β Spectrum of Bi^{210}(RaE) and the Coupling Constants of Scalar and Tensor Interactions in β Decay

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It is shown that the only pure states of spin 1 having magnetic moments in agreement with the measured upper limit for Bi^{210} are $(h_{9/2};g_{9/2})_{1-}$ and $(h_{9/2};i_{11/2})_{1-}$. The former is inconsistent with the results of an analysis of the β spectrum of Bi^{210} with a linear combination of scalar and tensor interactions, while the latter is consistent. It follows from the acceptance of the correctness of the latter that the relative sign of the scalar and tensor coupling constants is positive.

1. INTRODUCTION

VARIOUS authors[1–3] have now shown that it is extremely difficult to explain the non-allowed shape of the β spectrum of Bi^{210} with a mixture of tensor and pseudoscalar interactions and with the assumption that the transition is 0− to 0+. On the other hand Yamada[4] has demonstrated that it is possible to obtain this shape, with a mixture of scalar and tensor interactions, for the transition 1− to 0+. It thus appears that the decay of Bi^{210} is a case of a first forbidden transition with non-allowed shape of the type 1− to 0+. From a fitting of the theoretical to the observed correction factor it is possible to obtain relations involving the coupling constants of the interactions, G_S and G_T, and the various nuclear matrix elements concerned. For most nuclei it is extremely difficult to learn much about G_S and G_T from these relations because little is known about the relative magnitudes of the nuclear matrix elements. However, the ratio of $\int\beta\boldsymbol{\sigma}\times\mathbf{r}$ to $\int\beta\mathbf{r}$, one of the quantities needed for first forbidden transitions, can be calculated very easily if the initial and final states are spectroscopically pure. The nucleus Bi^{210}, having but one nucleon of each kind apart from closed shells, is one of the nuclei most likely to satisfy this condition. In addition, it is believed that for values of atomic number as high as that of bismuth jj coupling is a good approximation. We shall proceed, then, to find out what can be learned about G_S and G_T when it is assumed that the ground states of Bi^{210} and Po^{210} are spectroscopically pure.

Before one can calculate ratios of nuclear matrix elements it is necessary to identify the configurations concerned. The most useful piece of experimental information for this purpose is the demonstration, by measurement of hyperfine structure, that the magnetic moment of Bi^{210} is exceedingly small.[5] King and Peaslee[6] have shown that the state $(h_{9/2};g_{9/2})_{1-}$ satisfies this condition. A great objection to their choice, however, is that it is expected to lie above the state of spin 0 of the same configuration. Pryce[7] has calculated the interaction energy, as a function of resultant angular momentum, for jj-coupled neutron and proton, with the assumption of forces of infinitesimal range. Such calculations are very conveniently displayed on a diagram of the sort used by de-Shalit.[8] In this diagram the ordinate is energy and the abscissa, α, is a measure of the strength of the spin-spin forces in the interaction, attractive or repulsive as η is $+1$ or -1; reasonable values of α are believed to lie in the interval 0 to 0.25. The de-Shalit diagram for the configuration $(h_{9/2};g_{9/2})$ is shown in Fig. 1(a). For all plausible values of α, the state of spin 0 is far below that of spin 1.

In Sec. 2 the magnetic moments of all reasonable ground states for Bi^{210} are calculated. Apart from $(h_{9/2};g_{9/2})_{1-}$, the only other acceptable choice is $(h_{9/2};i_{11/2})_{1-}$. The de-Shalit diagram for this configuration is shown in Fig. 1(b). For most reasonable values of α the state of spin 1 is lowest. The low-lying state of spin 10 may well be the long-lived α-emitting isomer of Bi^{210} found by Neumann *et al.*[9]

In Sec. 3 the shape of the β spectrum of Bi^{210} is considered. This shape is explicable with the ratios of matrix elements calculated for the assumption that $(h_{9/2};i_{11/2})_{1-}$ is the ground state of Bi^{210}, but not for those ratios corresponding to the assumption of $(h_{9/2};g_{9/2})_{1-}$ for this ground state. Acceptance of the first choice requires that G_S/G_T be positive.

