

cate excited levels in  $\text{Al}^{26}$  at 6.655, 6.720, 6.769, 6.820, 6.840; and 6.860 Mev above the ground state.

Stähelin<sup>6</sup> and Moszkowski and Peaslee<sup>7</sup> have suggested that the level which decays by positron emission of 6.7-second half-life to  $\text{Mg}^{26}$  is not the ground state but the  $T_z=0$  component of the lowest triplet with isobaric spin  $T=1$  and ordinary spin  $J=0^+$ . The decay of the 6.769-Mev level by  $\gamma$  emission to this state therefore appears to be forbidden because of the absence of positron yield from the 436.5-keV resonance. This is consistent with Kluyver's observation of a strong  $6.77 \pm 0.08$  Mev  $\gamma$  ray from the 6.769 level. The  $\gamma$ -ray de-excitation is presumed to be preferential to the ground state of  $\text{Al}^{26}$ , which Kluyver postulates to have a spin of  $J=5^+$ . This in turn decays by a second forbidden positron transition to  $\text{Mg}^{26}$ . His estimated half-life of  $4 \times 10^4$  years would insure that the positron yield was much too small to be detected in our observations.

It is of interest to note that the ratio of positron yield to  $\gamma$ -ray yield of the other resonances does not vary by more than a factor of 2 (Table I). If the probability of

TABLE I. Relative positron and  $\gamma$ -ray yields from the  $\text{Mg}^{26}(p,\gamma)\text{Al}^{26}$  resonances.

Resonant proton energy	$\gamma$ -ray yield at peak	$\beta^+$ yield at peak	Ratio $\beta^+/\gamma$ -ray yield
316.7	70	100	1.4
391.5	140	400	2.8
436.5	200	<20	<0.1
495.6	180	350	1.9
513.4	155	320	2.0
530.4	92	220	2.4

de-excitation of one of these levels to the 6.7 second  $\beta^+$  activity level is  $\Gamma_1$ , and the total transition probability from the level is  $\Gamma_T$ , the above observation would seem to indicate that the ratio  $\Gamma_1/\Gamma_T$  is approximately constant for all the excited  $\text{Al}^{26}$  states listed here, except the 6.769 level. This conclusion may be modified slightly because of the possibility of de-excitation of some of these levels by cascade  $\gamma$ -ray emission as is indicated by Kluyver's detection of low-energy  $\gamma$  radiation.

Positrons from  $\text{Al}^{26}$  have been found to have an end-point energy approximately equal to those from  $\text{Al}^{26}$ ,<sup>8</sup> and the half-lives of the two activities are approximately equal.<sup>9</sup> It is therefore of interest to note that while the  $\gamma$ -ray yield of the 391.5-keV  $\text{Mg}^{25}(p,\gamma)\text{Al}^{26}$  and 418.4-keV  $\text{Mg}^{24}(p,\gamma)\text{Al}^{25}$  resonances are approximately equal,<sup>2,3</sup> the positron yield from the  $\text{Mg}^{24}$  resonances is approximately ten times higher than that from the  $\text{Mg}^{25}$  resonance (unseparated magnesium targets of comparable thickness were used for both measurements).

This could indicate that  $\Gamma_1/\Gamma_T$  for the excited  $\text{Al}^{26}$  levels is small compared with  $\Gamma_1/\Gamma_T$  for the  $\text{Al}^{25}$  levels. That is if we assume that  $\Gamma_1/\Gamma_T$  is unity for the  $\text{Al}^{25}$  level produced by the 418.4-keV resonance, an upper limit of  $\frac{1}{10}$  is placed on the  $\Gamma_1/\Gamma_T$  ratio for the  $\text{Al}^{26}$

levels listed here. Alternatively, it could be postulated that there is a competing transition from the  $\text{Al}^{26}$  short-lived positron state itself, possible  $\gamma$ -ray transition to the ground state, which would also produce the short half-life of this state, and reduce the positron yield.

The authors would like to thank Dr. J. C. Kluyver, who initially drew our attention to the difference between his  $\gamma$ -ray yield curve and our positron yield curve from the  $\text{Mg}^{25}(p,\gamma)\text{Al}^{26}$  reaction. The only remaining discrepancy between the two results is the existence of the resonance of 530.4 keV, which we find in both the positron and  $\gamma$ -ray yield curves.

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<sup>2</sup> S. E. Hunt and W. M. Jones, Phys. Rev. **89**, 1283 (1953).

<sup>3</sup> R. Tangen; Kgl. Norske. Videnskab. Selskabs, Skrifter No. 1 (1946).

