory.

[11] J. K. Knipp and G. E. Uhlenbeck, Physica **3**, 425 (1936).
[12] F. Bloch, Phys. Rev. **50**, 272 (1936).

The expression for $S(k)$ was derived for allowed transition and Fermi polar vector interaction. Chang and Falkoff[13] extended the theoretical calculations to forbidden transitions and different types of interactions between the nucleons and the electron-neutrino field, still under the assumption $Z=0$.

When comparing theory with experiment, however, it is necessary to take into account the nuclear charge. As an approximation, Eq. (1a) makes it natural simply to replace $P(W_e,0)$ by the $\beta$ distribution corrected for the Coulomb effect. Thus

$$S(k)=\int_{mc^2+k}^{W_0}\phi(W_e,k)P(W_e,Z)dW_e \qquad (1b)$$

with $P(W_e,Z)=P(W_e,0)F(Z,W_e)$. [A lower limit of this influence is $F(Z,W_0)=1.7$ for $S^{35}$.] This is the expression usually employed. It also permits the insertion of the experimental $\beta$ distribution and thereby avoids the difficulties otherwise encountered in classifying the $\beta$ decay.

The integrated expression for $S(k)$ evaluated by Bloch does not lend itself readily to a correction for the nuclear charge.

In Fig. 4 and Table II the values calculated from the theoretical expression (1b) are reproduced. For $P(W_e,Z)$ the allowed $\beta$ spectrum of Fermi is used, with the nonrelativistic Coulomb factor $F(Z,W_e)$ given by Kurie *et al.*[14]

From Fig. 4 and Table II it is also clear that the experimental values were much higher than the theoretical. At low energies the experimental values approach the theoretical curve.

The limits of error in Fig. 4 are those given in Table I. The error in the determination of the absolute activity of the source, about ±5 percent, is not included. The total yield of IB energy per $\beta$ decay $2.23\times10^{-5}$ $mc^2$ is 35 percent greater than the theoretical value.

In order to ascertain whether the approximation involved in going over from Eq. (1a) to Eq. (1b) might account for the discrepancy, Nilsson[15] calculated $S(k)$ with a more accurate consideration of the Coulomb influence. In the case of $S^{35}$ the correction terms to be added to Eq. (1b) at $k=92$ kev was calculated by Nilsson using the nonrelativistic approximation for the Coulomb factor and was found to be about 28 percent of the value obtained from Eq. (1b). At low energies the expression given by Nilsson approaches the curve in Fig. 4. As is clear from the figure, this improvement is not sufficient to explain the whole discrepancy.

TABLE I. Estimated errors in the $S^{35}$ internal bremsstrahlung spectrum (in percent). The total error does not include the inaccuracy, about ±5 percent, of the absolute calibration of the source.

| Energy kev | Channel width | Energy calibration | Resolving power+K x-ray escape | Crystal efficiency | External bremsstrahlung from the Be absorber | Counting statistics | Total (rms) |
|---|---|---|---|---|---|---|---|
| 20 | 3 | 1 | 2 | 1 | 2 | 5 | 7 |
| 40 | 3 | 3 | 3 | 1 | 1 | 6 | 8 |
| 70 | 3 | 4 | 1 | 1 | 1 | 6 | 8 |
| 100 | 3 | 6 | 1 | 1 | 1 | 10 | 12 |
| 120 | 3 | 10 | 2 | 1 | 1 | 14 | 18 |

TABLE II. The $S^{35}$ internal bremsstrahlung photon spectrum $S(k)_{calc}$ calculated according to the theory of Knipp and Uhlenbeck (on multiplication by $10^{-5}$ the figures give the number of photons per Mev per disintegration) and $S(k)_{exp}/S(k)_{calc}$, the ratio between the experimental and the calculated values.

| Energy kev | 15.0 | 25.6 | 51.1 | 76.7 | 102.2 | 127.8 |
|---|---|---|---|---|---|---|
| $S(k)_{calc}$ | 1180 | 506 | 109 | 25.9 | 5.75 | 0.814 |
| $S(k)_{exp}/S(k)_{calc}$ | 1.12 | 1.26 | 1.65 | 2.30 | 2.8 | 2.9 |

In this calculation Nilsson considered only the influence of the Coulomb field on the creation of the electron in the nucleus. Thus the radiation emitted when the $\beta$ particle is accelerated in the field of the daughter nucleus was still neglected in the theoretical expression.

When the present paper was being prepared for the press, Boehm and Wu[16] published a similar investigation of the IB.

Boehm and Wu did not mention the disturbance caused by the air round the source. With the geometry used in the present investigation, the EB disturbance from the air was found to be about 25 percent of the IB in the low-energy region. Boehm and Wu used no collimator in their apparatus; therefore the disturbance in their experiment was presumably greater. It might be mentioned that a 5-cm layer of air corresponds to approximately $\frac{1}{5}$ of the total range of a $\beta$ particle with maximum energy, while an electron with an initial energy of 40 kev is completely absorbed in about 2 cm air. The geometry without collimator used by Boehm and Wu can also give rise to contributions of scattered radiation from various parts of the apparatus.

This makes it possible to explain the discrepancy in the shape of the spectral distribution as found in the two determinations. The difference between the total photon energy yields is about the sum of the limits of error stated.

### B. External Bremsstrahlung

Bethe and Heitler[17,18] and Sauter[19] developed the theory of the EB in the Born approximation.

