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Effective Carrier Mobility in Surface-Space Charge Layers*

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Carriers held to a region near the surface by the potential well of a space charge layer may have their mobility reduced by surface scattering, if the width of the well is of the order of a mean free path. An effective mobility, which may differ from the bulk mobility by as much as a factor of ten, has been obtained from a solution of the Boltzmann equation. Solutions have been carried out for two types of potential functions: (*a*) a linear potential corresponding to a constant space-charge field, and (*b*) a solution of Poisson's equation including an external bias applied normal to the surface. The results have been used to calculate changes in surface conductance of germanium with changes in surface potential and predict the "field effect" and "channel effect" mobilities.

I. INTRODUCTION

THE surface of a semiconductor is the seat of a space-charge double layer produced by a surface charge distribution which is counterbalanced by a space-charge region consisting of holes, electrons, and impurity ions.[1,2]

The surface charge distribution arises from the trapping of holes or electrons at the surface of the material. There are three obvious types of surface traps. First, by adjusting the mathematics of the energy band solution for an ideal infinite crystal to take the surface into account, one may find allowed levels which correspond to states localized near the surface, and which lie in the forbidden gap of the energy level diagram. A second type of surface trap arises from the impurity ions found in increased quantity near the surface. A third type is that arising from chemisorbed material on the semiconductor surface. The chemisorption process in general requires a charge transfer from the body to the surface of the semiconductor. The chemisorption traps are often separated from the bulk material by an oxide layer. By changing the gaseous ambient surrounding the sample, one can change the density of chemisorption traps and therefore alter the magnitude of the space-charge double layer.

Morrison[3] has used this technique on a free germanium surface to displace the energy bands near the surface relative to the Fermi level. The conductance of the sample is changed by such a displacement because the carrier concentrations in the space-charge region are different from the bulk concentrations. Figure 1(a) shows a *p*-type semiconductor with a large positive charge on the surface causing an *n*-type surface layer, or "inversion layer," to be formed. Similarly, a large negative surface charge would cause the bands to rise and a strongly *p*-type layer would exist at the surface. The intermediate case is obtained when a small positive charge exists on the surface which is counterbalanced primarily by a space-charge of impurity ions, termed an "exhaustion layer." Since the hole concentration is reduced under such conditions while the electron concentration is still relatively small, the conductivity in an exhaustion layer will be smaller than that of the bulk semiconductor.

If an alternating field is applied normal to the surface, by arranging the semiconductor to be one plate of a condenser, an effective mobility of the carriers introduced by this field can be determined from the change in conductance of the sample. This type of effective mobility will be called the "field effect" mobility. Unlike the bulk carrier mobilities, the field effect mobility can change its sign, the positive value corresponding to electrons. Morrison finds in general that the sign of the field effect mobility changes when the ambient causes the conductance to go through a minimum value.[4]

Brown[5] and Kingston[6] have measured the conductance of a "channel" or *n*-type surface layer on a normally *p*-type base region of an *n*–*p*–*n* junction transistor, as a function of applied reverse bias. By fitting theoretical curves to experimental data, Brown estimates the mobility of electrons in the channel to be one-fifth to one-tenth the bulk mobility. Kingston carried out measurements in a water vapor atmosphere for several values of vapor pressure. The water vapor tends to form a positive surface charge distribution on the *p*-type base and creates an *n*-type inversion layer as described above. Kingston finds the surface conductance appears to vary inversely with applied bias, and has proposed a theory to explain this effect by taking into account the reduction in mobility of the channel electrons.

To estimate the change in conductance of a semiconductor due to a space-charge layer existing at the surface, one must consider at least two effects. The first is simply the change in the number of holes and electrons in the space-charge region. The second is the

* This work was supported by a grant from Motorola, Inc.
[1] J. Bardeen, Phys. Rev. **71**, 717 (1947).
[2] W. H. Brattain and J. Bardeen, Bell System Tech. J. **32**, 1 (1953).
[3] S. R. Morrison, Tech. Report No. 2, Electrical Engineering Research Laboratory, University of Illinois, and private communication.
[4] W. Shockley and G. L. Pearson, Phys. Rev. **74**, 232 (1948).
[5] W. L. Brown, Phys. Rev. **91**, 518 (1953).
[6] R. H. Kingston, Phys. Rev. **93**, 346 (1954).

