two nucleons, can lead to a cross section smaller than given by the previous calculations based on the impulse approximation, especially where the phase shift α_{33} is large, as is here the case. The experiments suggest that such effects may be important.[15]

[15] Noteworthy in this connection are the experiments of Arase, Goldhaber, and Goldhaber, Phys. Rev. **90**, 160 (1953). Their cross sections for elastic scattering are much smaller than the earlier calculation (see reference 12) would predict.

X. ACKNOWLEDGMENTS

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Landau Distribution and Density Effect at High Gas Pressures*

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Studies of the ionization energy losses of fast μ mesons have been carried out in a specially constructed proportional counter with argon gas at pressures up to 40 atmospheres. With increasing pressure, a strong reduction in the relativistic rise was observed; the extent of the reduction being at least as great as that expected from Sternheimer's calculation of the density effect. Furthermore, the distribution in energy losses, which at low pressures is much wider than expected from theoretical considerations, was observed to become narrower with increasing pressure. At our highest pressure, however, the energy loss distribution was still considerably wider than predicted by the Landau theory.

I. INTRODUCTION

PUBLISHED studies of the ionization energy losses of fast μ mesons confirm the existence of the relativistic rise in dispersed media[1–4] as well as the modifications of this rise (the density effect) caused by the dielectric polarization, when the medium is condensed.[5–10] Some observations also indicate the onset of the density effect, even at low pressures in gases, provided the meson energy is sufficiently high.[2,4,11]

The general features of the distribution in energy loss, as developed by Landau[12] have also been confirmed, for both dispersed and condensed media. In general, however, with small thicknesses of absorber, the widths of the observed distributions are found to be much greater than those predicted by the theory of Landau or by the modified theory of Blunck and Leisegang.[13] It should be noted, for the case of low-energy electrons traversing thin foils that a similar disagreement was reported although recent work appears to have eliminated the discrepancy.[14,15] The half-maximum widths observed with large thicknesses of absorber are generally in much better agreement with theory.[5,9]

This report deals with a study of μ-meson ionization energy losses, carried out in argon gas at pressures up to 40 atmospheres, in an attempt to observe the onset and the development of the density effect and the behavior of the Landau distribution in the transition region between the dispersed and condensed phases of matter.

II. DESCRIPTION OF APPARATUS

The apparatus used in this work, in particular the electronic and recording portions, was quite similar to that used in our previous studies,[3] in this laboratory. It consisted essentially of four parallel Geiger counter telescopes, with enough lead absorber to insure that four well collimated beams of mesons were obtained; two proportional counters located within the telescopes; lead absorber and range trays; plus the required re-

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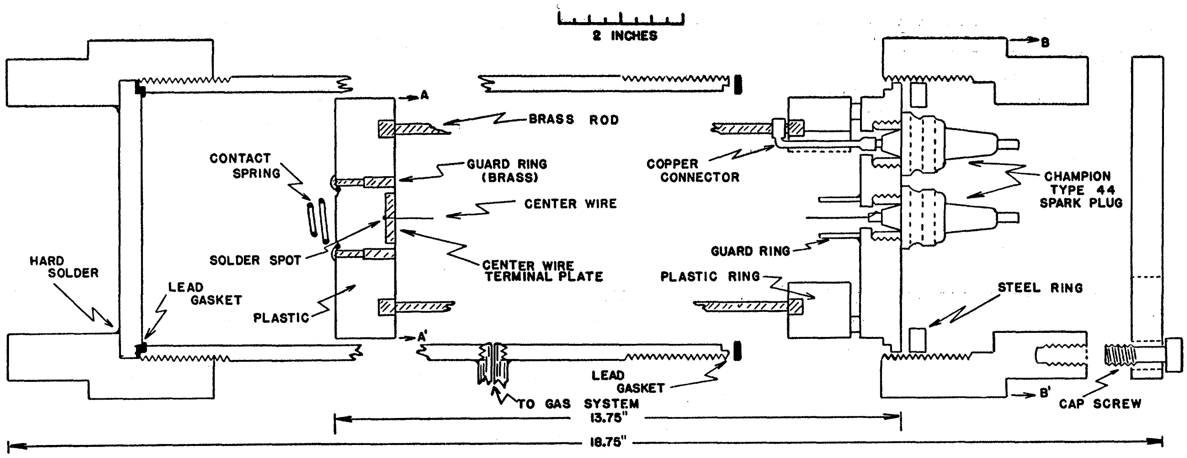

FIG. 1. Schematic drawing of high-pressure proportional counter.

cording circuits. The following main modifications were made to our original apparatus.