2. MAGNETIC MOMENTS

The magnetic moment of a system consisting of a proton and a neutron whose angular momenta, j_1 and j_2, are coupled to a resultant J is desired. The orbital angular momenta of the proton and neutron will be represented by l_1 and l_2 respectively. By the methods of Racah[10] it is possible to show that the required

[1] H. Takebe, Progr. Theoret. Phys. (Japan) **10**, 673 (1953).
[2] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1315 (1954).
[3] Alaga, Kofoed-Hansen, and Winther, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd, **28**, No. 3 (1953).
[4] M. Yamada, Progr. Theoret. Phys. (Japan) **10**, 252 (1953).
[5] Fred, Tomkins, and Barnes, Phys. Rev. **92**, 1324 (1953).
[6] R. W. King and D. C. Peaslee, Phys. Rev. **94**, 795 (1954).
[7] M. H. L. Pryce, Proc. Phys. Soc. (London) **A65**, 773 (1952).
[8] A. de-Shalit, Phys. Rev. **91**, 1479 (1953).
[9] Neumann, Howland, and Perlman, Phys. Rev. **77**, 720 (1950).
[10] G. Racah, Phys. Rev. **62**, 438 (1942).

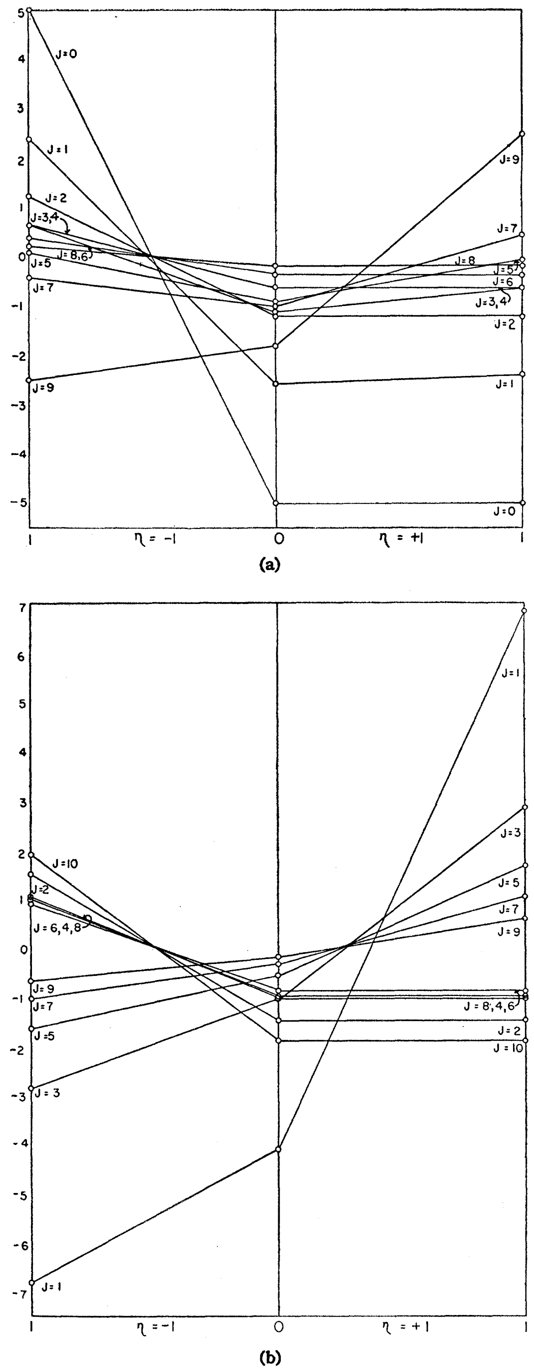

FIG. 1. De-Shalit diagrams (a) for the configuration ($h_{9/2}$; $g_{9/2}$); (b) for the configuration ($h_{9/2}$; $i_{11/2}$).

magnetic moment is

$$\mu=\frac{1}{J+1}\left[\frac{J(J+1)+j_1(j_1+1)-j_2(j_2+1)}{2j_1}\mu_p(l_1j_1)+\frac{J(J+1)+j_2(j_2+1)-j_1(j_1+1)}{2j_2}\mu_n(l_2j_2)\right],$$

where $\mu_p(l_1j_1)$ and $\mu_n(l_2j_2)$ are the magnetic moments, in the single-particle states indicated, of a proton and of a neutron respectively. The "Schmidt model" values of μ_p and μ_n will be used.[11]

The question of the choice of single-particle states has been discussed by Pryce;[7] several choices are almost equally probable. For the proton the shell-model suggests $h_{9/2}$, $f_{7/2}$, or possibly $i_{13/2}$; support for $h_{9/2}$ comes from the measured spin and magnetic moment of Bi^{209}. For the neutron we have $g_{9/2}$, $i_{11/2}$, or perhaps $k_{15/2}$; no spins of even-odd nuclei in this region are known. In Table I the magnetic moments of all states of spin 1 or 2 and of either parity which may be formed from the suggested single-particle states are listed.