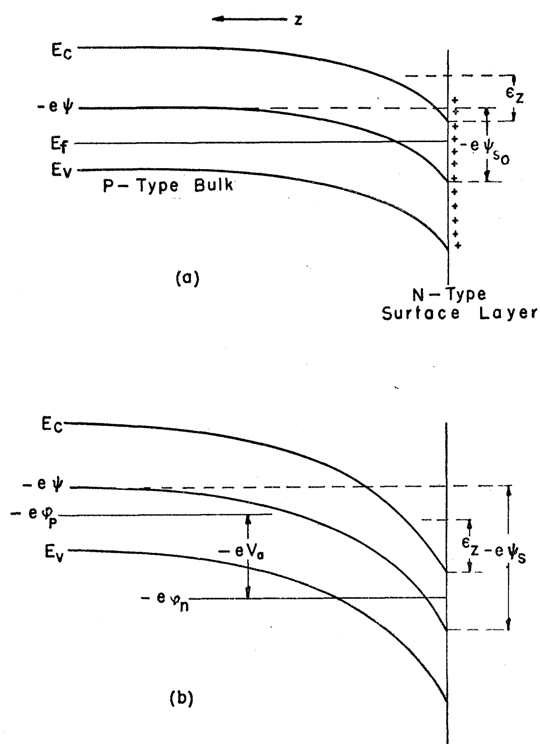

FIG. 1. (a) Energy level diagram of an n-type inversion layer existing at the free surface of a p-type material. (b) A voltage V_a applied across the surface shown in (a).

reduction of the carrier mobility by surface scattering if the width of the space-charge potential well is comparable to a mean free path or less. For example, if electrons in an n-type inversion layer are held near the surface by the potential well, the surface may scatter the electrons more frequently than the conventional bulk scattering mechanism, therefore reducing the electronic mobility appreciably. This reduction may be quite important in the channel effect where widths of the order of several hundred angstroms are attained. If a large dipole exists at the free surface of a semiconconductor, surface scattering will play a role in determining both the total change in conductance due to ambient and the results of the field effect measurement. The considerations presented here primarily deal with the influence of surface scattering on such measurements.

The increase in resistance of thin metallic films from a decrease in electronic mobility, in the absence of a magnetic field, has been discussed by Fuchs[7] and a generalization of the problem to include magnetic effects was carried out by Sondheimer.[8] The theory presented here in general follows these analyses, the essential difference arising from the spatial dependence of the unperturbed carrier distribution function due to the space-charge potential. The general solution for the surface conductance is given in Part II for diffuse surface scattering and applied to (a) a linear space-charge potential corresponding to a constant electric field, and (b) a solution of Poisson's equation including an externally applied potential, V_a, across the surface. The change in the number of holes and electrons in the space-charge layer as a function of ψ_{s0} (see Fig. 1), has been computed for impurity densities from intrinsic up to $10^{18}/\text{cm}^3$.[9] These results are combined with the carrier mobility obtained from the surface scattering considerations to discuss Morrison's work and predict the carrier mobility in the channel effect.

II. GENERAL THEORY

We consider a volume extending inward from a unit surface area of a semiconductor, subjected to an electric field E_x, parallel to the surface, and a field E_z along the inward normal to the surface, due to the space-charge layer. The carriers are regarded as free in the sense that the energy depends upon the absolute square of the wave vector only. Under steady state conditions, the distribution function for the holes or electrons is determined by the Boltzmann equation,[10]

$$\mathbf{v}\cdot\text{grad}_{\mathbf{r}}f+\mathbf{a}\cdot\text{grad}_{\mathbf{v}}\,f=-(f-f_0)/\tau, \qquad (1)$$

where $\mathbf{v}$ and $\mathbf{a}$ are the velocity and acceleration of a carrier, τ the relaxation time, and $f=f_0+f_1(v,z)$, f_0 being taken as the Maxwell-Boltzmann distribution function and f_1 a small perturbing function. Thus,

$$f_0=C\exp[-(mv^2/2+q\psi)/kT], \qquad (2)$$

where ψ is the potential associated with E_z, m the effective mass and q the charge of the carrier. When products of f_1 and E_x are neglected in (1), f_1 is found to satisfy the equation:

$$v_z\frac{\partial f_1}{\partial z}+\frac{qE_z}{m}\frac{\partial f_1}{\partial v_z}+\frac{f_1}{\tau}=\frac{qv_xE_xf_0}{kT}. \qquad (3)$$