(a) The dimensions of each of the four telescopes were set at 2 in.$\times$1.25 in.$\times$13 in., the greatest dimension lying in the vertical. This gave a total counting rate of 50 mesons per hour without destroying the collimation of the mesons or introducing a serious spread in the meson path length within the proportional counter. Thus for this arrangement, the extreme path lengths through the counter differed from the mean path length by less than three percent.

(b) The amount of lead absorber was greatly increased to attain greater momentum limits. The dimensions of the absorbers are given in Table I.

(c) An extensive set of guard counters was used to eliminate the effect of showers falling upon the apparatus.

(d) The lower proportional counter, which was run at a pressure of 2 atmospheres throughout the whole of the experiment, was one of the counters used in our previous experiment, modified by the substitution of a guard ring seal (Stupakoff Type 96.0019) in place of the previous seal. Although the type of seal used possesses excellent electrical insulation properties, it will not withstand a pressure differential of much more than two and one-half atmospheres.

(e) The upper proportional counter in which the high pressures were used, was constructed as shown in Figs. 1 and 2. It consisted of a core, which carried all the electrical components of the counter, and of a thick-walled steel shell into which the core fitted. The core was composed of six brass rods (forming a cylindrical equipotential surface concentric with the center wire which served as the outer electrical conductor), a center wire of tungsten and two end supports of Lucite, all of which were carried by a heavy steel end plate. The electrical connections to the outer conductor and the center wire, which were run at negative and ground potential, respectively, were carried through the steel end plate by means of two Champion Type 44 Diesel spark plugs. Guard rings were inserted as shown, one making direct contact with the end plate, the other making contact with the shell by means of the contact spring. The shell was grounded during the course of the experiment. A lead washer was used with the end plate. No gas leaks were observed with the counter at 120 atmospheres pressure during a 12-hour underwater test and excellent proportional counter pulses

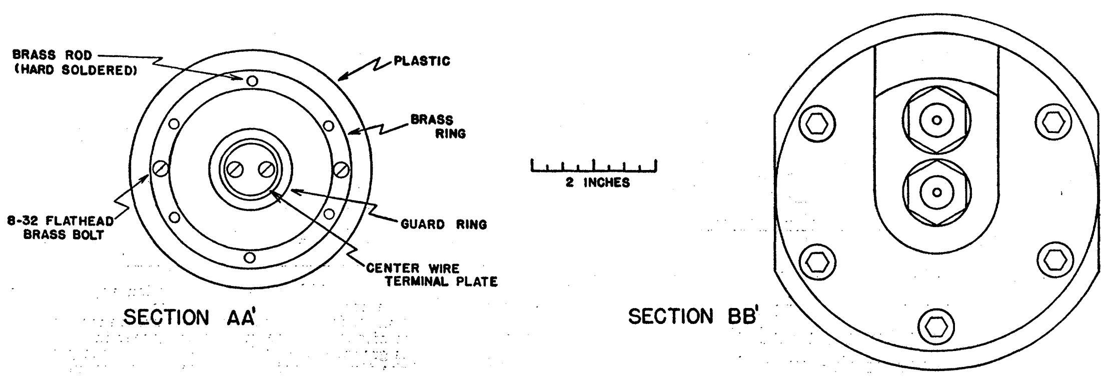

FIG. 2. Cross sections of high-pressure proportional counter.

TABLE I. Dimensions and energy limits for lead absorbers.

Designation of meson group	Absorber thickness (g cm^{-2} Pb)	Energy band (Bev)	Average energy (Bev)	Effective momentum (Bev/c)
A	120– 470	0.165–0.55	0.36	0.45
B	470– 930	0.55 –1.13	0.84	0.93
C	930–2590	1.13 –3.45	2.3	2.4
D	2590 or greater	3.45	13.0	13.0

were observed at a potential of 14 000 volts with an argon filling at 55 atmospheres pressure. No tests of this counter were run at higher pressures or voltages.

An important feature of the counter was the center wire which was constructed of 0.0007-in. diameter tungsten wire. Its positioning was achieved by threading through guide holes as shown in the figure and securing it in position by small wedges located well within the field-free region at the ends of the chamber. When the wire had been located within the core, the latter was placed in a bell jar and the wire was flashed in a vacuum for two seconds at 80 volts dc. This flashing procedure which raised the temperature of the wire to around 2000°C, according to our resistance measurements, was found to be necessary and quite effective in producing a center wire which operated consistently. Immediately upon flashing, the core was inserted into the shell and the counter was sealed.

