Internal and External Bremsstrahlung Accompanying the Beta Rays of P^{32}

K. LIDÉN AND N. STARFELT
Radiation Physics Department, University of Lund, Lund, Sweden
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The spectral distribution and the total energy of the internal bremsstrahlung of P^{32} were measured from 20 to 1000 kev with a NaI(Tl) scintillation spectrometer. Special care was taken to diminish and correct for bremsstrahlung from the apparatus and the surroundings. The experimental curve agrees with theory from 20 to 50 kev and in the region 50–120 kev shows a weak bump in a smoothed curve; from 120 to 500 kev the curve lies 15 percent, at 800 kev 45 percent, and at 1000 kev 70 percent above a theory based on allowed β transition; thus, above 120 kev, the experimental curve disagrees with the theory for both allowed and forbidden β transitions. The total energy per β decay in the internal bremsstrahlung was found to exceed the calculated value by 21 percent.

Using the same experimental arrangements the external bremsstrahlung emitted when the P^{32} β rays are completely stopped in carbon, aluminum, iron, tin, and lead was also measured. For light elements the observed spectrum was found to agree closely with the theory by Bethe and Heitler as improved by Elwert, but for heavy elements and high energies to exceed the calculated values considerably, e. g., for lead at 500 and 1000 kev by a factor of 1.4 and 2.0, respectively.

I. INTRODUCTION

A. Internal Bremsstrahlung (IB)

ALTHOUGH Aston[1] is widely credited with the discovery of this weak continuous nuclear γ radiation, the first to produce convincing experimental evidence of such radiation were Stahel and Kipfer,[2] v. Droste,[3] and Gray and Hinds.[4] It is true that Aston discussed the possibility of IB from RaE, but an analysis of his absorption measurements strongly suggested that the intensity of the external bremsstrahlung (EB) from the lead absorption foils and from the platinum wire of the source exceeded that of IB by a factor of 5 or more.

The first to measure the P^{32} IB was Sizoo *et al.*,[5] but the investigation by Wu[6] gave more reliable results. Wu studied EB+IB for various elements with an integration ion chamber. Extrapolation to $Z=0$ gave $IB/(IB+EB_{Al})=\frac{1}{4}$. When the experimental difficulties are considered, it appears that the agreement, claimed by Wu, between the calculated and the experimental total IB energy must be accepted with caution.

With the poor energy resolution of the ion chambers and G-M tubes used in these earlier investigations, it was not possible to study the spectral distribution with any degree of precision. Novey,[7] Madansky and Rasetti,[8] and Bolgiano *et al.*[9] were the first to use scintillation spectrometers in the investigations of IB from P^{32}, Y^{91}, and RaE for photon energies <250 kev. Renard[10] applied the proportional counter technique in the region of 3–30 kev. All of these authors found the shape of the spectrum to agree fairly well with theory.

The theory of IB put forward by Knipp and Uhlenbeck[11] and by Bloch[12] for allowed β transitions, was extended by Chang and Falkoff[13] and Madansky *et al.*[14] for other transitions.

Recently Michalowicz[15] and Goodrich and Payne[16] published IB measurements of P^{32} up to 400 and 900 kev, respectively. The shape of Goodrich and Payne's experimental spectrum agreed well with the theory for the allowed β transition, but not with the first forbidden scalar type given by Chang and Falkoff.[13]

In an earlier study of the scintillation spectrometer method used in the measurements of continuous γ and x-radiation, we stressed the importance of proper correction of the experimental results. The measurements made in the present investigation of the P^{32} IB were corrected in the way described in this earlier paper[17] (LS).

B. External Bremsstrahlung (EB)

This type of x-rays, emitted by β rays being stopped in matter, was first observed with certainty by Gray,[18] who used β rays from RaE. Chadwick[19] and Hess and Lawson[20] also studied EB from RaE. Later Gray[21] measured the fraction of the β energy emitted as EB when the β rays from RaE were completely stopped

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[9] Bolgiano, Madansky, and Rasetti, Phys. Rev. **89**, 679 (1953).
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[11] J. K. Knipp and G. E. Uhlenbeck, Physica **3**, 425 (1936).
[12] F. Bloch, Phys. Rev. **50**, 272 (1936).
[13] C. S. Wang Chang and D. L. Falkoff, Phys. Rev. **76**, 365 (1949).
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[15] A. Michalowicz, J. phys. et radium **15**, 156 (1954).
[16] M. Goodrich and W. B. Payne, Phys. Rev. **94**, 405 (1954).
[17] K. Lidén and N. Starfelt, Arkiv Fysik **7**, 427 (1954). This paper will be referred to as LS.
[18] J. A. Gray, Proc. Roy. Soc. (London) **A85**, 131 (1911); and **A86**, 513 (1912).
[19] J. Chadwick, Phil. Mag. **24**, 594 (1912).
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[21] J. A. Gray, Phys. Rev. **19**, 430 (1922); and **25**, 237 (1925).

in lead, iron, or paper. Stahel and Kipfer[2] observed that the IB of RaE was slightly more penetrating than the EB from Pb+RaE; they reported a total intensity ratio of $IB/EB_{Pb}=1/7$. Von Droste[3] who used a GM detector, was the first to estimate the spectral distribution of the RaE EB. Using high pressure ion chambers and UX β rays Staheland Mass a[22] studied the Z-dependence of the total EB energy emitted and found it to agree with Bethe and Heitler's[23] theory.

Wu[6] found, that the shape of the EB spectrum of P^{32} was independent of Z and that, roughly speaking, the total EB energy measured agreed with that calculated according to Bethe and Heitler.[23]

An experimental study of the P^{32} EB spectrum of Pb was made by Siegbahn and Slätis,[24] who used a β spectrograph and Pb converter. They also observed the superimposed characteristic x-rays from Pb. We studied these K x-rays quantitatively with a NaI scintillation spectrometer.[25] Similar measurements have since been reported by Michalowicz.[15]

Wyard[26] found reasonable agreement between approximate calculations from the Bethe-Heitler theory and absorption measurements of the EB spectrum of the P^{32} β rays stopped in brass. This spectrum was analyzed more closely by Goodrich *et al.*[27] who used a NaI(Tl) scintillation spectrometer. Applying Wyard's approximation they found, that from 50 to 1100 kev the experimental and calculated curves agreed within 10 percent.