By introducing an energy parameter,

$$\epsilon_z=\tfrac{1}{2}mv_z^2+q(\psi-\psi_s), \qquad (4)$$

where ψ_s is the value of the potential at the surface, Eq. (3) is reduced to

$$\frac{qE_z}{m}\frac{\partial f_1}{\partial v_z}+\frac{f_1}{\tau}=\frac{qv_xE_xf_0}{kT}. \qquad (5)$$

If the boundary condition of random scattering at the surface is imposed by making f_1 vanish at $v_z=v_{zs}$, where v_{zs} is the positive z component of velocity asso-

[7] K. Fuchs, Proc. Cambridge Phil. Soc. **34**, 100 (1938).

[8] E. H. Sondheimer, Phys. Rev. **80**, 401 (1950).

[9] This calculation has been carried out independently by R. H. Kingston (private communication). A somewhat similar calculation has been carried out by C. G. B. Garrett.

[10] See A. H. Wilson, *The Theory of Metals* (Cambridge University Press, London, 1953), Chap. VIII.

ciated with a carrier at the surface with energy parameter ϵ_z, Eq. (5) has the solution,

$$f_1=\frac{mv_xE_xf_0}{kT}\exp[-K(v_z,\epsilon_z)]\int_{v_z}^{v_{zs}}\frac{\exp[K(v_z',\epsilon_z)]}{E_z(v_z',\epsilon_z)}dv_z',\tag{6}$$

and where

$$K(v_z',\epsilon_z)=\frac{m}{q}\int_0^{v_z'}\frac{dv_z''}{\tau E_z(v_z'',\epsilon_z)}.\tag{7}$$

The current density can now be calculated as

$$i_x=q\int dv_xdv_ydv_zv_xf_1.\tag{8}$$

Introducing the distribution function, Eq. (6), performing the v_x and v_y integrations and integrating the current density over z to obtain the total current in the potential well for a unit surface area, we find

$$I_x=\frac{2\pi qE_xCkT}{m}\int_{-\infty}^{\infty}dv_z\int_0^{\infty}dz \times\exp\left\{\left[-\left(\frac{mv_z^2}{2}+q\psi\right)\Big/kT\right]-K\right\} \times\int_{v_z}^{v_{zs}}\frac{\exp(K')}{E_z'}dv_z'.\tag{9}$$

The z integration has been carried to infinity assuming the form of ψ insures a negligible contribution from the region beneath the space charge layer. Since

$$\partial K/\partial v_z=m/q\tau E_z,$$

Eq. (9) reduces to

$$I_x=2\pi q^2\frac{E_xCkT\tau^2}{m^3}\exp[-q\psi_s/kT]\int_0^{\infty}d\epsilon_z \times\exp[-\epsilon_z/kT][\exp2K(\epsilon_z)-2K(\epsilon_z)-1]=DA,\tag{10}$$

where $K(\epsilon_z)$ is evaluated from Eq. (7), the upper limit being $(2\epsilon_z/m)^{\frac{1}{2}}=v_{zs}$, and D the group of constants appearing before the integral. To evaluate the current explicitly, some form of the space-charge potential must be assumed.

(*a*) The first case considered is $E_z=E_{zs}$, a constant. The relaxation time is assumed constant and the surface scattering random. By introducing Eq. (7) into Eq. (10) and defining

$$\alpha=(qE_{zs}\tau)^{-1}(2mkT)^{\frac{1}{2}},\tag{11}$$

one obtains

$$I_x=\pi^{\frac{1}{2}}D\alpha kT[1-\exp(\alpha^2)(1-\text{erf}\alpha)],$$

$$\text{erf}\alpha\equiv\frac{2}{\pi^{\frac{1}{2}}}\int_0^{\alpha}\exp(-x^2)dx.\tag{12}$$

The reduction in the conductivity of the carriers in the well due to surface scattering can be taken into account by defining an effective mobility such that

$$I_x=Nq\mu_{\text{eff}}E_x,\tag{13}$$

where N is the total number of either holes or electrons in the well, depending upon whether a p-type or n-type surface is under consideration. If the zero of potential is taken at the surface, N is given by

$$N=C\int dv_xdv_ydv_zdz\exp\left[-\left(\frac{mv^2}{2}+q\psi\right)\Big/kT\right] =C\left(\frac{2\pi kT}{m}\right)^{\frac{3}{2}}\frac{kT}{qE_{zs}}.\tag{14}$$