III. THE OPERATING CONDITIONS

During the preliminary runs it was found that our previous practice of using tank CO_2 along with the argon in the high pressure counter caused electron attachment at the higher pressures (40 atmospheres) used in this study. This effect was quite noticeable by the wide spread introduced into the pulse height distribution curve and by the dependence of the pulse height distribution upon the location of the proportional counter with reference to the telescope, (upon the location of the meson path through the proportional counter). Whether this effect arose from the behavior of CO_2 at higher pressures or whether it was due to some impurity contained in our CO_2, we cannot say; it was found to be impossible to prevent this trouble by passage of the CO_2 over hot calcium turnings. It was found possible, however, to eliminate the effect to the extent that no pulse height dependence upon the location of the meson path could be observed (other than that expected from purely geometrical considerations) by completely eliminating the CO_2 and using only Linde tank argon purified by passage over hot calcium. This latter filling was used throughout with both counters.

During the course of the experiment, the standard tests and checks were performed to ensure the quality of the Geiger counter efficiencies and circuit linearities as well as the constancy of the operating voltages and to reduce spurious events to a minimum. In Table II the operating conditions are summarized. It will be observed that Runs 4 and 5 were performed at the same pressure but under different operating conditions of voltages and amplification. The fact that the operating voltage was varied by about 7.5 percent without any observable change in the histogram characteristics is considered to indicate that truly proportional operation of the apparatus was being achieved.

IV. TREATMENT OF THE DATA

The preliminary step in the analysis of the data was the rejection of certain spurious events which were readily recognizable and fell into three classes as follows:

(a) Shower events. These events were identifiable not only by the triggering of the guard tray but also by the simultaneous large outputs from both counters.[3] We believe that no appreciable error arises from a failure to detect such events.

(b) Events in nonlinear regions of the apparatus. Since the apparatus was found to be linear up to outputs of around 4 times the most probable pulse height output in all cases, and no events were used which were greater than 3.0 times the most probable, no errors will be encountered with this type of event.

(c) Spurious events of very small magnitude. A small number of small events were encountered which arose from a spurious triggering of our telescope combined with a small inefficiency in the clamping tube of the pulse lengthener circuit. These events were readily discernible since they only occurred with the A group of mesons (higher-order telescope coincidences were required with the other groups of mesons), since they always appeared as pulses of constant and small size simultaneously in both proportional counters, and since they always lay well below the low-energy end of the Landau pulse height distribution. These criteria all permit positive identification of such events. All other events were considered to be true events and were included in the histograms of the data.

TABLE II. Summary of experimental conditions.

Run	Pressure[a] (atmos)	Voltage	Amplifier gain	No. of mesons in group A	B	C	D
Low-pressure counter							
1	2	2130	4 $\times 10^4$	565	552	1055	637
High-pressure counter							
2	40	9700	1 $\times 10^4$	147	154	340	182
3	40	9700	1 $\times 10^4$	148	151	303	197
4	21	7000	0.9$\times 10^4$	155	182	288	210
5	21	6500	3.4$\times 10^4$	99	99	210	218
6	7	3950	2.0$\times 10^4$	133	127	252	146
7	1.25	2100	4 $\times 10^4$	116[b]	82[b]	219	146

[a] Pressure corrected to 0°C and slight correction applied where required to account for behavior of a Van der Waals gas.
[b] In this run a slight failure occurred in the first range tray. This caused about 20 percent of the B range mesons to appear in the A group. As a consequence, this group, while useful for estimating the pressure behavior of the widths of the distributions, is of no use whatsoever in a good measurement of the relativistic rise and has not been included in the results of Fig. 7.

In the determination of the main characteristics of the histograms, namely the most probable pulse height and the width at half-maximum, where the latter is defined as the ratio

$$\frac{\text{full width of distribution at half-height}}{\text{most probable value of specific ionization}},$$

the two following methods of analysis were used.

(a) Beherens method. This is an adaptation of the Gauss method of maximum likelihood[16] and is described fully elsewhere.[1]

(b) Quantile (or percentile) method. The details of this method are discussed in standard statistical treatises[17]; we shall discuss here only the application of the method to histograms of the type considered in in this study.