The theory of the EB at nonrelativistic energies was developed by Sommerfeld[28] and Elwert.[29] The relativistic case was treated by Bethe and Heitler[23] by the Born approximation. Heitler[30] pointed out that this theory does not give reliable results for high-energy photons and heavy elements. Considering the lack of accurate measurements of true EB spectra, we made a detailed study of the EB spectra of the P^{32} β rays stopped in various elements. The experimental results were compared with the Bethe-Heitler theory in its original form and with the improvement suggested by Elwert as well as with Wyard's spectrum.

A preliminary report of part of the present investigation has been published.[31]

II. EXPERIMENTAL MEASUREMENTS

In an earlier set of measurements a single-channel pulse analyzer as described by Elmore and Sands[32] was used in conjunction with an EMI 5311 photomultiplier and a NaI(Tl) crystal 1.5 cm in diameter and 1.5 cm thick. Later, for the purpose of comparison and control, some of the measurements were repeated with a new differential precision pulse-height analyzer, of the type described by Francis and Bell,[33] a Du Mont 6292 photomultiplier and the same crystal as before (Fig. 1). The crystal was optically coupled to the photocathode by a thin film of Dow Corning DC200 silicone oil (10^5 centistokes) and then enclosed hermetically in a 0.15-mm Al reflector coated on the inside with a thin film of MgO. The electronic components were connected to a mains stabilizer, the temperature of the laboratory was thermostatically controlled and the apparatus started several hours before measuring a spectrum. Despite these precautions, frequent checking of the energy calibration and the channel width was necessary to secure measurements of satisfactory accuracy.

In the IB measurements the direct β rays were absorbed in a 1.02-g/cm^2 beryllium disk halfway between the source and the detector, which were 16 cm apart. The cylindric hole in the lead collimator (50 mm thick and covered by 1.1 g/cm^2 Perspex) was 20 mm in diameter. The EB produced in the Perspex absorber and in the air by the β rays was almost completely eliminated by the lead collimator and the steel cylinder (wall thickness 8 cm) enclosing the photomultiplier. An estimation showed that even at the highest IB photon energy measured (1.0 Mev) such EB disturbance was well below 5 percent.

In order to avoid the K x-rays from the lead collimator, the hole was lined with 0.7 mm Sn+0.2 mm Cu and the detector side with 1.5 mm Sn+0.5 mm Cu.

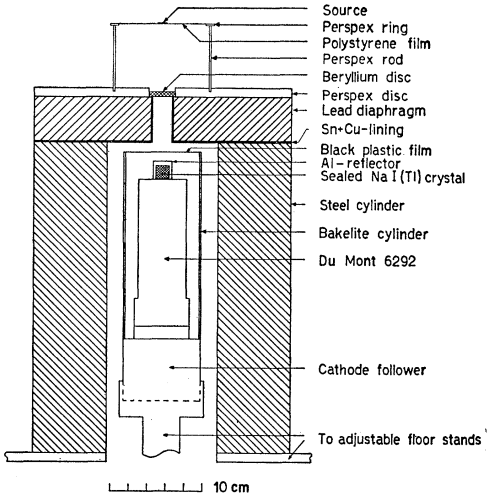


FIG. 1. Arrangement of source, detector, and shielding for measurement of the P^{32} internal and external bremsstrahlung spectra.

[22] E. Stahel and J. Massa, Helv. Phys. Acta **14**, 325 (1941).
[23] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).
[24] K. Siegbahn and H. Slätis, Arkiv Mat. Astron. Fysik **34A**, No. 6 (1948).
[25] K. Lidén and N. Starfelt, Arkiv Fysik **7**, 193 (1954).
[26] S. J. Wyard, Proc. Phys. Soc. (London) **A65**, 377 (1952).
[27] Goodrich, Levinger, and Payne, Phys. Rev. **91**, 1225 (1953).
[28] A. Sommerfeld, Ann. Physik **11**, 257 (1931).
[29] G. Elwert, Ann. Physik **34**, 178 (1939).
[30] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1954), third edition, Chap. V, Sec. 25.
[31] K. Lidén and N. Starfelt, Arkiv Fysik **7**, 83 (1954).
[32] W. C. Elmore and M. Sands, *Electronics Experimental Techniques* (McGraw-Hill Book Company, Inc., New York, 1949), first edition, p. 228.
[33] J. E. Francis and P. R. Bell (unpublished).

The P^{32} sources were prepared from absolutely calibrated P^{32} solutions, obtained from the Atomic Energy Research Establishment, Harwell, and containing about 5 mC/ml. The P^{32} was produced by $S^{32}(n,p)P^{32}$ and chemically processed. Before calibration, 10 μg/mC KH_2PO_4 was added as carrier to the originally carrier-free and chemically pure (no spectrographically detectable impurities) P^{32} solution (about 1 percent P^{33}). Using an infrared lamp a few drops were carefully evaporated on a 1.0-mg/cm^2 polystyrene film mounted on a Perspex ring of outer diameter 100 mm, inner diameter 90 mm, and thickness 0.5 mm, supported by three 2-mm Perspex rods. In this way a microscopically uniform spread of the sources (area 0.3–1 cm^2) was secured. Finally the sources were covered with 0.1 mg/cm^2 plastic films ("cocoon").

As the handling of a large source support on the balance offered some difficulties, the absolute amount of P^{32} was determined as follows. The bremsstrahlung emitted with the source between 1.0-g/cm^2 Al radiators was compared with the radiation from a similar source, prepared directly on an Al radiator by weighing and evaporating and covered with 1 mg/cm^2 polystyrene and another Al radiator.

The EB+IB spectra emitted from radiators of carbon, aluminum, iron, tin, and lead, which completely stopped the β rays from the P^{32} sources, were also measured.