<sup>4</sup> Supplied by Dr. R. H. V. M. Dawton and Dr. M. L. Smith, Atomic Energy Research Establishment, Harwell.

<sup>5</sup> The positron yield curve is reproduced by kind permission of the Physical Society.

<sup>6</sup> P. Stähelin, Helv. Phys. Acta **26**, 691 (1953); Phys. Rev. **92**, 1076 (1953).

<sup>7</sup> S. A. Moszkowski and D. C. Peaslee, Phys. Rev. **93**, 455 (1954).

<sup>8</sup> Hunt, Jones, and Churchill, Proc. Phys. Soc. (London) **A68**, 479 (1954).

<sup>9</sup> Hunt, Jones, Churchill, and Hancock, Proc. Phys. Soc. (London) **A68**, 443 (1954).

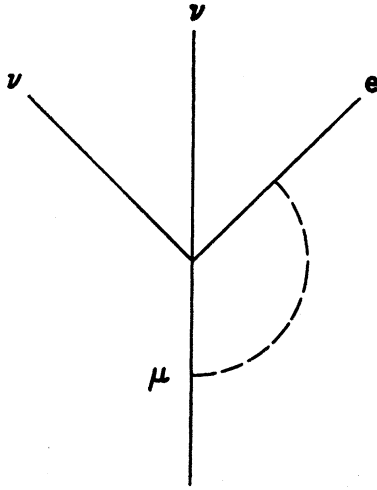
## Radiative Corrections to Muon Decay

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THE instabilities of the neutron and the muon may be attributed to Fermi interactions of approximately the same strength. To compare these couplings in more detail, one must take into account the fact that these two processes are influenced differently by other fields: in particular, mesonic<sup>1,2</sup> and electromagnetic<sup>3</sup> corrections are present in neutron decay while only electromagnetic corrections are present in muon decay. In this Letter, the electromagnetic corrections to the spectrum of the muon are given for three typical interactions ( $S+aP$ ,  $S+aT+bP$ ,  $S+aA+bP$ ,  $a=\pm 1$  and  $b=\pm 1$  in charge retention order). One might expect these corrections to be rather large since the energy of the electron near the end point is of the order of 100 rest masses. Further, without explicit calculation it is not possible to tell how seriously such an energy dependent correction distorts the observed spectrum. Since the nature of the interaction is inferred from this shape (especially near the end point, which is just where the radiative corrections are largest), it is important to consider this question carefully.

FIG. 1. A lowest order correction to  $\mu$  decay.

The process is described by the familiar three diagrams, one of which is shown in Fig. 1. In addition one has to take into account inner bremsstrahlung,<sup>4</sup> which is of the same order. As usual the infinite infrared terms associated with the inner bremsstrahlung exactly cancel the corresponding infinite nonradiative terms. The really divergent terms associated with high photon energies cancel for the vector and axial vector interactions as in electrodynamics but in the other cases there are uncompensated logarithmically divergent terms.<sup>5</sup> The results, however, are rather insensitive to these terms: with a cutoff corresponding to the proton Compton wavelength, they amount to about one-fifth of the net correction.

These remarks are illustrated by the following expressions, valid for the scalar case, which express the change in the probability of decay associated with inner bremsstrahlung and virtual photons respectively.

$$\left(\frac{\Delta P}{P}\right)_{\text{I.B.}} = \frac{e^2}{2\pi} \left\{ 4 \left( \ln \frac{2\epsilon}{m_2} - 1 \right) \ln \frac{m_1}{\lambda_{\text{min}}} + 4 + \ln 2 - \frac{2\epsilon_m}{\epsilon} \right. \\ \left. + 2 \sum_{n=1}^{\infty} \frac{1}{n^2} \left( \frac{\epsilon}{\epsilon_m} \right)^n - 2 \left( \frac{\epsilon + \epsilon_m}{\epsilon} \right) \ln \frac{\epsilon_m' - \epsilon}{\epsilon_m} \right. \\ \left. - \left[ 4 \ln \frac{\epsilon_m' - \epsilon}{2\epsilon_m} + \frac{\epsilon_m^2 - 4\epsilon_m\epsilon - 17\epsilon^2}{3\epsilon^2} \right] \ln \frac{m_2}{2\epsilon} \right\}, \quad (1)$$

$$\left(\frac{\Delta P}{P}\right)_{\text{V.}} = \frac{e^2}{2\pi} \left\{ -4 \left( \ln \frac{2\epsilon}{m_2} - 1 \right) \ln \frac{m_1}{\lambda_{\text{min}}} \right. \\ \left. + A + 2 \sum_{n=1}^{\infty} \frac{1}{n^2} \left( \frac{\epsilon}{\epsilon_m} \right)^n \right. \\ \left. + \left[ \frac{2\epsilon_m}{\epsilon_m' - \epsilon} + 2 \ln \frac{\epsilon_m' - \epsilon}{\epsilon} \right] \ln \frac{\epsilon}{\epsilon_m'} \right\}, \quad (2)$$

where

$$A = 6 \ln(\lambda/m_1) + \ln(m_1/m_2) - \frac{1}{2} \\ - \frac{1}{3}\pi^2 + 2[\ln(m_1/m_2)]^2 = 71.4,$$