Because the magnetic moments of all states of even parity are large, the ground state of Bi^{210} cannot have even parity. The assignment $1+$ would require a β spectrum of allowed shape, contrary to observation, but the assignment $2+$, with a second forbidden β-transition, was at one time thought to explain the spectrum shape.[12] The only states with small magnetic moments are those with an $h_{9/2}$ proton; all other states of odd parity have magnetic moments too large to be considered. Because of the well-known discrepancy between calculated and observed nuclear magnetic moments, none of the cases in the first column can be excluded, even if the magnetic moment is a little larger than the observed upper limit. Fortunately the states of spin $2-$ may be rejected because their β spectra, which have unique shape, are not in agreement with experiment. Only the two states of spin 1 in the first column need be given further consideration.

3. β SPECTRUM OF Bi^{210}

The experimental shape of the spectrum which we shall use is that given by Langer and Price.[13] From their curve the experimental correction factor was obtained; the maximum electron energy used was 3.29 relativistic units. Expressions for the direct and cross terms of the theoretical correction factor have been given by Greuling[14] and Pursey[15] respectively. If it is assumed that the interaction is of the form G_SS+G_TT, then in the correction factor there appear three different matrix elements, among which we write the following relations:

$$\int\beta\boldsymbol{\alpha}=\Lambda(\alpha Z/2\rho)\int\beta\boldsymbol{\sigma}\times\mathbf{r}\quad\text{and}\quad\int\beta\boldsymbol{\sigma}\times\mathbf{r}=i\epsilon\int\beta\mathbf{r}.$$

It is convenient to put $x=G_S/(G_T\epsilon)$. We shall assume that Λ and x are real; justification of this assumption

[11] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).
[12] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).
[13] L. M. Langer and H. C. Price, Phys. Rev. **76**, 641 (1949).
[14] E. Greuling, Phys. Rev. **61**, 568 (1942).
[15] D. L. Pursey, Phil. Mag. **42**, 1193 (1951).

has been given by Longmire and Messiah.[16] For ease in handling the cancellation of the large terms in the correction factor, it is convenient to examine its dependence on the two variables x and y, y being defined by the relations $y=\frac{1}{2}(1+S)\Lambda-1-x$ and $S=[1-(\alpha Z)^2]^{\frac{1}{2}}$. The correction factor C for the transition 1− to 0+ is given by expression (1).

$$C=2G_T^2(1+S)^{-1}(\alpha Z/2\rho)^2 D\epsilon^2\left|\int \mathbf{r}\right|^2, \qquad (1)$$

where

$$\begin{aligned}
D=\;&y^2+\zeta\{-2y[(2S+1)^{-1}(1+x)(W-S^2W^{-1})\\
&+(1+S)(1-x)K/6]-(1-S)(2S+1)^{-1}y^2\\
&\times[(2S+3)W+SW^{-1}]\}+\tfrac{1}{2}(1+S)\zeta^2\\
&\times[(F_1-F_3+F_5)x^2+(2F_5-F_3)xy+F_5y^2\\
&+(2m_0-F_3+F_4+2F_5)x+(F_4+2F_5)y\\
&+(F_2+F_4+F_5)+2(x-\tfrac{1}{2})^2L_1],\\
F_1=\;&(1+S)K^2/6-\tfrac{2}{3}(2S+1)^{-1}\\
&\times K(Sp^2W^{-1}-2\alpha^2Z^2W)+m_0,\\
F_2=\;&(1+S)K^2/12+\tfrac{2}{3}(2S+1)^{-1}\\
&\times K(Sp^2W^{-1}-2\alpha^2Z^2W)+m_0,\\
F_{3,4}=\;&2\alpha Z(1+S)^{-1}\{\tfrac{1}{3}K\alpha Z(2S+1)^{-1}\\
&\times[(2S+3)W+SW^{-1}]\mp n_0,\\
F_5=\;&(1-S)(1+S)^{-1}l_0,\quad \zeta=2\rho/\alpha Z.
\end{aligned}$$

For the definitions of L_0, M_0, N_0, L_1, and K see reference 12; m_0, n_0 and l_0 are the coefficients of the zeroth, first and second powers of ρ in the expansions of M_0, N_0, and L_0 (see reference 4). Values of L_1 were taken from the tables of Rose *et al.*[17] The expression for D is correct as far as the term in ρ^2. For $x=\frac{1}{2}$ expression (1) reduces to Yamada's Eq. (13), except for some small terms in ρ^2. In discussing the shape of the spectrum one need only consider D, the other factors in C being independent of energy.