The aforementioned IB geometry was also used for the EB measurements, which permitted most of the corrections to be made as for a parallel beam. All the IB and the EB spectra were studied with at least two different sources. The channel width employed varied between 4.5 kev for low photon energies and 80 kev for the highest energies measured. The intensity of the background spectrum, observed with the source removed, was low, 10 percent or less of the IB spectrum at 1000 kev. The statistical error for all experimental points was less than 3 percent. This required 8–10 hours' counting time per channel at the highest energies.

The energy calibration was frequently checked with a suitable set of the following lines: Na^{22} 1277 kev, Cs^{137} 662, Na^{22} 511, Au^{198} 412, I^{131} 364, Cr^{51} 330, Ce^{141} 145, Pb K_α 75 (P^{32}+Pb radiator), and Ce K_α 35 kev (P^{32}+Ce radiator).

III. CALCULATION OF THE TRUE SPECTRA FROM THE OBSERVED PULSE-HEIGHT DISTRIBUTIONS

A. Detector Corrections

The observed pulse-height distributions were corrected for background, dead-time of the analyzer, resolving power, Compton distribution, K x-ray escape, backscatter from the photomultiplier, and γ efficiency (LS).[17]

The ratio $k(E_\gamma)$ (LS, p. 450) between the number of pulses in the photopeak and the total number of observed pulses of a single γ-ray line with the energy E_γ was now determined with greater accuracy (Fig. 2). The result agrees within a few percent with values calculable from data given by Maeder *et al.*[34]

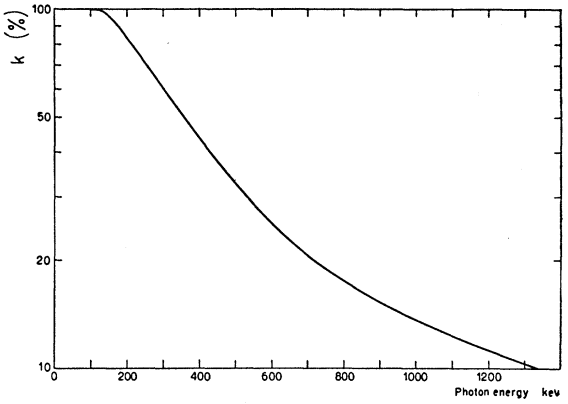


FIG. 2. The experimental ratio $k(E_\gamma)$ of the number of pulses in the photoelectric peak to the total number of pulses of a single γ ray line with the energy E_γ.

The correction for K x-ray escape and resolving power were made as follows. When corrected for K x-ray escape the distribution $N_3(\epsilon_\gamma)$ of the scintillations in the crystal gives the distributions $N_4(\epsilon_\gamma)$. A first approximation $N_4^I(\epsilon_\gamma)$ of $N_4(\epsilon_\gamma)$ was estimated. The corresponding distribution $N_3^I(\epsilon_\gamma)$ was then calculated. After that, using the experimentally determined resolving power, the resulting pulse-height distribution $N_2^I(\epsilon)$ was calculated according to LS. When $N_2^I(\epsilon)$ thus obtained deviates only slightly from the experimental curve, $N_2(\epsilon)$, the following equation is valid:

$$N_4(\epsilon_\gamma)=N_4^I(\epsilon_\gamma)N_2(\epsilon)/N_2^I(\epsilon).$$

Otherwise $HN_4^I(\epsilon_\gamma)$ was improved upon and the calculations were repeated. The experimentally determined line widths at half-height at 35 and 75 kev were 37 and 28 percent, respectively.

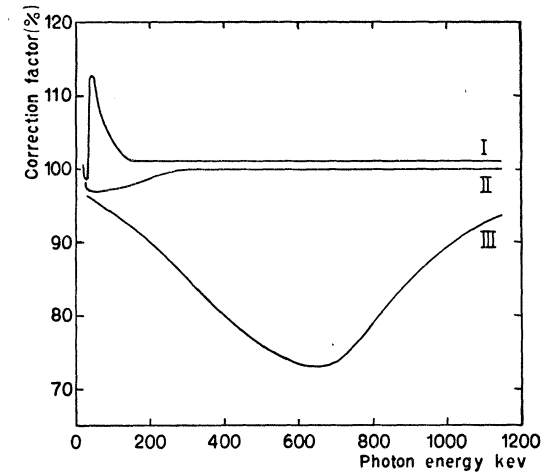
The calculated correction factors of the IB spectrum are shown in Fig. 3. The factors of the EB spectra were of roughly the same shape.

If two pulses ϵ' and ϵ'' coincide, the size of the resulting pulse ϵ is given by $\epsilon \leq \epsilon'+\epsilon''$. In a spectrum where the intensity decreases rapidly with increasing photon energy this effect can increase the counting rate at high energies considerably, if the total number of pulses is too high. In the present investigation with a pulse length of about 3 μsec this source of error was less than 0.5 percent.

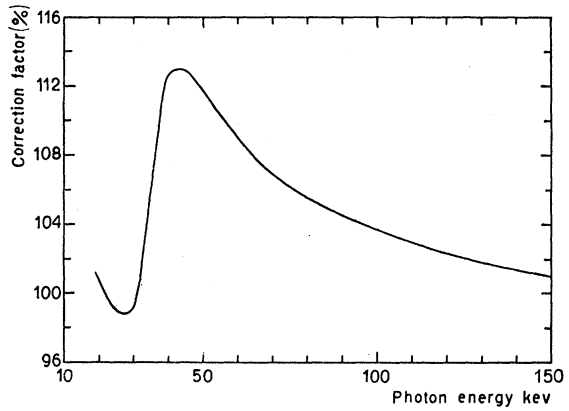
B. Correction for Scattering and Absorption

The contribution of scattered radiation from the Be absorber and from the hole of the collimator was calculated by the Klein-Nishina formula on the assumption that all scattering processes (single scat-

[34] Maeder, Müller, and Wintersteiger, Helv. Phys. Acta **27**, 3 (1954).