They calculated the probability, $\phi(W_e,k)$, that an electron with the initial energy $W_e$ emits a photon with

[13] C. S. Wang Chang and D. L. Falkoff, Phys. Rev. **76**, 365 (1949).
[14] Kurie, Richardson and Paxton, Phys. Rev. **49**, 368 (1936).
[15] S. B. Nilsson (to be published).
[16] F. Boehm and C. S. Wu, Phys. Rev. **93**, 518 (1954). In this paper there is a drafting error (as confirmed by F. Boehm, private communication): In Fig. 3, for $10^{-6}$ read $10^{-5}$ and for $10^{-5}$ read $10^{-4}$.
[17] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).
[18] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1954), third edition, Chap. V, Sec. 25.
[19] F. Sauter, Ann. Physik **20**, 404 (1934).

TABLE III. The $S^{35}$ external bremsstrahlung photon spectra $S(k)_{\text{calc}}$ for stopping the $\beta$ particles in C, Al, Cu, Ag, and Pb, calculated according to the theory of Sommerfeld and Elwert (on multiplication by $10^{-5}$ the figures give the number of photons per Mev per disintegration) and $R=S(k)_{\text{exp}}/S(k)_{\text{calc}}$, the ratio between the experimental and the calculated values.

| Energy kev | Carbon $S(k)_{\text{calc}}$ | Carbon $R$ | Aluminum $S(k)_{\text{calc}}$ | Aluminum $R$ | Copper $S(k)_{\text{calc}}$ | Copper $R$ | Silver $S(k)_{\text{calc}}$ | Silver $R$ | Lead $S(k)_{\text{calc}}$ | Lead $R$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 30.7 | 666 | 0.92 | 1910 | 1.06 | 5050 | 1.19 | 9030 | ... | 17300 | ... |
| 51.1 | 172 | 0.91 | 486 | 1.17 | 1370 | 1.33 | 2480 | 1.36 | 4740 | 1.38 |
| 71.5 | 46.1 | 0.89 | 140 | 1.28 | 405 | 1.48 | 754 | 1.59 | 1470 | 2.13 |
| 92.0 | 11.7 | ... | 34.9 | 1.34 | 102 | 1.78 | 202 | 2.0 | 408 | 2.7 |
| 112.4 | 2.24 | ... | 7.43 | ... | 24.2 | ... | 48.1 | 1.9 | 90.2 | 2.8 |

the energy between $k$ and $k+dk$ when it is accelerated in the field of the nucleus.

From Sommerfeld's theory,[20] Elwert[21] deduced a correction factor $f(\xi_0,\xi)$ which makes it possible to use the expression $\phi(W_e,k)$ as given by Bethe and Heitler also in the nonrelativistic case.

$$f(\xi_0,\xi)=\frac{\xi}{\xi_0}\frac{1-e^{-2\pi\xi_0}}{1-e^{-2\pi\xi}}, \tag{2}$$

where $\xi=Ze^2/hv$ and $v$ is the velocity of the electron. The symbols $\xi_0$ and $\xi$ refer to the electron in the initial and the final state, respectively. When $\xi$ is of the order of unity the expression 2 takes the form $f(\xi_0,\xi)=\xi/\xi_0$.

Numerical calculations by Kirkpatrick and Wiedmann[22] confirmed the result obtained by Elwert.

In the present calculations only the EB from the field of the nucleus was treated. As to the bremsstrahlung originating from the field of the atomic electrons, no calculations are available in the nonrelativistic case. The screening of the Coulomb field by the atomic electrons can be neglected as far as $S^{35}$ betas are concerned.

For the spectral distribution, $n(k,W_e)$, of the EB, emitted when an electron of the energy $W_e$ is stopped in matter, Bethe and Heitler gave the expression

$$n(k,W_e)=\int_{mc^2+k}^{W_e}\frac{N\cdot\phi(k,W)dW}{-(dW/dX)}, \tag{3}$$

where $N$ is the number of atoms per cm$^3$ and $-dW/dX$ the energy loss per cm, here calculated according to Aron *et al.*[23] The values of the mean excitation potentials were taken from Segrè.[24]

[20] A. Sommerfeld, *Atombau und Spektrallinien* (Friedr. Vieweg und Sohn, Braunschweig, 1951), second edition, Vol. II, Chap. 7.
[21] G. Elwert, Ann. Physik **34**, 178 (1939).
[22] P. Kirkpatrick and L. Wiedmann, Phys. Rev. **67**, 321 (1945).
[23] Aron, Hoffman, and Williams, Atomic Energy Commission Report AECU-663, May 28, 1951 (unpublished).
[24] E. Segrè, *Experimental Nuclear Physics* (John Wiley and Sons, Inc., New York, 1953), Vol. 1, p. 203.

The spectral distribution $S(k)$ of the EB, obtained when the $\beta$ particles from $S^{35}$ are stopped, is given by

$$S(k)=\int_{mc^2+k}^{W_0} n(k,W_e)P(W_e,Z)dW_e. \tag{4}$$

In Fig. 6 and Table III the values of $S(k)$ for C, Al, Cu, Ag, and Pb are given as number of photons per Mev per $\beta$ disintegration *vs* energy. The experimental values for the different elements are presented in the same figure. For the experimental points for which no limits of error are given, the limits lie within the symbols.

Good agreement was found between the experimental and the theoretical values for carbon. For aluminum, however, the experimental values at high energies tended to exceed the theoretical curve, but still lay within the limits of error. For copper and silver the experimental values above about 50 kev deviated widely from the theoretical. For lead the experiments gave considerably higher values than did theory. As far as this element is concerned, however, it should be pointed out that no correction was made for the $K$ x-rays.

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