We next ask what values of x and y correspond to correction factors of the observed form. To answer this question we equate the theoretical and experimental

TABLE I. Magnetic moments of possible *jj*-coupled ground states of Bi^{210}. In each compartment of the table are inserted the spins, parities and calculated magnetic moments of certain states formable from the single-particle states indicated.

Neutrons \ Protons	$h_{9/2}$	$f_{7/2}$	$i_{13/2}$
$g_{9/2}$	1−, 0.080 2−, 0.16	1−, −4.07 2−, −1.89	2+, 8.04
$i_{11/2}$	1−, −0.36 2−, +0.34	2−, −2.59	1+, 4.26 2+, 3.94
$k_{15/2}$			1−, −5.48 2−, −2.92

[16] C. L. Longmire and A. M. L. Messiah, Phys. Rev. **83**, 464 (1951).

[17] Rose, Perry, and Dismuke, Oak Ridge Laboratory Report ORNL-1459, 1953 (unpublished).

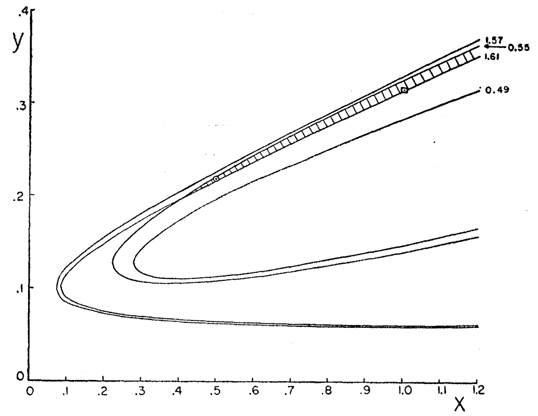

FIG. 2. The hatched area is the region of the x-y plane in which the theoretical and observed correction factors are in agreement. At any point between the hyperbolas labelled 1.57 and 1.61 the ratio of the values of the correction factor at $W=1.2mc^2$ and $2mc^2$ is in agreement with experiment; between the ellipses marked 0.49 and 0.55 the ratio of the values of the correction factor at $3mc^2$ and $2mc^2$ is in agreement with experiment. The point marked ○ corresponds to the correction factor found by Yamada, (see reference 4) that marked □ to the one found by the author.

ratios of the correction factors at $W=1.2$ and $W=2$. Since D is a quadratic function of x and y, any such relation between the values of the correction factor at two different energies must give a conic section in the x-y plane. The points in the x-y plane satisfying this particular equation lie on an hyperbola. Because there is some uncertainty in the measured ratio, we have used two values, 1.57 and 1.61. The curves labelled 1.57 and 1.61 in Fig. 2 are the corresponding hyperbolas. If a point (x,y) is to correspond to a correction factor whose value at $W=1.2$ is in correct ratio to its value at $W=2$, it must lie in the region bounded by the two hyperbolas. A second condition on x and y may be obtained by equating theoretical and observed values of the ratio of the correction factor at $W=3$ and $W=2$. If the point (x,y) is to correspond to a correction factor whose value at $W=3$ lies between 0.49 and 0.55 times the value at $W=2$, then this point must lie between the two ellipses labelled 0.49 and 0.55. To satisfy both conditions a point must lie in the hatched region of Fig. 2. Not all points in this region need, however, correspond to good correction factors; the correction factor must be tested over the whole range of W.

The fit found by Yamada[4] corresponds to the point $(x=\frac{1}{2}, y=0.219)$; the corresponding correction factor is shown in Fig. 3. We have obtained an equally good correction factor for the point (1,0.316), also shown in Fig. 3. A reasonable fit is probably possible anywhere in the hatched area.