(a)



(b)

FIG. 3(a). Correction factors of the scintillation pulse-height distribution of the internal bremsstrahlung of P^{32} for: (I) resolving power+K x-ray escape effect, (II) backscatter from the photomultiplier, (III) Compton electron distribution. (b). Correction factor (I) of Fig. 3(a) in the region 20–150 kev.

tering) in the hole occur only in a transverse plane of the wall exactly intermediate between the end openings (Fig. 4).

The IB and EB spectra were also corrected for absorption in the β ray absorber and the reflector (Fig. 4).

The true distribution of the P^{32} bremsstrahlung EB+IB, escaping perpendicularly from the radiator, is shown in Fig. 5. The lead spectrum was not corrected for resolving power below 150 kev.

C. Correction of the IB Measurements for EB

The IB pulse-height distribution includes EB emitted non-isotropically from the β absorber. Since the imperfection of the theory for the angular distribution of the thick target bremsstrahlung could not permit calculation of the corresponding correction, it was deduced from measurements obtained with β absorbers of Be, C, and Al. The values for $N_Z(E_\gamma)/\delta_Z$, where $N_Z(E_\gamma)$ is the measured photon intensity at energy E_γ and δ_Z the transmission of the absorber, were plotted as a function of Z for a number of energies. A linear extrapolation to $Z=0$ gave the correct value of the IB pulse-height distribution $N_{Z=0}(E_\gamma)$ for E_γ. The correction factor $N_{Z=0}(E_\gamma)/N_{Z=4}(E_\gamma)$ is given in Fig. 4; it decreased gradually from 7 percent at low energies to zero at 1000 kev.

The total correction factor for conversion of the pulse-height distribution into the true IB spectrum is given in Fig. 6. Above 200 kev its shape is dictated mainly by the effective photoelectric absorption of the crystal.

The P^{32} IB photon spectrum from 20 to 1000 kev is shown in Fig. 7.

D. Correction of the EB for Absorption and Scattering in the Radiator and for IB

It was assumed that all the photons emanate from the center of the source. The error introduced by this approximation was negligible at those energies where the absorption in the radiator was small and probably did not exceed 5 percent, even when the absorption was as much as 75 percent.

In the energy region around the peak at 75 kev in the bremsstrahlung spectrum of lead the correction for the absorption was more complicated, as this peak consists mainly of K x-rays, superimposed on a bremsstrahlung spectrum strongly attenuated by absorption in the radiator. These K x-rays are emitted not only after photoelectric absorption of bremsstrahlung photons but also after ionization in the K shell by β particles.[25] Because of the limited resolving power of the spectrometer this K x-ray peak could not be separated with sufficient accuracy from the continuous spectrum. This peak was instead regarded as a single line and was substituted by a Gaussian distribution which was then subtracted from the spectrum. In this way pulses caused by photons of

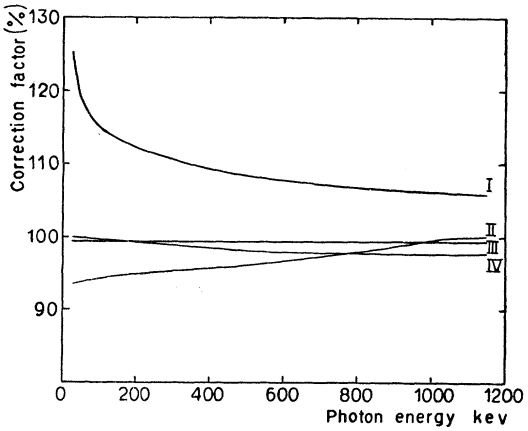


FIG. 4. Correction factors of the scintillation pulse-height distribution of the internal bremsstrahlung of P^{32} for: (I) absorption in the β absorber and the reflector, (II) external bremsstrahlung from the β absorber, (III) scattering from the β absorber, and (IV) scattering from the wall of the collimator hole.

energies below the K absorption edge of lead were removed from the spectrum above 100 kev.

The probability of a photon being scattered in the radiator towards the crystal was calculated for various primary photon directions according to the Klein-Nishina formula. The spectral distribution of the singly scattered radiation originating in the continuous spectrum under consideration was calculated by a procedure similar to that used in the calculation of the Compton electron distribution in the crystal.

The EB spectra (Fig. 9) were then obtained by subtraction of the true IB spectrum.

IV. SURVEY OF THE ERRORS

The accuracy of the measurements and of the calculations of the corrections for the IB are apparent from Table I. As to the spectra of EB+IB the various errors were approximately of the same order; in addition to the errors presented in Table I those in the calculations of scattering and absorption in the radiator must be taken into account. The calculated total rms deviation (Table I) does not include the statistical error and the error in the absolute calibration (2 percent).

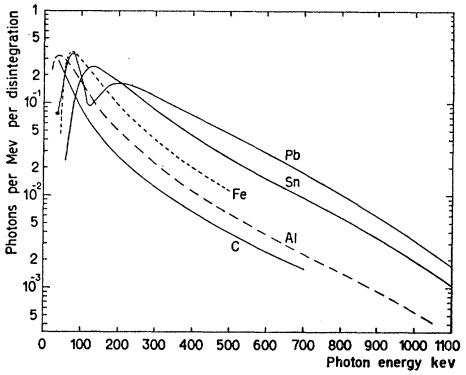


FIG. 5. The true spectra of the P^{32} bremsstrahlung photons escaping perpendicularly from radiators of about 1.0 g/cm² enclosing the source; number of photons per β disintegration per 1-Mev energy internal *vs* photon energy.

The error of the calculation of the Compton electron continuum was due mainly to the approximation of the Compton electron distribution for a certain γ-ray energy E_γ to a straight line. The estimation of the error was based on calculations performed using a slope of this line representing the greatest possible deviation from the approximation used.

The error in the crystal efficiency was due mainly to the inaccuracy of the measurement of $k(E_\gamma)$ caused above all by the radiation scattered from the collimator and the photomultiplier. The values, according to White,[35] of the absorption coefficients of sodium and iodine were taken as correct, but, as pointed out by Maeder *et al.*,[34] the possibility of systematic errors cannot be excluded.