$e^2 = 1/137$ ,  $\epsilon_m' = \epsilon_m + m_2^2/2m_1$ ,  $\epsilon =$  electron energy  $= (p^2 + m_2^2)^{1/2}$ ,  $\epsilon_m =$  maximum electron energy  $= m_1/2$ ,  $m_2 =$  mass of electron,  $m_1 =$  mass of muon,  $\lambda =$  maximum virtual photon energy, and  $\lambda_{\text{min}} =$  infrared cutoff.

The dominant term in (2) is  $2[\ln(m_1/m_2)]^2$ . The net correction is plotted in Fig. 2, which shows that  $\Delta P/P$

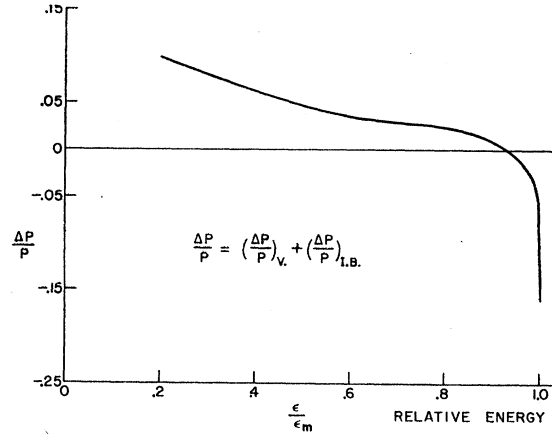


FIG. 2. Relative change in probability of decay for scalar case.

decreases from +7 percent at low energies to -17 percent at the endpoint. The downward trend of  $\Delta P/P$  may be attributed to the fact that inner bremsstrahlung is relatively more important at low energies while virtual photons dominate at higher energies.

In Fig. 3, the results are given for  $S+aP$ ,  $S+aT+bP$ , and  $S+aA+bP$ . The experimental points are those of Sagane *et al.*<sup>6</sup> The changes in the lifetime in the three cases are about 5 percent, 2.5 percent, and 3 percent respectively, which are rather large for electromagnetic corrections and must be taken into account in comparing

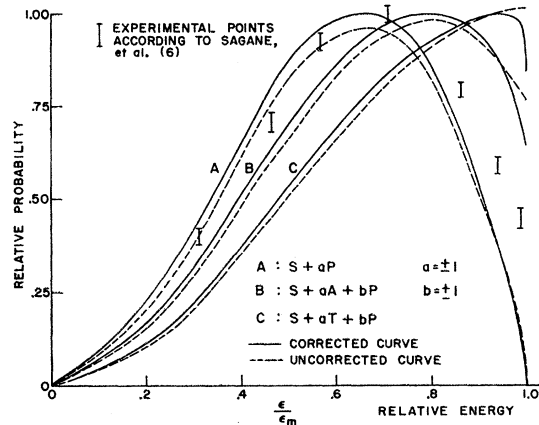


FIG. 3. Relative probabilities of decay with and without radiative corrections.

the neutron and muon decay. On the other hand, these corrections do not distort the shape of the spectrum in a significant way. Detailed calculations will be published.

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<sup>2</sup> R. J. Finkelstein and S. A. Moszkowski, Phys. Rev. **95**, 1695 (1954).

<sup>3</sup> T. Nakano *et al.*, Progr. Theoret. Phys. (Japan) **5**, 1014 (1950).

<sup>4</sup> A. Lenard, Phys. Rev. **90**, 968 (1953). The differential cross

sections given by Lenard were integrated to evaluate the total cross section of inner bremsstrahlung.

<sup>5</sup> S. Hanawa and T. Miyazima, Progr. Theoret. Phys. (Japan) **5**, 459 (1950); T. Nakano *et al.*, see reference 3. The first of these treats the correction to the decay of a meson with integral spin, the second to the decay of a neutron. The dominant term in the boson case is the same as in (2). The finite terms in the neutron case are not given. The same divergences appear in both of these calculations.

<sup>6</sup> R. Sagane *et al.*, Phys. Rev. **95**, 863 (1954).