states is small.[10] A time-dependent perturbation causes the transition of the isotopic spin state, and if H_c is the Coulomb interaction and $\hbar\omega = E_1 - E_2$ is the difference in energy between the mixing states, the amplitude of mixing is

$$a(t) = \frac{H_c \sin\omega t}{\hbar\omega}$$

$$\approx \frac{H_c t}{\hbar} = \frac{H_c}{\Gamma} \cdot \frac{t}{T} \text{ (when } t \ll 2\pi/\omega).$$

[10] L. A. Radicati, Proc. Phys. Soc. (London) A66, 139 (1953); A67, 39 (1954).

Here $\Gamma = h/T$, where T is the life of the decaying state; if this is much larger than H_c, decay takes place before mixing proceeds. Γ is of order 1 Mev or more and H_c is probably a fraction of one Mev. Hence the isotopic spin purity is not affected by close-lying intermixable levels, which would cause considerable mixing in the case of a stationary perturbation.

Further studies on this subject are in progress.

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Energy of the Ground State of Li^6

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The binding energy of Li^6 is calculated by using a wave function of the exponential type with a central Yukawa interaction, both neutral and symmetric exchange characters being considered. The neutral interaction leads to a large excess binding energy whereas the symmetric case gives much too small a value, for a particular set of nuclear parameters. The contribution to the energy from the central part of the neutral and symmetric types of Pease-Feshbach interaction is also determined.

1. INTRODUCTION

In this paper, the energy of the ground state of Li^6 is determined by using a wave function of the exponential type for a central Yukawa interaction, both the neutral and symmetric cases being considered. The contribution to the energy of the central part of the Pease-Feshbach[1,2] type of interaction—neutral and symmetric—is also evaluated. The justification for dealing only with the central part of the interaction at this stage arises from the treatment of H^3 and He^4, where it is necessary to construct as good a wave function as possible for the S-state,[3] before taking into account the tensor part of the interaction.

It has previously been established that a two-body interaction, involving a mixture of central and tensor forces with a Yukawa well-shape, can give, for a set of nuclear parameters which fits the low-energy two-body data, reasonable values for the binding energies of both the triton[1,2] and the alpha particle.[3] Since the results for the two-body problem are independent of the exchange nature of the forces and the three- and four-body energy values differ very little if a neutral or symmetric interaction is used, it is of importance to determine whether the effect of both neutral and symmetric interactions of the above type is the same for the lightest bound p-shell nucleus, Li^6.

[1] R. L. Pease and H. Feshbach, Phys. Rev. 81, 142 (1951).
[2] R. L. Pease and H. Feshbach, Phys. Rev. 88, 945 (1952).
[3] J. Irving, Proc. Phys. Soc. (London) A66, 17 (1953).

If the nuclear interaction is assumed to be of the two-body type, saturation requirements for heavy nuclei indicate that the interaction is of a symmetric character. Kronheimer[4] has in fact shown that the exchange nature of the interaction is evident in the case of the light nucleus Be^9. Using single-particle Gauss wave functions and taking only the lowest state (^2P) of highest orbital symmetry of the $(1s)^4(2p)^5$ configuration, he has obtained an excess binding energy with the neutral Pease-Feshbach type of interaction. For the charge-symmetric interaction on the other hand, the $(1s)^4(2p)^5$ term does not describe a bound state. Edwards[5] has found, in the case of the Be^8 nucleus, that for a symmetric central interaction with a Gauss well-shape the system is not bound. Morpurgo,[6] using a similar interaction, has calculated the energy of Li^6, treating the system as composed of a deuteron and an alpha particle. He finds that the energy is a minimum when the deuteron is at infinity, that is, the system is not bound.

Other calculations on the binding energy of Li^6 have been carried out by Inglis,[7] Margenau,[8] and Tyrrell,[9] using a central two-body Gauss interaction, and by

[4] E. H. Kronheimer, Phys. Rev. 90, 1003 (1953).
[5] S. F. Edwards, Proc. Cambridge Phil. Soc. 48, 652 (1952).
[6] G. Morpurgo, Nuovo. cimento 10, 473 (1953).
[7] D. Inglis, Phys. Rev. 51, 531 (1937).
[8] H. Margenau and K. Carroll, Phys. Rev. 54, 705 (1938).
[9] W. Tyrrell, Jr., Phys. Rev. 56, 250 (1939).