[35] G. R. White, Natl. Bur. Standards Rept. No. 1003, May 13, 1952 (unpublished).

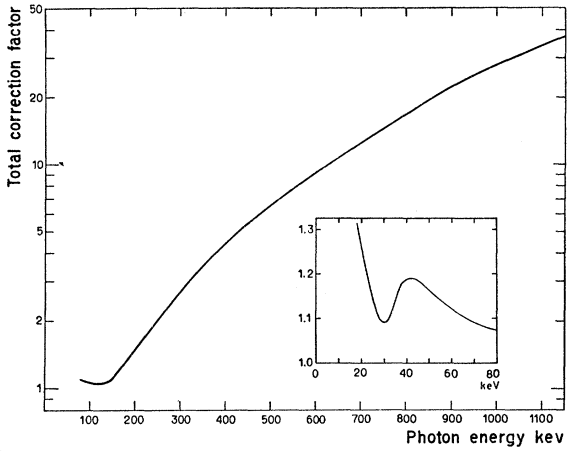


FIG. 6. The total correction factor converting the scintillation pulse-height distribution of the internal bremsstrahlung of P^{32} into the true spectrum.

Some of the aforementioned errors (backscatter, crystal efficiency, EB disturbance) can be diminished by using better geometry. However, the geometry is determined by the strength of the source which cannot be increased too much because of EB then produced in the source.

The resolving time of the pulse analyzer and the related maximum pulse rate is also a limiting factor. Finally, it may be mentioned that the accuracy of the measurements may be further improved upon by the use of such electronic components in the spectrometer as provide perfect long-term stability.

V. RESULTS AND DISCUSSION

A. Internal Bremsstrahlung

According to Knipp and Uhlenbeck's theory,[11] β decay and radiation are regarded as independent processes. The probability $S(k)dk$ for the emission of a

TABLE I. Sources of error in the P^{32} internal bremsstrahlung spectrum (in percent). Errors in the corrections for backscatter (<1 percent), external bremsstrahlung from the β absorber (1 percent), and scattering in the collimator (<1 percent) are included in the total error, which does not, however, include counting statistics or the inaccuracy of the absolute calibration of the source (2 percent).

E_γ kev	Channel width	Energy calibration	Resolving power + K x-ray escape	Compton electron continuum	Crystal efficiency	Total (rms)	Counting statistics
30	2	1	2	0.5	0.5	3.3	1.5
50	2	1	1	0.5	0.5	2.9	2
100	2	0.5	0.5	0.5	1	2.6	2
200	2	0.5	0.5	0.5	2	3.2	3
500	1	0.5	0.5	2	3	4	3
700	1	1	0.5	5	3	6	3
1000	1	3	0.5	3	4	6	3

photon with energy between k and $k+dk$ is then

$$S(k)=\int_{1+k}^{E_m} P(E_0)\Phi(E_0,k)dE_0, \qquad (1)$$

where $P(E_0)dE_0$ is the probability for the emission of a β particle with total energy E_0; E_m is the end-point energy of the β spectrum. The expression for Φ is

$$\Phi=\frac{\alpha p}{\pi p_0 k}\left[\frac{E_0^2+E^2}{E_0 p}\ln(E+p)-2\right], \qquad (2)$$

where $E=E_0-k$; p_0 and p are the momenta of the radiating electron corresponding to the energies E_0 and E, respectively, and α is the fine structure constant, 1/137. All energies are in units of mc^2 and all momenta in units of mc. However, Chang and Falkoff[13] pointed out "that there appears to be no good reason to suppose that the probability for the overall process should be simply the product of the probability for each," but they prove that it yields exactly the same result as the more rigorous second-order perturbation method for allowed transitions. In both methods Born's approximation is used, and the effect of the nuclear charge on the radiation is thus neglected. According to Novey,[7] Hellund has shown that this effect is unimportant in the low-energy photon region investigated by Novey. However, if the experimentally determined β spectrum is used as $P(E_0)$ in (1), the effect of the nuclear charge is, to a certain extent, taken into consideration.

The solid curve in Fig. 7 shows $S(k)$ calculated from (1) by taking $P(E_0)$ from the β spectrum of P^{32} according to Jensen *et al.*[36] The experimental points are the result of four runs with the same geometry, using two P^{32} sources of 2.5 and 1.0 mC, prepared from two different Harwell solutions.

From 20 to 50 kev the experimental and calculated curves agree with an accuracy of ± 5 percent. The weak bump in the region 50–120 kev with its maximum about 15 percent above a smoothed curve is more readily recognized in an IB energy *vs* photon energy representation. This bump is much weaker than in Novey's IB spectrum,[7] because he did not eliminate the Pb K x-rays from the collimator hole. Such a bump can also be discerned in the spectra given by Bolgiano *et al.*[9] and by Michalowicz.[15] It is as yet not possible to say whether the bump is ascribable to an impurity or to P^{32} itself. Apart from this, no evidence of nuclear γ rays was found. From 120 to 500 kev the shapes of the experimental and calculated spectra show close agreement but disagree definitely above 600 kev. On an absolute basis the experimental points in the region 120–500 kev lie about 15 percent above theory, at 800 kev 45 percent, and at 1000 kev 70 percent. These differences are inconsistent with the experimental accuracy. Since the same general results have been obtained here[37] for the IB spectrum of S^{35} (end point energy 168 kev), it seems reasonable to suppose, that the improvement of the theory worked out by Nilsson[38] for the S^{35} IB spectrum is also of some importance for P^{32}.

In contrast to the present result Goodrich and Payne[16] found the shape of the experimental spectrum to agree with theory from 100 to 900 kev. The discrepancy can be partly ascribed to an additional contribution of low-energy EB photons, from the air, from the Lucite β absorber, and from the 5-mg/cm^2 P^{32} source used by these authors. Furthermore, the experimentally measured spectral distribution of the EB contribution from the β absorber differs from ours.