By combining Eqs. (12), (13), and (14) and writing $\mu_{\text{bulk}}=q\tau/m$, one obtains

$$\mu_{\text{eff}}/\mu_{\text{bulk}}=1-\exp(\alpha^2)(1-\text{erf}\,\alpha).\tag{15}$$

Figure 2 shows this ratio plotted as a function of α. For the limit of large E_{zs}, the expression reduces to

$$\mu_{\text{eff}}/\mu_{\text{bulk}}\cong2\alpha/\pi^{\frac{1}{2}}.\tag{16}$$

Thus the effective mobility is inversely proportional to the space charge field for large constants fields. For small fields, the effective mobility reduces to the bulk value as expected.

(*b*) The second case considered is that in which E_z is obtained from a solution of Poisson's equation for the space-charge layer. For definiteness, the problem will be solved for a p-type material with an n-type inversion layer existing at the surface and an external bias, V_a, applied in the z direction such that the energy bands are depressed further at the surface as shown in Fig. 1(b). The results are valid for an n-type material with a p-type inversion layer with an appropriate change in the definition of quantities involved.

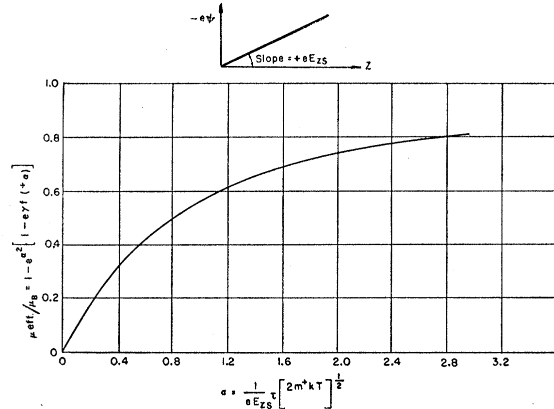

FIG. 2. The effective mobility for the constant field case, as a function of the parameter α.

We define ψ to be the mid-gap potential relative to the mid-gap potential deep within the sample. φ_n and φ_p are the quasi Fermi potentials and assumed constant over the region of interest. From Fig. 1(b), we note

$$\varphi_n = \varphi_p + V_a.$$

The charge density is taken as

$$\rho = e[-N_a + p_0 \exp(-e\psi/kT) - n_0 \exp(e\psi/kT)], \quad (17)$$

where

$$n_s = n_0 \exp[e\psi_s/kT] = \text{electron density at the surface},$$
$$n_0 = n_i \exp[(\Delta E_F - eV_a)/kT],$$
$$p_0 = n_i \exp[-\Delta E_F/kT], \quad (18)$$
$$\Delta E_F = E_{F(\text{sample})} - E_{F(\text{intrinsic})},$$

n_i is the intrinsic carrier density, N_a the acceptor density, and κ the dielectric constant.

Poisson's equation becomes

$$\frac{d^2\psi}{dz^2} = -\frac{4\pi e}{\kappa}[-N_a + p_0 \exp(-e\psi/kT) - n_0 \exp(e\psi/kT)]. \quad (19)$$

Let $d\psi/dz = \psi = 0$ at some point within the sample as an origin for integration. Multiplying Eq. (19) by $d\psi/dz$ and integrating over z, we obtain the electric field as a function of the potential.

$$E_z^2 = \frac{8\pi e N_a}{\kappa}\left[\psi + \frac{kT}{e}\left\{\left(\frac{p_0}{N_a}\right)(\exp[-e\psi/kT] - 1) + \left(\frac{n_0}{N_a}\right)(\exp[e\psi/kT] - 1)\right\}\right]. \quad (20)$$

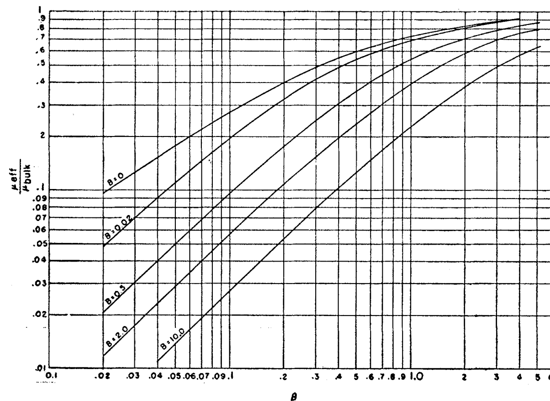

FIG. 3. The effective mobility corresponding to the potential obtained from a solution of Poisson's equation for the space charge layer, plotted as a function of the parameter β for several values of the parameter B.