Humblet[10] for a Yukawa interaction. Feingold[11] has calculated the level spacing in Li^6 using a central tensor interaction of the Gaussian type. He obtains qualitative agreement with the experimental values for the spacing, but he has not shown that his wave function gives a reasonable value for the binding energy of the Li^6 nucleus. Finally, Morita and Tamura[12] have, independently of the authors, calculated the binding energy of Li^6 with the same exponential wave function for the neutral interaction only, using an elegant adaptation of the method of Jahn and Van Wieringen.[13]

2. METHOD OF CALCULATION

Since Li^6 has spin 1, the ground state on the basis of the shell model is taken to be $^3S_1$, in the usual notation. This state is represented by the wave function, of a similar form to that introduced by Feingold,

$$\begin{aligned}\Psi(^3S_1)=\{&[S(34)\alpha_4A(56)+S(35)\alpha_5A(64)\\&+S(36)\alpha_6A(45)]A(12)\alpha_3+[S(14)\alpha_4A(56)\\&+S(15)\alpha_5A(64)+S(16)\alpha_6A(45)]A(23)\alpha_1\\&+[S(24)\alpha_4A(56)+S(25)\alpha_5A(64)\\&+S(26)\alpha_6A(45)]A(31)\alpha_2\}\phi=\chi\phi,\end{aligned}\quad(2.1)$$

where particles 1, 2, 3 are neutrons and 4, 5, 6 are protons; $S(ij)=\mathbf{r}_{Gi}\cdot\mathbf{r}_{Gj}$, G being the center of mass of the system; $A(ij)=\alpha_i\beta_j-\alpha_j\beta_i$, α and β denoting the usual spin wave functions; ϕ denotes the radial part of the wave function, which is assumed to have the form

$$\phi=N_S^{\frac{1}{2}}\exp\left[-\alpha\left(\sum_{i>j}^{6}{}_1\,r_{ij}^2\right)^{\frac{1}{2}}\right],\quad(2.2)$$

$N_S^{\frac{1}{2}}$ being the normalizing factor for the complete wave function (2.1) and α a variation parameter. The choice (2.2) is suggested by the success of a wave function of this type in accounting for the energy of He^4. Feingold assumes a Gaussian radial dependence.

We consider the interaction of the form

$$\mathcal{V}^c(r_{ij})=PV_0\{[(1-\tfrac{1}{2}g)+\tfrac{1}{2}g(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)]\times\exp(-\kappa_cr_{ij})/(\kappa_cr_{ij})\},\quad(2.3)$$

where $P=-1$ and $g=g_N$ for the neutral case; $P=(\mathbf{T}_i\cdot\mathbf{T}_j)/3$ and $g=g_S=2-3g_N$ for the symmetric case. This interaction is of the same form as the central part of the more general interaction involving the tensor force, considered by Feshbach and Schwinger[14] in the two-body problem and by Pease and Feshbach in the triton problem.

The variational formula,

$$E\leqslant W=\int\Psi^*H\Psi d\tau\Big/\int\Psi^*\Psi d\tau,\quad(2.4)$$

where

$$H=-\frac{\hbar^2}{2M}\sum_{i=1}^{6}\Delta_i+\sum_{i>j}^{6}{}_1\,\mathcal{V}^c(r_{ij})$$

and $d\tau$ includes summation over the spins and spatial integrations, is used to obtain an approximate value W to the true energy E. Substituting the wave function (2.1) and the interaction (2.3) in (2.4), the spin matrix elements are first of all evaluated in the usual way. This is tedious but straightforward. The space integrations are then carried out, by means of the transformation of coordinates given by

$$\begin{aligned}\mathbf{x}_1&=(\mathbf{r}_2-\mathbf{r}_1)/\sqrt{2},\\\mathbf{x}_2&=(\mathbf{r}_6-\mathbf{r}_5)/\sqrt{2},\\\mathbf{x}_3&=(\mathbf{r}_6+\mathbf{r}_5-\mathbf{r}_2-\mathbf{r}_1)/2,\\\mathbf{x}_4&=(2/\sqrt{3})\{(\mathbf{r}_3+\mathbf{r}_4)/2-(\mathbf{r}_6+\mathbf{r}_5+\mathbf{r}_2+\mathbf{r}_1)/4\},\\\mathbf{x}_5&=(\mathbf{r}_4-\mathbf{r}_3)/\sqrt{2},\\\mathbf{X}&=(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4+\mathbf{r}_5+\mathbf{r}_6)/6,\end{aligned}\quad(2.5)$$