In this connection it might be mentioned that earlier measurements of the shape of the P^{32} IB spectrum with poorer geometry described elsewhere[25] showed good agreement with theory from 50 to 800 kev, but the experimental spectrum was about 40 percent above the calculated. Later, closer analysis indicated, that there was a greater amount of EB from the Perspex β absorber than that found before. As the ratio EB/IB is greatest at low energies, this additional contribution happened to smooth out the shape of the IB spectrum with consequent good agreement with theory.

In other papers of the P^{32} IB spectrum, only the low-energy end was studied. The results reported by Novey[7] and by Bolgiano *et al.*[9] were discussed to some extent in LS. Furthermore, these authors as well as Michalowicz[15] underestimated the EB contribution from the

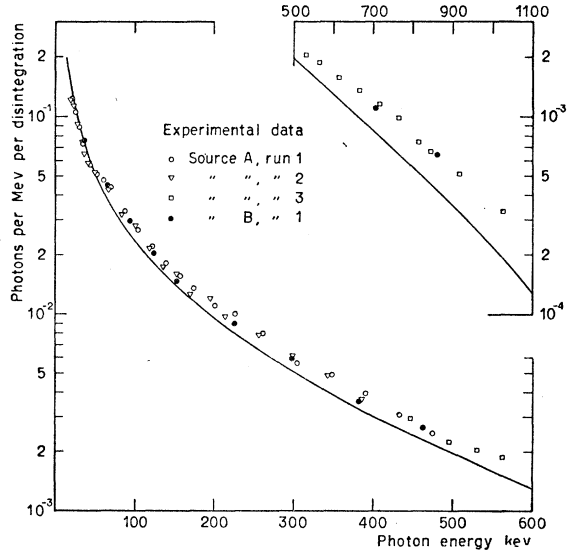


FIG. 7. The P^{32} internal bremsstrahlung spectrum given as photons per β disintegration per 1-Mev energy interval. The solid line represents the theoretical spectrum according to Knipp and Uhlenbeck (see reference 11). The statistical error is smaller than the size of the points. Source $A=2.5$ mC; source $B=1.0$ mC.

[36] Jensen, Nichols, Clement, and Pohm, Phys. Rev. **85**, 112 (1952).
[37] N. Starfelt and N. L. Svantesson, Phys. Rev. **97** (1955) (in press).
[38] S. B. Nilsson (to be published).

β absorber in assuming isotropic distribution (Sauter,[39] Sesemann[40]). Novey and Michalowicz ignored the Compton electron distribution, the back-scatter from the photomultiplier, and the energy dependence of the geometrical crystal efficiency, which may easily give errors of anything up to 15 percent.

It may also be mentioned, that the good agreement from 3 to 30 kev reported by Renard[10] is based upon calculated values, which are about 50 percent smaller than those calculated by Novey, Michalowicz, and by us.

The present measurements gave a total amount of IB energy per β disintegration of 2.87×10^{-3} mc^2/β, i.e., 21 percent greater than the calculated value, 2.37×10^{-3} mc^2/β. The result reported by Bolgiano *et al.*[9] agrees with theory, but their value was estimated only from

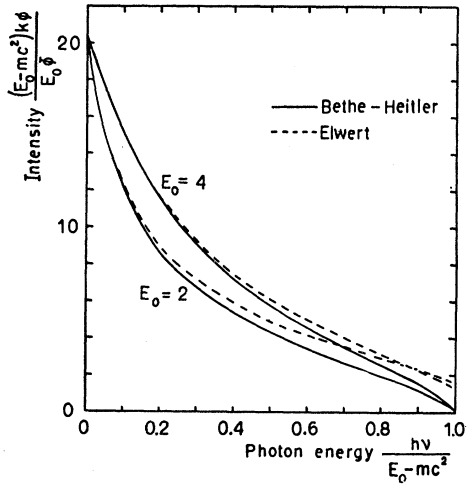


FIG. 8. Energy distributions $k\phi(E_0-mc^2)/E_0\bar{\phi}$ of the thin-target bremsstrahlung of monochromatic electrons with total energy $E_0=2mc^2$ and $E_0=4mc^2$ as a function of $k/(E_0-mc^2)$, calculated for lead according to Bethe and Heitler (see reference 23) (solid lines) and according to Elwert (see reference 29) (dotted lines). Here k is the photon energy in units of mc^2, ϕ the cross section integrated over all photon directions for the emission of a photon with energy between k and $k+dk$ $\phi=Z^2r_0^2/137$, and r_0 the classical electron radius.

measurements of all pulses above 90 kev. Michalowicz[15] obtained a value 10 percent above theory in the region 0–400 kev, in agreement with our measurements.

B. External Bremsstrahlung

The cross section ϕ of a single atom (thin target) for the emission of an EB photon with energy between k and $k+dk$, integrated over all photon directions, was calculated by Bethe and Heitler[23] using Born's approximation, which is valid for low Z and relativistic velocities of the radiating electron before and after the process. For nonrelativistic energies Elwert[29] and later

[39] F. Sauter, Ann. Physik **20**, 404 (1934).
[40] G. Sesemann, Ann. Physik **40**, 66 (1941).

TABLE II. The P^{32} external bremsstrahlung photon spectra for complete stopping of the beta rays in C, Al, Sn, and Pb, calculated according to the theories of Bethe and Heitler and of Elwert, respectively (references 23 and 29). After multiplication by 10^{-3} the figures give the number of photons per Mev per disintegration.

Energy Mev	Carbon Elwert	Aluminum Elwert	Aluminum Bethe-Heitler	Tin Elwert	Lead Elwert	Lead Bethe-Heitler
0.0511	160	380		1760	3000	
0.1022	54.7	132	114	603	1016	844
0.1533	27.4	65.1	56.5	298	509	430
0.2555	9.54	23.3	20.9	107.2	184	155
0.358	4.33	10.5	9.55	49.4	85.5	68.6
0.511	1.50	3.80	3.45	17.8	30.6	24.3
0.639	0.697	1.73		8.28	14.0	
0.767	0.321	0.810	0.670	3.93	6.87	4.75
0.894	0.140	0.350		1.76	3.07	
1.022	0.057	0.144	0.120	0.76	1.33	0.82

Guth[41] gave a correction factor, $f(\xi,\xi_0)$, to ϕ

$$f(\xi,\xi_0)=\frac{\xi}{\xi_0}\frac{1-e^{-2\pi\xi_0}}{1-e^{-2\pi\xi}},\quad \xi=\frac{Ze^2}{hv},\quad \xi_0=\frac{Ze^2}{hv_0},\tag{3}$$

where v and v_0 are the velocities of the electron before and after the process. Furthermore, they proposed as a plausible assumption this factor to hold also for relativistic energies.