In calculating correction factors the nuclear radius was assumed to be 8.3×10^{-13} cm. A slightly different choice for this radius would not affect the positions of the hyperbolae and ellipses of Fig. 2 very much; in particular, we have shown that when ρ lies between 4.7×10^{-13} cm and 10.7×10^{-13} cm the minimum value

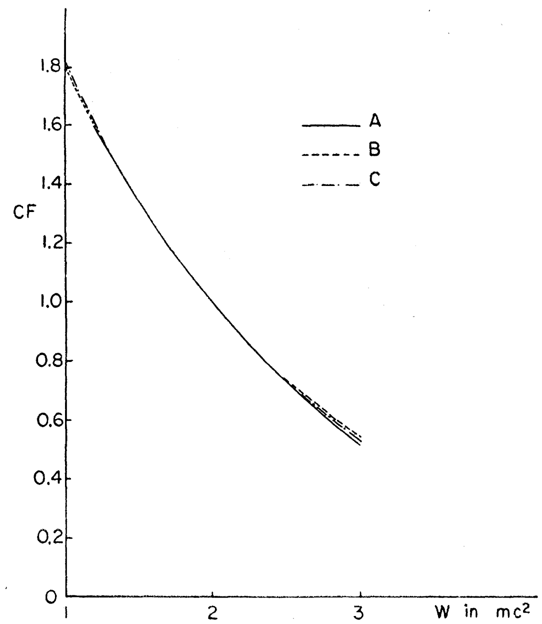

FIG. 3. Correction factor for the spectrum of Bi^{210}. The experimental curve is referred to as A, the fit found by Yamada as B, that by the author as C.

of x attained by the ellipse labelled 0.55 is greater than 0.19.

4. DISCUSSION

Examination of Fig. 2 leads to the conclusion that the theoretical correction factor will not explain the observed spectrum unless x is greater than 0.35. This limit will now be used to show that the assignment of $(h_{9/2}; g_{9/2})_{1-}$ to the ground state of Bi^{210} is inconsistent with its spectrum. Recall that $x=G_S/(G_T\epsilon)$. We are assured by the work of Kofoed-Hansen and Winther[18] and by that of Blatt[19] that $|G_S/G_T|$ is not appreciably greater than unity. It has been proved by Pursey[15] and by Ahrens and Feenberg[20] that

$$\epsilon=j_p(j_p+1)-j_n(j_n+1)-l_p(l_p+1)+l_n(l_n+1).$$

In this formula l and j are the orbital and total angular momentum quantum numbers of a single-particle state; the subscripts p and n refer to proton and neutron respectively. Since for the configuration $(h_{9/2}; g_{9/2})$ the value of ϵ is -10, the corresponding value of x cannot possibly be as large as 0.35, and therefore this choice of configuration cannot explain the β spectrum. For the assumption $(h_{9/2}; i_{11/2})$, ϵ is $+1$, and the observed spectrum is obtainable for a wide range of values of G_S/G_T above 0.35. Thus, if the ground state of Bi^{210} is a pure state, then that pure state must be $(h_{9/2}; i_{11/2})_{1-}$; the state required by the spectrum analysis is the same as that preferred by the energy-level calculation.

A comparison of values of Λ predicted with those found by spectrum-fitting is useful. Rose and Osborn[21] expect Λ to lie in the range 1 to 3, with upper value considered somewhat more plausible. Pursey[15] and Ahrens and Feenberg[20] came to a similar conclusion. The value of Λ obtained from fitting the spectrum of Bi^{210} depends on the value of x assumed; for $x=\frac{1}{2}$ we find $\Lambda=1.92$ and for $x=1$ we find $\Lambda=2.59$. Since the two values of Λ are in equally good agreement with prediction, it is not possible to use them to determine G_S/G_T. On the other hand, the agreement suggests that the fit found for the spectrum is the correct one.

The positiveness of the ratio G_S/G_T, which follows from the positiveness of x and ϵ, is not in agreement with earlier determinations.[22,23] Insofar as the work of Peaslee is based on the spectrum of Bi^{210} it is invalidated by the work of Rose and Osborn.[2] We find too that the first forbidden correction factor for G_SS+G_TT used by this author does not agree with expression (1). The Japanese authors reach the same conclusion as Peaslee from a study of almost the same decaying nuclei, but without finding it necessary to correct the nuclear matrix elements for the presence of pseudoscalar forces. Of the four nuclei which Morita *et al.* consider, only Cs^{137} definitely requires a negative value of G_S/G_T. We believe that ratios of nuclear matrix elements calculated with pure jj-coupled wave functions are more likely to be correct for Bi^{210} than for Cs^{137}. If it is accepted as certain that Λ must be positive, then one of these four spectra, that of Fe^{59}, actually requires a positive value for the ratio G_S/G_T.