If x represents pulse height and $f(x)$ represents the normalized population from which the sample distribution is to be obtained, the quantile $q(x_0)$ of the distribution associated with a given pulse height x_0 is the fraction of events satisfying the requirement $x \leq x_0$. Thus

$$q(x_0)\int_v^u f(x)dx = \int_v^{x_0} f(x)dx,$$

where u and v are the upper and lower limits, respectively, of the range of x. While the value of v is a property of the Landau distribution, that of u must be chosen arbitrarily. We have truncated our histograms at u equal to 2.4 times the most probable pulse height. This procedure is quite convenient and has been used by previous workers.[1]

When a value of $q(b)$ has been chosen, for example, for the most probable pulse height (b), and the quantile procedure is applied to a sample distribution containing n events to determine b for the sample, the standard deviation of the latter quantity is given by

$$[q(b)p(b)/nf^2]^{\frac{1}{2}},$$

where $p(b) = 1 - q(b)$ and f (the ordinate of the parent distribution at x_0) is to be taken as the frequency per unit interval at x_0 in the sample distribution,[17] normalized over the range v to u.

With a more complex quantity such as the half-width of the distribution, the procedure is as follows. If b, as before, represents the most probable pulse height and the half-maximum values of x are denoted by a and c for the lower and upper values, respectively, then the half-width $(c-a)/b$ possesses a standard deviation which is a function of the variance and covariance of these related quantities and is given by the square root of the following expression:

$$\frac{\text{var}c}{b^2} + \frac{\text{var}a}{b^2} + \frac{(c-a)^2\,\text{var}b}{b^4} - \frac{2\,\text{cov}(ac)}{b^2} + \frac{2(c-a)}{b^3}\{\text{cov}(ab) - \text{cov}(bc)\},$$

where

$$\text{var}x = \frac{q(x)p(x)}{nf^2(x)},$$

and

$$\text{cov}(x_1,x_2) = \frac{q(x_1)p(x_2)}{nf(x_1)f(x_2)},$$

where

$$x_1 < x_2.$$

If the three quantities a, b, and c had been independent of one another, the terms containing the expressions for covariance would have disappeared and the expression for the standard deviation would reduce to the usual form.[18] The same remarks as given previously apply here with respect to the choice of f and it is well to note that care must be taken to obtain the correct selection of p and q in the expressions for the covariance.[17]

Before either of these methods can be applied, a function must be chosen with which to obtain a best fit of the data. The curve used with the Beherens method was obtained from the combined B and C groups for Run 1 (1607 mesons) and was determined by use of the probit method[16] for the lower half of the histogram and a trial and error adjustment to a least-square fit for the upper half. The histogram and our basic curve are shown in Fig. 3. From a study of this curve as well as of all the histograms available to us from the published literature and our own results, it has been possible to determine that the best values for the

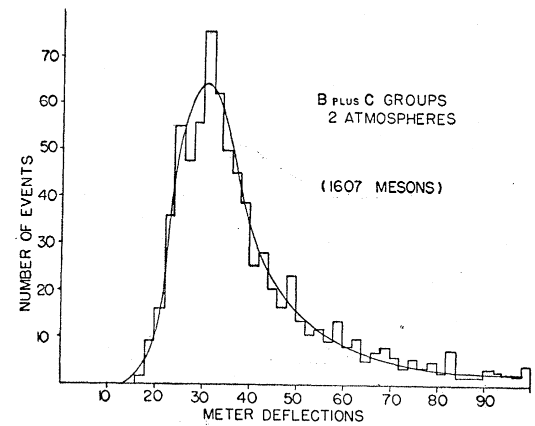

FIG. 3. Basic curve used to fit data and histogram of combined groups from which it was derived.