from which

$$\sum_{i>j}^{6}{}_1\,r_{ij}^2=6(x_1^2+x_2^2+x_3^2+x_4^2+x_5^2).\quad(2.6)$$

The method of reducing the resulting spatial integrals and their subsequent evaluation is described in the Appendix. The formulas for W for the neutral and symmetric interactions are now given.

(a) Neutral Interaction ($P=-1$)

$$\begin{aligned}\left\langle\chi^*\left|\sum_{i>j}^{6}{}_1\,\mathcal{V}^c(r_{ij})\right|\chi\right\rangle=&-36V_0[\exp(-\kappa_cr_{12})/(\kappa_cr_{12})]\\&\times\{[S^2(12)+6S^2(34)+8S^2(13)-12S(13)S(34)\\&-6S(34)S(35)-6S(13)S(14)+8S(14)S(35)\\&+4S(13)S(24)-2S(14)S(24)-4S(12)S(13)\\&+2S(12)S(34)+S(34)S(56)]+2g_N[-3S^2(34)\\&-2S^2(13)+6S(13)S(34)+3S(34)S(35)\\&+2S(13)S(14)-6S(14)S(35)+S(13)S(24)\\&-S(14)S(24)]\}.\end{aligned}\quad(2.7)$$

Using the transformation (2.5) and integrating over the

[10] J. Humblet, Physica 14, 285 (1948).
[11] A. M. Feingold, thesis, Princeton University, 1952 (unpublished).
[12] M. Morita and T. Tamura, Tokyo University of Education (private communication).
[13] H. A. Jahn and H. Van Wieringen, Proc. Roy. Soc. (London) **A209**, 502 (1951).
[14] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951).

angles, we then obtain the potential energy term,

$$\left\langle \Psi^* \left| \sum_{i>j}^{6} {}_1\, \mathcal{V}^c(r_{ij}) \right| \Psi \right\rangle$$

$$= -[N_S V_0 (4\pi)^5/(\sqrt{2}\kappa_c)]$$

$$\times \int \{[9x_1^4+42x_1^2x_2^2+267x_2^4/2-199x_2^2x_3^2/2]$$

$$-6g_N[2x_1^2x_2^2+21x_2^4-17x_2^2x_3^2]\}$$

$$\times \exp\left[-\beta\left(\sum_{i=1}^{5} x_i^2\right)^{\frac{1}{2}} - \sqrt{2}\kappa_c x_1\right] x_1 x_2^2 x_3^2 x_4^2 x_5^2$$

$$\times dx_1 dx_2 dx_3 dx_4 dx_5 \quad (\text{with } \beta = 2\alpha\sqrt{6})$$

$$= -\frac{5\cdot 11\cdot 13\cdot 17}{9\cdot 2^{14}} V_0 c^{19}\{[369A_1+588B_1+504C_1] -6g_N[54A_1+28B_1]\}, \quad (2.8)$$

on reducing the multiple integrals. Here $c=2\sqrt{3}\alpha/\kappa_c$ and A_1, B_1, C_1 are single integrals which are given in the Appendix.

The kinetic energy [App. (b)] reduces to

$$3\hbar^2\alpha^2/M = \hbar^2\kappa_c^2 c^2/(4M). \quad (2.9)$$

The Coulomb energy [Appendix (c)] becomes

$$(5^2\cdot 11\cdot 13\cdot 17 e^2 \kappa_c c)/2^{17}. \quad (2.10)$$

(b) Symmetric Interaction $[P=(\mathbf{T}_i\cdot\mathbf{T}_j)/3]$

$$\left\langle \chi^* \left| \sum_{i>j}^{6} {}_1\, \mathcal{V}^c(r_{ij}) \right| \chi \right\rangle$$