The identification of the configuration of the ground state of Bi^{210} enables one to make other shell-model assignments. The transition Pb^{209} to Bi^{209} has $\log ft$ equal to 5.51, calculated for a maximum electron energy of 0.620 Mev[24] with the aid of tables of the Fermi function.[25] A first forbidden transition with such a small ft-value cannot correspond to cancellation of terms in the correction factor as in Bi^{210}. Such cancellation, if we accept the assignment $(h_{9/2}; i_{11/2})_{1-}$ to Bi^{210}, occurs for $\epsilon=+1$. The ground state of Bi^{209} is known to be $h_{9/2}$. The values of ϵ for the two possible assignments to Pb^{209}, $g_{9/2}$ and $i_{11/2}$, are -10 and $+1$ respectively. The latter must be rejected. This means that the 127th is in different states in Pb^{209} and in Bi^{210}. We expect, therefore, that the $i_{11/2}$ level in Pb^{209} should have a very small excitation energy, and that the radial integral fixing the magnitude of the neutron-proton interaction energy[7] should be larger for the pair $(h_{9/2}; i_{11/2})$ than for the pair $(h_{9/2}; g_{9/2})$. Since the h and i radial wave functions are those which have no nodes and since the

[18] O. Kofoed-Hansen and A. Winther, Phys. Rev. **86**, 428 (1952).
[19] J. M. Blatt, Phys. Rev. **89**, 83 (1953).
[20] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).
[21] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1326 (1954).
[22] D. C. Peaslee, Phys. Rev. **91**, 1447 (1953).
[23] Morita, Fujita, and Yamada, Progr. Theoret. Phys. (Japan) **10**, 630 (1953).
[24] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).
[25] *Tables for the Analysis of Beta Spectra*, Applied Mathematics Series No. 13 (National Bureau of Standards, Washington, D. C., 1952).

g has a node, the required behavior of the radial integral is not unreasonable.

Another β transition of interest is that of Pb^{210}(RaD) into Bi^{210}. This decay goes, as far as is known, entirely to an excited state of Bi^{210} at 47 kev. It is consistent with the conclusions of the preceding paragraph to make the assignment $(h_{9/2}; g_{9/2})_{0-}$ to this excited state. The ft-value for this transition should, then, be very near that for the decay of Pb^{209}. Using a maximum electron energy of 18 kev,[26] we find $\log ft = 5.49$. This same assignment will also explain the absence of a transition to the ground state of Bi^{210}, for it requires that the ground state of Pb^{210} be $(\ ; g_{9/2}^2)_0$. A transition from this initial state to the ground state of Bi^{210} would require a two-particle jump.

From a more detailed examination of the ft-values of transitions between nuclei in the neighborhood of Pb^{208} an estimate of the values of Λ and of G_S/G_T can be obtained. Because the ft-values for the transitions $Pb^{209} \rightarrow Bi^{209}$ and $Pb^{210} \rightarrow Bi^{210*}$ are almost equal, we shall equate the expressions for the correction factors corresponding to the two transitions. For the former the correction factor is given by (1) with the addition of the term corresponding to the matrix element $\int \boldsymbol{\sigma}\cdot\mathbf{r}$ occurring in the tensor interaction. The correction factor for this matrix element was given by Konopinski and Uhlenbeck;[12] we shall use only the term of order ρ^{-2}, *viz.*,

$$\frac{2}{1+S}(\alpha Z/2\rho)^2 G_T^2 \left|\int \boldsymbol{\sigma}\cdot\mathbf{r}\right|^2.$$

It is well known that only the matrix element $\int \boldsymbol{\sigma}\cdot\mathbf{r}$ contributes to transitions of the type $0+ \rightarrow 0-$. As is shown in the appendix, the correction factor corresponding to this matrix element for the two-particle system is exactly twice its value for the same particle jump in the one-particle system. Methods for the calculation of $\int \boldsymbol{\sigma}\cdot\mathbf{r}$ and $\int \mathbf{r}$ in the single-particle system are well known; convenient formulas are given by King and Peaslee.[27] After cancellation of common factors including the radial integrals, the equation obtained by equating the correction factors for the two transitions becomes

$$1+\frac{100}{99}\left(\frac{1+S}{2}\Lambda-1+\frac{1}{10}\frac{G_S}{G_T}\right)^2 \simeq 2. \tag{2}$$