The cross section ϕ for lead calculated both according to Bethe-Heitler and to Elwert is shown in Fig. 8 with the intensity $k\phi(E_0-mc^2)/E_0\bar{\phi}$ as a function of the photon energy $h\nu/(E_0-mc^2)$.

According to Bethe and Heitler[23] the EB spectrum, $n(k,E')$, obtained when completely stopping an electron of total primary energy E' is

$$n(k,E')=\int_{1+k}^{E'}\frac{N\phi\,dE_0}{(-dE_0/dx)},\tag{4}$$

where N is the number of atoms per cm^3 and $-dE_0/dx$ the energy loss of the electron per cm. A set of values of $n(k,E')$ for various k was calculated by numerical integration of (4), using values of $-dE_0/dx$ calculated according to Aron *et al.*[42] with values of the mean excitation potential from Segrè.[43] The P^{32} EB spectrum $S(k)$ at complete absorption of the β rays is then

$$S(k)=\int_{1+k}^{E_m}P(E')n(k,E')dE'.\tag{5}$$

Numerical integration of (5) for various k gave $S(k)$ for the elements investigated (Table II).

Here the contribution of the atomic electrons to the EB was not taken into account. A proper consideration of this effect would probably increase the calculated

[41] E. Guth, Phys. Rev. **59**, 325 (1941).
[42] Aron, Hoffman, and Williams, U. S. Atomic Energy Commission Report AECU-663, May 28, 1951 (unpublished).
[43] E. Segrè, *Experimental Nuclear Physics* (John Wiley & Sons, Inc., New York, 1953), Vol. 1, p. 203.

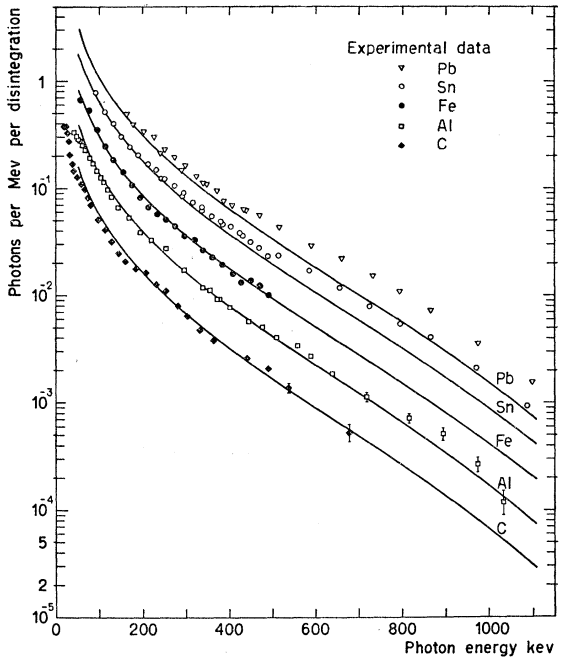


FIG. 9. The P^{32} external bremsstrahlung spectra produced when completely stopping the β rays in radiators of C, Al, Fe, Sn, and Pb, given as photons per β disintegration per 1-Mev energy interval. The solid lines are calculated according to Elwert (see reference 29).

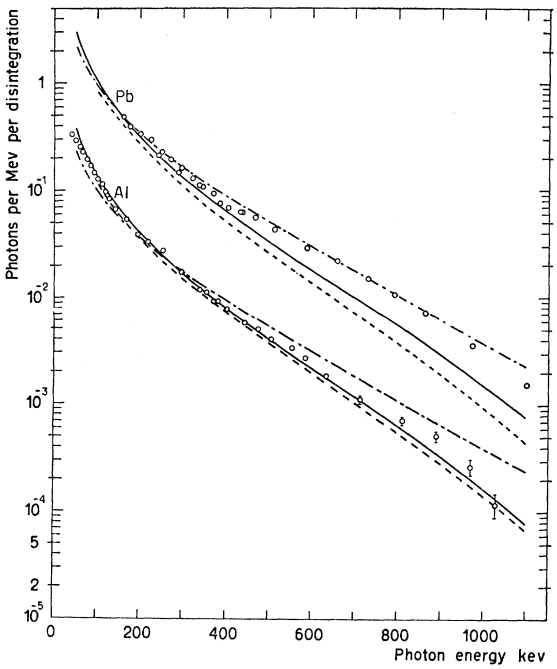


FIG. 10. The photon spectra of the P^{32} external bremsstrahlung from Al and Pb. The circles represent the experimental points, the solid lines and the dotted lines the calculated spectra according to Elwert (see reference 29) and to Bethe and Heitler, (see reference 23) respectively. The dot dashed lines represent the result of Wyard's (see reference 26) approximation of the Bethe and Heitler theory; it is normalized to the experimental spectrum at 0.5 mc^2.

bremsstrahlung intensity by a factor of the order of $1+1/Z$, at least for extreme relativistic energies.[30]

The solid curves in Fig. 9 represent $S(k)$ according to Elwert for C, Al, Fe, Sn, and Pb and the various sets of points the respective experimental spectra.

The EB spectra of the elements C, Al, and Fe agree almost exactly with theory in the photon energy region studied. Generally, smoothed curves through the experimental points deviate less than 10 percent. Being calculated as differences between two large numbers, EB+IB and IB, the aluminum points above 800 kev are less accurate.

As to Sn and Pb, the disagreement between experiment and theory above 300 kev increases with energy. At 300 kev the difference is 8 percent for Sn and 15 percent for Pb, at 600 kev 43 and 58 percent, and at 1000 kev 90 and 95 percent, respectively.