$$= -36V_0[\exp(-\kappa_c r_{12})/(\kappa_c r_{12})]$$

$$\times\{[2S^2(34)+S^2(12)-4S(12)S(13)$$

$$+2S(12)S(34)+2S(14)S(15)+2S(13)S(23)$$

$$-4S(13)S(34)-2S(34)S(35)+S(34)S(56)]$$

$$+2g_S[S^2(34)-S(14)S(25)-2S(13)S(34)$$

$$+2S(14)S(35)-S(34)S(35)$$

$$+S(13)S(23)]\}. \quad (2.11)$$

Hence

$$\left\langle \Psi^* \left| \sum_{i>j}^{6} {}_1\, \mathcal{V}^c(r_{ij}) \right| \Psi \right\rangle$$

$$= -\{N_S V_0 (4\pi)^5/(\sqrt{2}\kappa_c)\}$$

$$\times \int \{[9x_1^4-30x_1^2x_2^2+99x_2^4/2-63x_2^2x_3^2/2]$$

$$+2g_S[-6x_1^2x_2^2+21x_2^4-17x_2^2x_3^2]\}$$

$$\times \exp\left[-\beta\left(\sum_{i=1}^{5} x_i^2\right)^{\frac{1}{2}} - \sqrt{2}\kappa_c x_1\right] x_1 x_2^2 x_3^2 x_4^2 x_5^2$$

$$\times dx_1 dx_2 dx_3 dx_4 dx_5 \quad (\text{with } \beta = 2\alpha\sqrt{6})$$

$$= -\frac{5\cdot 11\cdot 13\cdot 17}{9\cdot 2^{14}} V_0 c^{19}[(153A_1-420B_1+504C_1) +6g_S(18A_1-28B_1)]. \quad (2.12)$$

The kinetic and Coulomb energy terms are given by (2.9) and (2.10), respectively.

3. RESULTS AND CONCLUSIONS

For the central Yukawa interaction with $1/\kappa_c=1.17\times 10^{-13}$ cm, $V_0=67.3$ Mev, and $g_N=0.155$ (i.e., $g_S=1.535$), we obtain

$$W=-101 \text{ Mev and } E_{\text{Coul}}=4 \text{ Mev, i.e., } E=-97 \text{ Mev}$$

for the neutral case; and

$$W=-4.5 \text{ Mev and } E_{\text{Coul}}=2 \text{ Mev, i.e., } E=-2.5 \text{ Mev}$$

for the symmetric case. E(experimental) $=-32.0$ Mev.

It is evident that the neutral interaction leads to a collapsed nucleus. The very small value for the energy obtained for the symmetric interaction indicates that the assumed form of the wave function is rather poor. The introduction of a tensor force term into the two-body interaction has the effect of reducing the central well depth and consequently the central force binding energy. For the Pease-Feshbach interaction with

$$1/\kappa_c=1.184\times 10^{-13} \text{ cm}, \quad V_0=46.1 \text{ Mev}, \quad g=-0.004, \quad \gamma=0.54, \quad 1/\kappa_t=1.67\times 10^{-13} \text{ cm},$$

the central force contribution is

$$W=-17.7 \text{ Mev and } E_{\text{Coul}}=2.8 \text{ Mev, i.e., } E=-14.9 \text{ Mev},$$

in the neutral case, but the symmetric form of this interaction does not give a bound state.

The tensor force will, of course, contribute to the binding energy. The work of Lyons and Feingold,[15] in which the D state of maximum symmetry, involving a mixture of configurations, is considered, indicates a contribution ~12 Mev to the binding energy. Other D states will give additional binding so that the above neutral central-tensor interaction probably gives excess binding. The symmetric interaction, on the other hand, will still yield little, if any, binding energy for the above wave function. Morita and Tamura claim a fit for the Li^6 ground state energy with a neutral interaction, for which the central depth is 49.3 Mev. Such an interaction will not only give an excess binding energy for the triton and the alpha particle, but will also give an excess for Li^6, if a D state of the same form as that of Feingold is assumed. Morita and Tamura take the D state from the $(1s)^4(2p)^2$ configuration alone and find that it slightly reduces the total energy, in contrast with the result of Feingold and Lyons. Moreover, the work of Cohen[16] on the binding energy of He^4 indicates that the Pease-Feshbach interaction, of a symmetric

[15] D. H. Lyons and A. M. Feingold, Phys. Rev. 95, 606 (1954).
[16] L. Cohen, thesis, University of Manchester, 1953 (unpublished).

character, with $V_0=42.7$ Mev is the more correct for a reasonable fit in the two-, three- and four-body problems. It should be emphasized that calculations of the Li^6 ground state energy should be carried out with a symmetric interaction.