A similar pair of decaying nuclei are Tl^{207} and Tl^{206}, for which the calculated $\log ft$-values are 5.16 and 5.18 respectively. In each case a $p_{\frac{1}{2}}$ neutron changes into an $s_{\frac{1}{2}}$ proton; calculations of the Pryce type indicate that the spin of the ground state of Tl^{206} is $0-$. Equation (3) for the two Tl isotopes corresponds to Eq. (2) for the two Pb isotopes.

$$1+\frac{4}{3}\left(\frac{1+S}{2}\Lambda-1-\frac{1}{2}\frac{G_S}{G_T}\right)^2 \simeq 2. \tag{3}$$

If it may be assumed that Λ has roughly the same value in the two cases, then (2) and (3) may be solved simultaneously. There are four solutions. Two of these require $|G_S/G_T|$ to be greater than 3, contrary to the evidence from allowed transitions. A third may be excluded because of the smallness of the value of Λ, *viz.*, 0.025. For the remaining solution we have $\Lambda = 2.2$ and $G_S/G_T = 0.23$. This value of Λ, which is not very sensitive to the approximations involved, is in good agreement with the values obtained from fitting the spectrum of Bi^{210}. The smallness of the value of G_S/G_T found by this method is of little significance, because the approximations introduce a very large uncertainty in this value; even the sign of the ratio is not really well determined.

5. POSSIBILITY OF CONFIGURATION MIXING

All the arguments and discussion in the preceding parts of this paper are founded on the assumption that the ground state of Bi^{210} is spectroscopically pure. Though there is, we believe, evidence for the assumption, it must be admitted that the possibility that this ground state is a mixture of $(h_{9/2}; i_{11/2})_{1-}$ and $(h_{9/2}; g_{9/2})_{1-}$ cannot be excluded. Such a mixture is not inconsistent with the spectrum shape. Let a and b refer to $(h_{9/2}; i_{11/2})_{1-}$ and $(h_{9/2}; g_{9/2})_{1-}$ respectively; we write $a\psi_a + b\psi_b$ for the wave function of the ground state of Bi^{210}. Instead of having $x = G_S/G_T$ as we did for the pure state ψ_a, we now have $x = (G_S/G_T)F$ for the mixed state. The function F is $(1+t)(1-10t)^{-1}$, where t is the ratio of $b(f|r|b)$ to $a(f|r|a)$; the letter f represents the final state $(h_{9/2}^2;\ )_0$. If we are allowed complate freedom in the choice of t, then F may take any value, and nothing can be said about the ratio G_S/G_T. If, on the other hand, it can be shown that t is negative, then the only negative values for F lie in the range $-1/10$ to 0; since these values are inconsistent with what is known of the magnitudes of x and G_S/G_T, it can then be said that G_S/G_T must be positive. By well known methods the expression for t can be reduced to $(3\sqrt{6})^{-1}bI_1/aI_2$, where

$$I_1 = \int_0^\infty R(1h)R(2g)r^3dr, \quad I_2 = \int_0^\infty R(1h)R(1i)r^3dr;$$

the R's stand for normalized radial wave functions. It may also be shown, by consideration of configuration interaction in a system consisting of a jj-coupled proton and neutron, that b/a has the same sign as $-(a|V|b)$, where V is the interaction potential between the proton and the neutron. Using the theory of Pryce,[7] one may

[26] Insch, Balfour, and Curran, Phys. Rev. 85, 805 (1952).
[27] R. W. King and D. C. Peaslee, Phys. Rev. 94, 1284 (1954).

show that $(a|V|b)$ has the same sign as

$$I_3=\int_0^\infty R^2(1h)R(2g)R(1i)r^2dr.$$

Thus t is negative if I_1, I_2 and I_3 all have the same sign. These integrals all do have the same sign when computed with wave functions belonging to either a square well of infinite depth or to an harmonic oscillator. It is unlikely, therefore, that the sign of G_S/G_T is negative, even if there is an appreciable admixture of ψ_b in the ground state of Bi^{210}.