However, to evaluate K, Eq. (7), E_z must be expressed as a function of v_z and ϵ_z by using the defining relationship for ϵ_z, Eq. (4). The expression for K becomes

$$K = -\frac{m}{e\tau}\left(\frac{\kappa}{8\pi N_a kT}\right)^{\frac{1}{2}} \int_0^{v_z} dv_z \Big/ \left\{\frac{e\psi_s}{kT} + \frac{n_0}{N_a}\exp(e\psi/kT) + \left[-\frac{(p_0+n_0)}{N_a} - \left(\epsilon_z - \frac{mv_z^2}{2}\right)\Big/ kT + \frac{p_0}{N_a}\exp(-e\psi/kT)\right]\right\}^{\frac{1}{2}}, \quad (21)$$

where

$$e\psi = e\psi_s + \tfrac{1}{2}mv_z^2 - \epsilon_z.$$

We now make the approximation that the bracketed portion in the denominator of Eq. (21) is negligible compared to the terms retained, over the region of major contribution to the current integral. This approximation is equivalent to setting the field due to the impurity ions equal to a constant value, given by its value at the surface, and neglecting the contribution of holes to the space charge. If $e\psi_s$ is greater than several kT, the population of states in the region where $\epsilon_z - \frac{1}{2}mv_z^2$ is comparable to $e\psi_s$, is very small compared to the population at the bottom of the well. The bracketed expression also indicates that the maximum possible hole contribution to the denominator corresponds to a decrease of $e\psi_s$ by kT. Thus for most problems of interest, the above approximation appears valid and K reduces to

$$K = -(2/\pi)^{\frac{1}{2}}\beta \int_0^{\sqrt{(\epsilon_z/kT)}} \frac{dy}{B + \exp[-(\epsilon_z/kT) + y^2]}, \quad (22)$$

where

$$\beta = \frac{1}{e\tau}\left[\frac{m\kappa}{8n_0}\exp(-e\psi_s/kT)\right]^{\frac{1}{2}} = \frac{1}{e\tau}\left[\frac{m\kappa}{8n_s}\right]^{\frac{1}{2}} \quad (23a)$$

$$B = \frac{N_a}{n_0}\frac{e\psi_s}{kT}\exp(-e\psi_s/kT) = \frac{N_a}{n_s}\frac{e\psi_s}{kT}. \quad (23b)$$

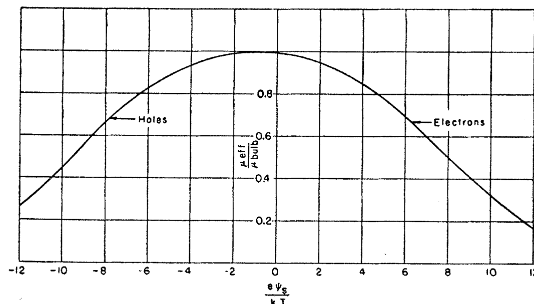

FIG. 4. The effective mobility plotted as a function of the depression of the energy bands at the surface.

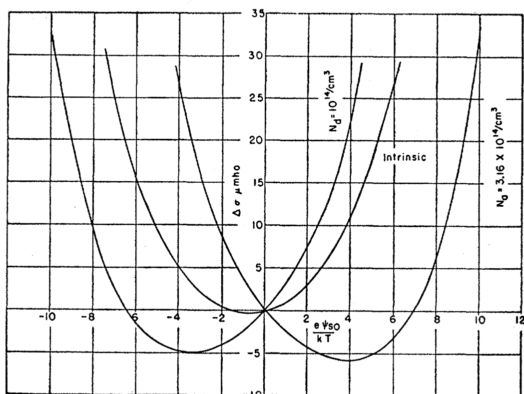

Fig. 5. The change in surface conductivity as a function of the depression of the energy bands at the surface.