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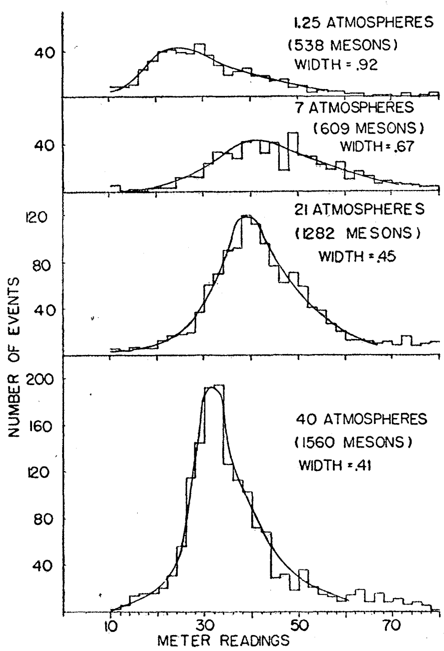

FIG. 4. Combined histograms for entire meson spectrum for various gas pressures in the high-pressure proportional counter.

quantile method are given by the following expressions:

$$q(a)=0.10,\quad q(b)=0.315,\quad q(c)=0.70,$$

for $u=2.4b$, where a, b, and c are, respectively, the lower half height, most probable, and upper half height value of the abscissa.

These values are very close to those obtained from the truncated theoretical curves,[12] with the exception of the lower quantile point. Here the theory indicates a value $q(a)=0.05$. Since it is just in this region that previous workers[19] have reported the greatest discrepancy between theory and experiment we have chosen to work with the experimentally determined best value of the lower quantile point.

The application of the quantile method is exceedingly simple and rapid, several successive approximations generally being sufficient to locate all points. We have subjected our procedure to the following thorough checks. With the data of Run 5, the effect upon the observed relativistic rise, of defining the quantile value of the most probable pulse height at various points within the range from 0.25 to 0.375 was studied. It was found that this rise was unaffected, within the limits of statistical accuracy of our data, over this entire range.

With the data of Run 1, in which the largest numbers of mesons were observed under any given set of conditions, an extensive comparison was made of the two methods. The results of this study, which are shown in Table III, indicate the accuracy with which the two methods check one another.

A further check was made with a synthetic distribution, constructed to be almost identical with the average shape of all our distributions, with a half-width of 0.71 and a quantile value of 0.315 for the most probable value of the abscissa. A set of 100 histograms, each containing 100 events, were obtained from this distribution by a random sampling method and the distributions in the observed values of the most probable and the half-width were obtained. The results achieved, which again were well within the standard deviation, indicated that for samples of the order of 100 events or greater the quantile method leads quickly to the same results as Beherens method and that the complicated expression previously given for the variance of the width at half-maximum is correct.

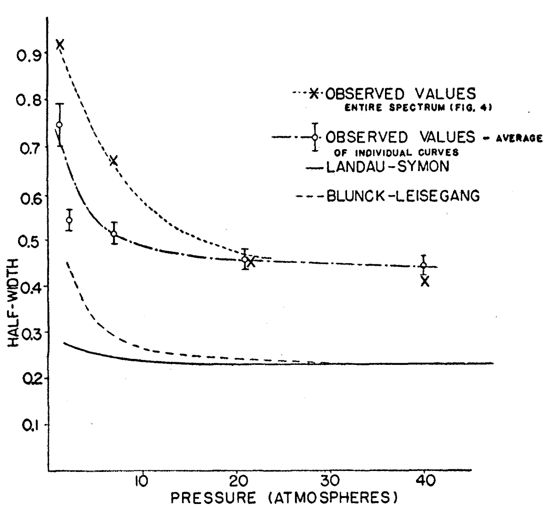

FIG. 5. Comparison of the theoretical and observed values of the width at half-maximum of the Landau distribution as a function of gas pressure.

V. RESULTS AND CONCLUSIONS

In Fig. 4 are shown four histograms for the pulse height distribution of the entire meson spectrum at four different pressures. Since each of these histograms is composed of many Landau distributions, shifted with respect to one another, no accurate curve fitting

TABLE III. Comparison of the results of the Beherens and quantile methods of analysis for the low-pressure proportional-counter data.

(Run 1) Group	No. of mesons in truncated histogram	Most probable pulse height (Arbitrary scale) Beherens	Quantile	Half-width Beherens	Quantile
A	463	25.5±0.5	26.0±0.5	0.55 ±0.05	0.54±0.06
B	503	28.8±0.5	28.5±0.5	0.515±0.04	0.53±0.05
C	950	32.0±0.5	32.3±0.4	0.50 ±0.03	0.52±0.04
D	591	34.0±0.5	35.0±0.5	0.48 ±0.04	0.53±0.05
			Weighted average	0.51 ±0.02	0.53±0.025

[19] P. Rothwell, Proc. Phys. Soc. (London) **B64**, 911 (1951).

procedure has been attempted, and the curves drawn in each case are simply the average of the most probable hand drawn curves as judged by nine independent observers. In this crude manner we have estimated the half-widths shown with each histogram in Fig. 4. Although the strong pressure dependence of the entire meson pulse height spectrum is quite apparent, the nature of the dependence does not become clear until the features of the histograms of the different energy bands are studied and the separate behaviors of the Landau distribution and of the density effect are determined.