The evidence against there being an appreciable amount of ψ_b in this ground state is the nonexistence of a transition to it from Pb^{210}. Though the transition to the ground state has a forty-fold energy advantage over the transition to the 47-kev level, the former has not been observed; the upper limit on its intensity is about a tenth of the intensity of the latter. How can one explain why the transition to the excited state is at least 400 times as strong as the transition to the ground state? Some of the difference is certainly caused by the selection rule which permits $\int\boldsymbol{\sigma}\cdot\mathbf{r}$ to be operative in the former and not in the latter, but the effect of the other matrix elements is of roughly the same magnitude, unless there is a cancellation of the large terms in the correction factor. According to our analysis of the spectrum of Bi^{210}, this cancellation takes place for some value of x greater than 0.35. If the ground state of Pb^{210} is pure $(\ ;g_{9/2}{}^2)_0$, then only the component ψ_b in the wave function of Bi^{210} has nonzero matrix elements, and x, therefore, is less than 0.1. To produce the value of x corresponding to cancellation one must form the wave function of Pb^{210} of just the right proportions of $(\ ;i_{11/2}{}^2)_0$ and $(\ ;g_{9/2}{}^2)_0$. Such a coincidence is, we believe, unlikely. Furthermore, it is not certain that the low ft-value for the decay of Pb^{210} to Bi^{210*} is consistent with an appreciable amount of $(\ ;i_{11/2}{}^2)_0$ in Pb^{210}.

A possible test of the amount of ψ_b in the ground state of Bi^{210} is to be found in the measurement of the half-life of the 47-kev state of Bi^{210}. From a comparison of the calculated[28] and measured[29] ratios of the numbers of L_{I}, L_{II} and L_{III} conversion electrons, it follows that the 47-kev γ radiation is undoubtedly $M1$. The usual selection rules permit $M1$ transitions only between states of the same configuration. The half-life of the 47-kev state will be, then, inversely proportional to b^2. The Weisskopf[11] half-life for a 47-kev $M1$ transition, corrected for an internal conversion coefficient of about 20 and amended to include a better estimate of the matrix element, is 0.3×10^{-11} sec. If the nonappearance of the Pb^{210} to Bi^{210} ground state transition is taken to mean that $b^2\leq1/400$, then the half-life of the 47-kev state $\geq1.2\times10^{-9}$ sec, which is perhaps just measurable. A shorter half-life will not necessarily mean that b^2 is greater than 1/400, however, because the transition may be caused by the sort of process suggested by Austern and Sachs.[30]

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APPENDIX

A comparison of the matrix elements for the transition

$$;\ (l_1j_1)^2{}_{J_1}\rightarrow(l_2j_2;\,l_1j_1)_{J_2}+\beta^- \quad (4)$$

with those for the transition

$$l_1j_1\rightarrow l_2j_2+\beta^- \quad (5)$$

is desired. As is customary we shall use the symbol $|\int A|^2$ to represent the sum over all possible final states of the square of the matrix element of the operator A. In general A will be composed of a sum of terms each one of which depends on the coordinates of only one particle of the system. Each term is the product of an operator on the isotopic spin wave function and of a tensor operator of order L. The wave function of the initial state of (4) is constructed precisely as was done in reference 21. For the final state, however, we do not choose an eigenfunction of the isotopic spin operator. Instead we taken that combination of isotopic spin eigenfunctions corresponding to isotopic spins 0 and 1 which ensures that the particle in the state l_1j_1 is always a proton and that in the state l_2j_2 is always a neutron. Such a wave function, rather than any eigenfunction of isotopic spin, we believe to be appropriate to a proton-neutron system in a nucleus in the neighborhood of Pb^{208}.

Using the methods of Racah,[10] it is not difficult to show that

$$\left|\int A(2)\right|^2=2(2j_1+1)(2J_2+1)\times W^2(j_2J_2j_1J_1;\,j_1L)\left|\int A(1)\right|^2. \quad (6)$$

The arguments 1 and 2 refer to the one-particle and to the two-particle systems respectively. For the special case $J_1=0$, (6) becomes

$$\left|\int A(2)\right|^2=2\left|\int A(1)\right|^2\delta_{J_2L}. \quad (7)$$

Formulas (6) and (7) differ from the similar formulas of Brysk[31] and of Rose and Osborn[21] in the presence of a 2 as a factor on the right-hand side. This difference has its origin in the different definitions of the final state.

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