The current is given by substituting Eq. (22) into Eq. (10) and after some calculation gives

$$I_x = DkT\left\{2\sqrt{2}\beta[(B+1)^{\frac{1}{2}}-B^{\frac{1}{2}}] + \int_0^\infty \exp\left[-x-2(2/\pi)^{\frac{1}{2}}\beta \times \int_0^{\sqrt{x}} [B+\exp(-x+y^2)]^{-\frac{1}{2}}dy\right]dx-1\right\}. \quad (24)$$

An effective mobility is again defined by Eq. (13) and proceeding exactly as in the constant field case we obtain the expression,

$$\frac{\mu_{\text{eff}}}{\mu_{\text{bulk}}} = 1+\frac{1}{2\sqrt{2}\beta[(B+1)^{\frac{1}{2}}-B^{\frac{1}{2}}]} \times\left\{\int_0^\infty \exp\left[-x-2(2/\pi)^{\frac{1}{2}}\beta \times \int_0^{\sqrt{x}} (B+\exp(-x+y^2))^{-\frac{1}{2}}dy\right]dx-1\right\}. \quad (25)$$

This ratio is plotted in Fig. 3 as a function of β for several values of B. Figure 4 shows $\mu_{\text{eff}}/\mu_{\text{bulk}}$ for intrinsic germanium.

III. APPLICATIONS

The results of Sec. II may be applied to the problem of determining the position of the energy bands at the free surface of a semiconductor if the conductance of the sample is known relative to the conductance at a definite band position. The conductance due to the space-charge layer is given by

$$\Delta\sigma = e[\mu_{(n)\text{eff}}\Delta n + \mu_{(p)\text{eff}}\Delta p], \quad (26)$$

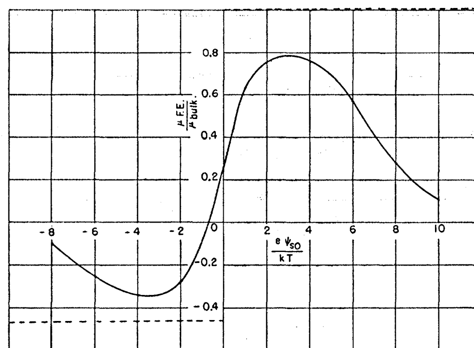

Fig. 6. The "field effect" mobility as a function of the depression of the energy bands at the surface, assuming the charge associated with surface traps is independent of the applified field.

where

$$\Delta n = n_0 \int_0^{\psi_{s0}} \left[\frac{\exp(e\psi/kT)-1}{d\psi/dz}\right]d\psi, \quad (27a)$$

$$\Delta p = p_0 \int_0^{\psi_{s0}} \left[\frac{\exp(-e\psi/kT)-1}{d\psi/dz}\right]d\psi, \quad (27b)$$

and n_0 and p_0 are defined as in Eqs. (18) by setting $V_a=0$. The effective mobility is to be considered different from the bulk mobility when the carrier is constrained to conduct in the potential well, thus neglecting the small correction due to some carriers of opposite sign scattering from the potential barrier rather than the surface. Since all quantities in Eq. (26) are expressible as functions of ψ_{s0}, this relationship and the sign of the field effect give the band position. There is some question as to what values to assume for the effective mass and relaxation time for both holes and electrons. Assuming $m_n=m_p=0.25m_e$, we have plotted $\Delta\sigma$ in Fig. 5 for intrinsic, n-type and p-type germanium. The relaxation times have been determined from the relationship $\mu_{\text{bulk}}=e\tau/m$. It should be noted that the conductance decreases and goes through a minimum as the surface conductance tends to become inverted. For the p-type material this is due to the hole density near the surface decreasing to a small value before the electron density has increased appreciably. The conductance minimum can be used to establish the definite band position needed above. Changes in conductance may be interpreted as moving along this conductance curve and the energy band position is read directly once the minimum has been established.