In Fig. 5 the percentage widths at half-maximum are plotted against pressure for all our data along with the Landau-Symon and the Blunck-Leisegang theoretical widths. While the general shape of the experimental curve agrees well with the Blunck-Leisegang curve, the two appear to differ by a relatively constant amount even at our highest pressures. We know of no experimental defect in our work which would lead to an error of this type and magnitude and, as far as we are able to ascertain, all published results exhibit this tendency. This is demonstrated in Fig. 6 where we have plotted all such data available to us, the abscissa now being expressed as the logarithm of the amount of material traversed. While these data are for several gases for different experimental arrangements and for electrons as well as mesons, it is apparent that the prediction of decrease of width with increase of pressure is confirmed, but that a strong disagreement between the theoretical and observed magnitudes of the widths still exists.

The behavior of the density effect is exhibited in Fig. 7 where the most probable energy loss is plotted as a function of the mean momentum of the meson group for various values of gas pressure. The solid lines are the density-corrected curves as obtained from Sternheimer's[20] calculations. Within the limit of accuracy of our results the agreement is excellent; there may be some slight tendency towards a greater density effect than expected but we no longer consider this difference significant, contrary to a preliminary report emanating from this laboratory.[21]

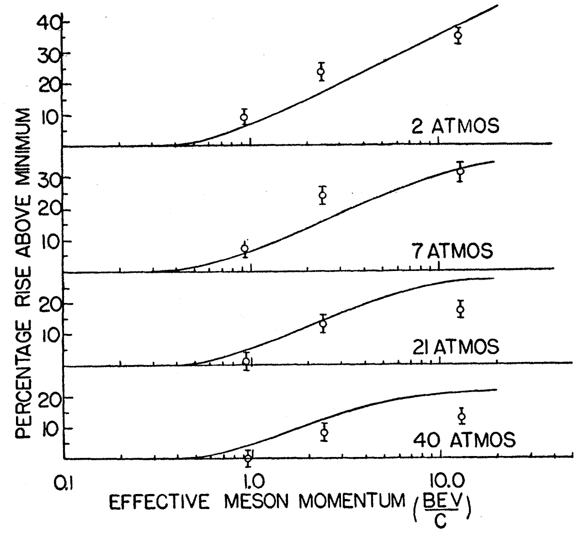

FIG. 7. Comparison of the theoretical rise of the most probable ionization energy loss (based on Sternheimer's calculations for the density effect) with the observed value at various gas pressures.

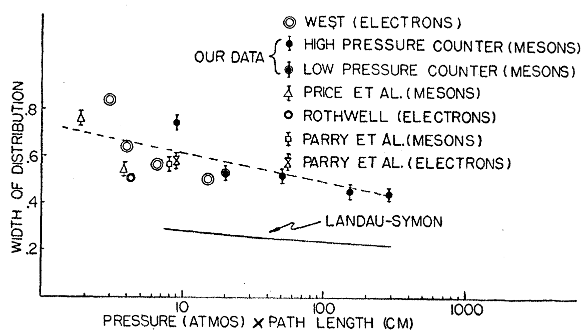

FIG. 6. Comparison of theoretical and all observed values of the width at half-maximum of the Landau distribution as a function of the logarithm of the amount of material traversed.

Finally, it should be noted that both of the above pressure effects are required to explain the very strong pressure dependence of the pulse height distribution for the entire meson spectrum (top curve of Fig. 5 and all curves of Fig. 4) and that the rapidity with which the top curves of Fig. 5 converge indicates that the Fermi plateau is reached at relatively low gas pressures.

In conclusion the authors wish to express their thanks to Mr. Nestore Di Costanzo for his invaluable assistance in the construction of the high-pressure proportional counter, to Dr. W. A. Bowers for his helpful advice in the interpretation of the results and to Dr. G. E. Nicholson, Jr. and Dr. W. Hoeffding of the Department of Statistics for their invaluable aid in the methods of the treatment of the data. Finally, we wish to thank the Oak Ridge National Laboratories of the U. S. Atomic Energy Commission for the loan of the large quantity of lead required to perform the experiment.

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