For comparison, the solid curves in Fig. 10 show $S(k)$ of Al and Pb according to Elwert, the dashed curves $S(k)$ according to Bethe and Heitler, and the circles the experimental points. Concerning Al, the two theories differ but slightly, only 15 percent in the whole energy region investigated, while the lead curves diverge with increasing energy. Thus the Elwert-factor displaces Bethe and Heitler's EB spectrum in the desired direction, but the discrepancy is still considerable. For electron energies between nonrelativistic and extreme relativistic values the EB cross section for high photon energies and large Z comes out too small in the theories discussed above.

The dot-dashed curves in Fig. 10, normalized to the experimental points at 0.5 mc^2, were calculated from the rather rough approximation of the Bethe-Heitler-Sommerfeld EB spectrum recently proposed by Wyard.[26] This approximation gives the shape of the EB spectrum of Pb surprisingly well up to 900 kev, but deviates at higher energies (the EB of Pb was also measured at 1217 and 1330 kev, not included in Figs. 9 and 10). However, the representation of the EB spectrum of Al by Wyard's spectrum is less successful.

From Fig. 9 it is obvious that the practically linear relation between the EB intensity and the atomic number Z, which follows from the Bethe-Heitler theory, is only approximately fulfilled; the deviation increases with increasing photon energy and increasing Z.

The observed total EB energy, EB_{obs}, as well as the ratio EB_{obs}/EB_{calc}, where EB_{calc} is calculated according

TABLE III. The experimentally determined energy, EB_{obs}, of the P^{32} external bremsstrahlung of various elements completely stopping the β rays, and the ratio EB_{obs}/EB_{calc}, where EB_{calc} is the calculated external bremsstrahlung according to Elwert (see reference 29).

Element	C	Al	Fe	Sn	Pb
Energy region, Mev	0 −0.5	0 −1.7	0 −0.5	0 −1.7	0.1 −1.7
EB_{obs}, mc^2/β	0.00310	0.00888	0.0173	0.0471	0.0558
EB_{obs}/EB_{calc}	0.896	0.946	0.944	1.119	1.301

to Elwert, is given in Table III. As appears from Table III, the deviation from theory increases with increasing atomic number of the radiator element.

In the literature there is a lack of accurate observations of EB spectra excited by β rays. Goodrich *et al.*[27] investigated the shape of the EB+IB spectrum from P^{32} surrounded by 1.1-g/cm^2 brass, and found agreement within 10 percent from 50 to 1100 kev with Wyard's approximation discussed above. Michalowicz[15] has recently investigated the P^{32} EB spectrum of various elements in connection with corrections to a measurement of the IB spectrum. He made no comparison with theory, and details of the experimental results are given only for the K x-ray peaks.

The present investigation was carried out mainly at the Department of Physics, University of Lund. The authors wish to express their gratitude to Professor B. Edlén and Professor S. v. Friesen for their interest and support in this work and to J. Cederlund, B. Forkman, and N. L. Svantesson for valuable assistance.

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Excitation Function for Na^{22} from Deuterons on Aluminum*

S. O. Ring and L. M. Litz
Livermore Research Laboratory, California Research and Development Company, Livermore, California
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The cross section for the reaction $Al^{27}(d,d\alpha n)Na^{22}$ has been determined as a function of deuteron energy in the energy range from 30 to 190 Mev. This reaction may be used for monitoring a high-energy deuteron beam.

INTRODUCTION

In order to calculate cross sections of cyclotron-induced activities, it is necessary to know the integrated beam intensity, i.e., the number of particles impinging on the target. This intensity can be obtained by the use of a monitor material which undergoes a nuclear reaction for which the cross section as a function of energy is accurately known.

A standard monitor reaction used in deuteron bombardment work is $Al^{27}(d,\alpha p)Na^{24}$. The disintegration rate of radioactive Na^{24} obtained by counting the thin aluminum foils is used to monitor the beam. Na^{24} is a 15-hour, β^- emitter.[1] In those cases where the monitor foils cannot be counted soon after bombardment a monitor with a longer half-life would be desirable. The reaction $Al^{27}(d,d\alpha n)Na^{22}$ occurs simultaneously with the reaction given above. Na^{22} is a 2.6-year[1] positron emitter.

In the following experiments, thin aluminum foils were irradiated with deuterons in the 184-inch Berkeley cyclotron. The aluminum foils were counted and the cross section for Na^{22} was obtained as a function of deuteron energy by using the Na^{24} as an internal monitor.

EXPERIMENTAL

For the irradiations, aluminum foils 0.002-inch thick were used. A one-half mil aluminum foil was placed on either side of the 0.002-inch foil to eliminate any errors caused by recoils. These aluminum stacks were placed in a target holder mounted on the end of the probe of the 184-inch cyclotron. Various deuteron energies were obtained by varying the distance of the foils from the center of the cyclotron tank. The foils were mounted in such a way that the deuteron beam hit perpendicular to the surface of the foils so that a single energy was obtained at each irradiation. (The attention in energy in passing through such a thin target is negligible.) Twelve irradiations were made with two separate aluminum target foils at each irradiation energy. The irradiations were from 15 to 30 minutes with a deuteron beam of approximately 0.5 microampere. After irradiation, each foil was cut in half and both pieces mounted side by side on a thick aluminum backing plate so as to give a source of small area. These were counted under constant geometry with an Amperex type 100C, end-window Geiger tube. The tube and sample were contained in an aluminum-lined, lead counting chamber.

RESULTS AND DISCUSSION

The cross section as a function of energy for Na^{24} from aluminum bombarded by deuterons has been accurately established.[2] The activity of the aluminum foils was followed until the activity from the Na^{24} had died out leaving the activity caused by Na^{22} alone. (No other activities are formed with half-lives greater than several minutes.) The activities at the end of the bombardment were obtained for the two isotopes by resolution and extrapolation of the decay curves. Since the foils were counted under constant geometry

* This work was carried out under a U. S. Atomic Energy Commission contract.

[1] Hollander, Peckman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

[2] Batzel, Crane, and O'Kelley, Phys. Rev. **91**, 939 (1953).