APPENDIX

(a) Normalization of the Wave Function

The normalization of the wave function involves integrals of the form

$$I=\int_0^\infty\cdots\int_0^\infty f(x_1^2+x_2^2+\cdots+x_n^2)\times x_1^{\alpha_1}x_2^{\alpha_2}\cdots x_n^{\alpha_n}dx_1dx_2\cdots dx_n,$$

which, on using the transformation

$$u_r=x_r^2,\quad r=1,2,\cdots,n, \tag{1}$$

becomes

$$\frac{1}{2^n}\int_0^\infty\cdots\int_0^\infty f(u_1+u_2+\cdots+u_n)\times u_1^{\beta_1}u_2^{\beta_2}\cdots u_n^{\beta_n}du_1du_2\cdots du_n,$$

where

$$\beta_r=(\alpha_r-1)/2,\quad r=1,2,\cdots,n.$$

The further transformation,

$$u_1+u_2+\cdots u_{n-r+1}=y_1y_2\cdots y_r,\quad r=1,2,\cdots,n, \tag{2}$$

yields, with elementary integration,

$$I=\frac{\Gamma(\beta_1+1)\cdots\Gamma(\beta_n+1)}{2^n\Gamma(\beta_1+\cdots+\beta_n+n)}\int_0^\infty y_1^{(\beta_1+\cdots+\beta_n+n-1)}f(y_1)dy_1. \tag{3}$$

For the particular form

$$f(x_1^2+\cdots+x_n^2)=f(y_1)=\exp(-\beta y_1^{\frac{1}{2}}),$$

we have, from (3),

$$I=\pi^{-\frac{1}{2}}\beta^{-n}\left(\frac{2}{\beta}\right)^{\alpha_1+\cdots+\alpha_n}\Gamma\left(\frac{\alpha_1+\cdots+\alpha_n+n}{2}\right)\times\Gamma\left(\frac{\alpha_1+1}{2}\right)\cdots\Gamma\left(\frac{\alpha_n+1}{2}\right). \tag{4}$$

Now for Li^6,

$$\langle\Psi^*|\Psi\rangle=36\int[S^2(34)-2S(34)S(35)+S(34)S(15)]\phi^2dv$$
$$=36(4\pi)^5N_S\int_0^\infty\cdots\int_0^\infty\exp[-(2\alpha\surd6)(x_1^2+\cdots+x_5^2)^{\frac{1}{2}}](x_1^4-x_1^2x_2^2)x_1^2dx_1\cdots x_5^2dx_5=1.$$

Hence, using (4) with $n=5$, we obtain

$$N_S=\frac{36\alpha^{19}\surd6}{35\pi^7}. \tag{5}$$

(b) Kinetic Energy

Using the transformation of coordinates given by (2.5), the kinetic energy is

$$T=(\hbar^2/2M)\sum_{j=1}^{5}\int\{\nabla_{\mathbf{x}_j}(\chi\phi)\}^2d\tau,$$

where ϕ is the radial, and χ the orbital-spin part of the wave function.

Since $\nabla^2\chi=0$, it follows that

$$T=(\hbar^2/2M)\sum_{j=1}^{5}\int\chi^2(\nabla_{\mathbf{x}_j}\phi)^2d\tau.$$

On carrying out the spin summations we obtain

$$T=(\hbar^2/2M)36\int\{S^2(34)-2S(34)S(35)+S(34)S(15)\}(\nabla_{\mathbf{x}_j}\phi)^2dv.$$

Now, if ϕ has the form (2.2), then

$$(\nabla_{\mathbf{x}_j}\phi)^2=6\alpha^2\phi^2,$$

i.e.,

$$T=(\hbar^2/2M)36\int\{S^2(34)-2S(34)S(35)+S(34)S(15)\}\phi^2dv\times6\alpha^2.$$

Hence

$$T=3\hbar^2\alpha^2/M \tag{6}$$

since

$$36\int\{S^2(34)-2S(34)S(35)+S(34)S(15)\}\phi^2dv$$

is the normalization integral.