The magnitude of the field effect can be estimated if one assumes the charge associated with surface traps is unchanged when the field is applied. Since there can exist no net field deep within the semiconductor, Gauss's law applied to the free surface gives the total charge, Q, in the space charge region by the value of the field just inside the surface, Eq. (20). Defining a

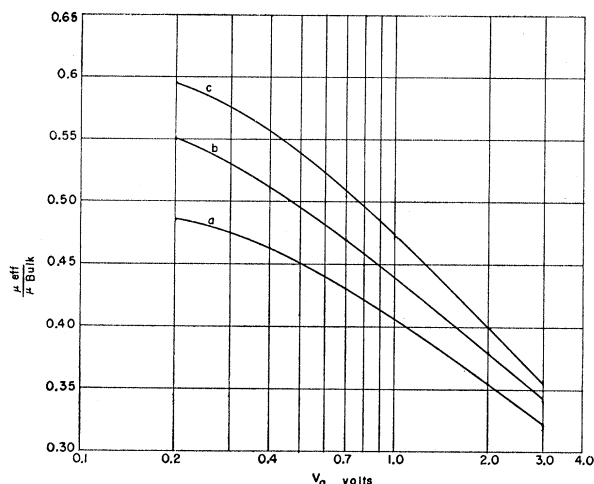

FIG. 7. The theoretical effective mobility corresponding to Kingston's data on the channel effect, for ambient water vapor pressure (*a*) 19.8 mm of Hg, (*b*) 14.5 mm, and (*c*) 4.6 mm.

field-effect mobility, $\mu_{\mathrm{F.E.}}$, one finds

$$\mu_{\mathrm{F.E.}} \equiv d\Delta\sigma/dQ$$

$$= \Big\{ \mu_{(n)\mathrm{eff}} n_0 [\exp(e\psi_s/kT)-1] + \mu_{(p)\mathrm{eff}} p_0 [\exp(-e\psi_s/kT)-1] + \left(\frac{d\psi}{dz}\right)\bigg|_{\psi_s} \left[\Delta n \frac{\partial\mu_{(n)\mathrm{eff}}}{\partial\psi_s} + \Delta p \frac{\partial\mu_{(p)\mathrm{eff}}}{\partial\psi_s}\right] \Big\} \Big/ [N_a + n_0 \exp(e\psi_s/kT) - p_0 \exp(-e\psi_s/kT)], \quad (28)$$

where $(d\psi/dz)_{\psi_s}$ is given in Eq. (20), the derivatives of the effective mobility are obtained from Eq. (25), and Δn and Δp as defined above have been tabulated as functions of $e\psi_s/kT$. The first two terms in the numerator of Eq. (28) are due to the number of electrons and holes in the space-charge layer changing, while the last terms arise from the fact that the effective mobility of the carriers already present in the space charge layer changes. In general both types of terms must be considered. The field-effect mobility is shown in Fig. 6 for intrinsic germanium. If surface scattering were not included, the mobility for a large surface dipole would approach the bulk values indicated by the dotted lines. Morrison's observation that the sign on the field effect changes when the ambient forces the conductance to go through a minimum is explained by realizing the field effect essentially senses the slope of the n-type curve in Fig. 5. By applying the external field, the bands are either lowered or raised at the surface, depending upon the sign of the applied field. If the sample is at a conductance minimum, the conductance will increase in either case however the sign of the induced charge is opposite, thus accounting for the observed effect.

An estimate of the effective mobility to be expected in the channel effect can be derived from Kingston's data of channel conductance *versus* reverse bias on n-p-n junction transistors by extrapolating his conductance curves to zero bias. Since Kingston's data give directly the conductance due to the existence of the channel, the depression of the bands at zero bias is given by the value of ψ_{s0} corresponding to the intercept. The values obtained in this way for several values of ambient water vapor pressure are

	Vapor pressure	$\Delta\sigma(V_a=0)$	ψ_{s0}
a	19.8 mm of Hg	26 micromhos	0.25 volt
b	14.5 mm of Hg	10 micromhos	0.22 volt
	8.6 mm of Hg	6.6 micromhos	0.21 volt
c	4.6 mm of Hg	4.0 micromhos	0.20 volt

If one assumes the Fermi level at the surface is stabilized relative to the energy bands, as Kingston has proposed,[11] the effective mobility is then given by Eq. (25), where β assumes a fixed value for each value of ψ_{s0} and B depends linearly upon V_a. Figure 7 indicates the mobility is reduced to about one third the bulk mobility for a reverse bias of several volts.

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[11] R. H. Kingston, Phys. Rev. **94**, 1416 (1954).