(c) Coulomb Energy

The Coulomb energy is given by

$$E_{\mathrm{Coul}}=3\langle\Psi^*|e^2/r_{56}|\Psi\rangle$$
$$=36e^2\int\{2S^2(35)+S^2(34)-S(35)S(36)-2S(34)S(35)-2S(35)S(15)-S(34)S(14)+S(35)S(16)+2S(35)S(14)\}(\phi^2/r_{56})dv,$$

which gives, when one uses the preceding integrals,

$$E_{\mathrm{Coul}}=(5^2\cdot11\cdot13\cdot17e^2c\kappa_c)/2^{17}. \tag{7}$$

(d) Potential Energy

The potential energy terms involve integrals of the form

$$J=\int_0^\infty\cdots\int_0^\infty f(x_1^2+\cdots+x_n^2)g(x_n)\times x_1^{\alpha_1}\cdots x_n^{\alpha_n}dx_1\cdots dx_n. \quad (8)$$

When one uses the transformations (1) and (2), Eq. (8) reduces to

$$\frac{\Gamma(\beta_1+1)\Gamma(\beta_2+1)\cdots\Gamma(\beta_{n-1}+1)}{2^n\Gamma(\beta_1+\cdots+\beta_{n-1}+n-1)}\times\int_0^\infty dy_1 f(y_1)y_1^{(\beta_1+\beta_2+\cdots+\beta_n+n-1)}\times\int_0^1 dy_2 g\{y_1^{\frac{1}{2}}(1-y_2)^{\frac{1}{2}}\}(1-y_2)^{\beta_n}, \quad (9)$$

where

$$\beta_r=(\alpha_r-1)/2.$$

For the particular forms,

$$f(y_1)=\exp(-\beta y_1^{\frac{1}{2}}),$$

$$g\{y_1^{\frac{1}{2}}(1-y_2)^{\frac{1}{2}}\}=\exp\{-\kappa' y_1^{\frac{1}{2}}(1-y_2)^{\frac{1}{2}}\}/\{\kappa' y_1^{\frac{1}{2}}(1-y_2)^{\frac{1}{2}}\},$$

we obtain from (9), on carrying out the integration with respect to y_1,

$$J=\frac{\Gamma(\beta_1+1)\cdots\Gamma(\beta_{n-1}+1)\Gamma\{2(\beta_1+\cdots+\beta_n+n)-1\}}{\kappa' 2^{n-1}\Gamma(\beta_1+\cdots+\beta_{n-1}+n-1)}\times\int_0^1 dy_2\frac{y_2^{\beta_1+\cdots+\beta_{n-1}+n-2}(1-y_2)^{\beta_n-\frac{1}{2}}}{[\beta+\kappa'(1-y_2)^{\frac{1}{2}}]^{[2(\beta_1+\cdots+\beta_n+n)-1]}}. \quad (10)$$

Using (10), with $n=5$, the potential energy is determined and involves the integrals A_1, B_1, C_1 where

$$A_1=\int_0^1(1-y^2)^7 y/(y+c)^{18}dy, \quad (11)$$

$$B_1=\int_0^1(1-y^2)^6 y^3/(y+c)^{18}dy, \quad (12)$$

$$C_1=\int_0^1(1-y^2)^5 y^5/(y+c)^{18}dy, \quad (13)$$

with $c=2\sqrt{3}\alpha/\kappa_c$. Evaluating A_1, B_1, C_1 by elementary methods, we obtain

$$(5\cdot 9\cdot 11\cdot 13\cdot 17\cdot 2^4)c^{16}(1+c)^{10}A_1=[109\,395c^8+504\,126c^7+1\,103\,040c^6+1\,472\,130c^5+1\,293\,930c^4+759\,330c^3+288\,288c^2+64\,350c+6435],$$

$$(3\cdot 5\cdot 7\cdot 11\cdot 13\cdot 17\cdot 2^4)c^{14}(1+c)^{11}B_1=[34\,465c^7+106\,203c^6+147\,213c^5+123\,735c^4+67\,155c^3+23\,265c^2+4719c+429],$$

$$(7\cdot 9\cdot 11\cdot 13\cdot 17\cdot 2^4)c^{12}(1+c)^{12}C_1=[7293c^6+13\,788c^5+12\,303c^4+6504c^3+2115c^2+396c+33].$$

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