

## Scattering of 0.6-, 1.0-, and 1.7-Mev Electrons from Aluminum and Gold\*†

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The relative differential cross sections for the scattering of electrons by gold and aluminum has been measured at 0.6, 1.0, and 1.7 Mev over the angular range  $30^\circ$  to  $150^\circ$ . No significant deviation from the theoretically expected angular distribution was observed. Absolute differential cross sections were measured at  $60^\circ$  for aluminum at the same energies. The data are in agreement with the theoretical predictions within the experimental error.

### I. INTRODUCTION

THE differential cross sections for the scattering of relativistic electrons by nuclei has been evaluated by Mott.<sup>1</sup> The earlier experimental results have been summarized by Mott and Massey.<sup>2</sup> These early results present a rather confused picture. More recent measurements by Van de Graaff and co-workers,<sup>3</sup> however, gave good agreement with the theoretical predictions for the absolute differential cross section at energies from 1.27 to 2.27 Mev and for a variety of elements. Their measurements extended only over an angular range of  $20^\circ$  to  $50^\circ$ . Measurements extending from  $30^\circ$  to  $150^\circ$  on 0.245-Mev electrons were reported in 1952 by Kinzinger and Bothe,<sup>4</sup> and from  $60^\circ$  to  $120^\circ$  on 2.2-Mev electrons by Paul and Reich.<sup>5</sup> Subsequent measurements at  $120^\circ$  were reported by Kinzinger<sup>6</sup> between 0.15 and 0.4 Mev. In these last experiments good agreement with theory at all angles was found for aluminum, but rather sizable deviations at large angles were found for the heavier elements gold and platinum. Since the theory is expected to describe the situation reasonably well at these energies, and since no cause could readily be found to explain the experimental deviations, it seemed desirable to make another measurement of relative electron scattering.

In the experiment of Kinzinger and Bothe, the relative scattering cross section ratios in aluminum at  $30^\circ/50^\circ$ ,  $50^\circ/70^\circ$ — $130^\circ/150^\circ$  were first measured, and then the cross sections for gold, silver and nickel at various angles were compared with those obtained for

aluminum. The measurements were normalized by measuring the target foil thicknesses and keeping the electron beam current constant. It was assumed that at all angles the backscattering from the interior walls of the scattering chamber into the detectors would be negligible. Paul and Reich used a somewhat similar experimental arrangement.

It seemed desirable in the present experiment to avoid errors in the relative scattering measurements due to uncertainties in the foil thickness determinations and in the constancy of the beam current. Therefore relative scattering measurements were made directly in gold, and independently, in aluminum. The measurements in aluminum were made principally to check the operation of the apparatus. The scattering intensities at  $30^\circ$ ,  $60^\circ$ ,  $120^\circ$ , and  $150^\circ$  were each compared with that at  $90^\circ$  in order to avoid the cumulative errors associated with a sequential comparison of scattering ratios  $30^\circ/60^\circ$ ,  $60^\circ/90^\circ$ , etc. The energies for which measurements were made were 0.6, 1.0, and 1.7 Mev. It was found that the assumption made by Kinzinger and Bothe, that backscattering of electrons into the detectors was unimportant, could not be made for the present experimental arrangement. Multiple-scattering corrections could not be neglected entirely at small angles, but they were made small through the use of thin target films.

In order to provide another measurement of the absolute scattering cross section, as well as to check the operation of the experimental apparatus, a determination of absolute scattering was made in aluminum at  $60^\circ$  for electrons of the three energies used in the relative scattering measurements.

### II. APPARATUS

#### A. Electron Source

The source of electrons used was a 2-Mev Van de Graaff generator. The beam was brought to a focus at the scattering foil location by means of a lens-type focusing coil. The diameter of the major part of the focused beam varied from about 5 mm for 0.6 Mev to 3 mm for 1.0 and 1.7 Mev. These diameters were determined by permitting the beam to pass through a piece of clear Scotch tape, which turns white in the

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<sup>1</sup> N. F. Mott, Proc. Roy. Soc. (London) **A124**, 425 (1929).

<sup>2</sup> N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1949), second edition.

<sup>3</sup> Van de Graaff, Buechner, and Feshbach, Phys. Rev. **69**, 452 (1946); Buechner, Van de Graaff, Sperduto, Burrill, and Feshbach, Phys. Rev. **72**, 678 (1947).

<sup>4</sup> E. Kinzinger and W. Bothe, Z. Naturforsch. **7a**, 390 (1952).

<sup>5</sup> W. Paul and H. Reich, Z. Physik **131**, 326 (1952).

<sup>6</sup> E. Kinzinger, Z. Naturforsch. **8a**, 312 (1953).

irradiated region. Another determination of beam spread was made by measuring the fraction of the beam which would fail to pass through holes of various diameters drilled through aluminum plates and centrally located with respect to the beam. It was determined that no more than about 0.05 percent of the beam fell outside a  $\frac{3}{8}$ -in. diameter circle, regardless of the energy. Because of the large distance from the collimating magnet to the point of focus, the paths of the electrons in the beam were very nearly parallel, the maximum deviation being about  $0.6^\circ$  (for 0.6 Mev). The energy of the beam was determined with a generating voltmeter which was also used to control the beam energy. Calibration was achieved by measuring the beryllium and deuterium gamma-neutron thresholds of  $(1.666 \pm 0.002)$  Mev and  $(2.227 \pm 0.003)$  Mev.<sup>7</sup> The beam energy would fluctuate in some cases as much as  $\pm 1$  percent, although it was usually more stable. The soft component of the beam, as determined by the current measured with the electron emitting filament off, was not measurable with a meter which could detect currents of the order of  $10^{-10}$  amperes. The beam currents used varied from a few times  $10^{-9}$  amperes to  $10^{-6}$  amperes.

### B. The Scattering Chamber and Accessories

A sectional view of the scattering chamber is shown in Fig. 1. Some of the details in this figure are not strictly to scale. The chamber and all parts exposed to the direct or scattered beam are aluminum. The main body of the chamber is an aluminum ring  $1\frac{3}{4}$  in. thick and 11 in. i.d. with machined ports and faces located every  $30^\circ$ . This central ring has two aluminum coverplates which are sealed by means of O-ring gaskets. The ports used for entry and exit of electrons during measurements have appropriate fittings attached and sealed by O-ring gaskets, while those ports not in use have aluminum covers.

The purpose of the collimators to which the detectors are attached is to reduce the number of electrons which can reach the detectors without having been scattered directly from the foil. The diaphragms of the collimators are  $\frac{1}{8}$ -in.-thick aluminum and the holes are  $\frac{3}{8}$  in. in diameter. The face and shoulder which mate with the face and port of the scattering chamber are machined with respect to the diaphragm holes so that when the collimator is bolted in place, the centerline of the diaphragms passes through the center of the chamber. The diaphragm farthest from the scattering foil, i.e., that adjacent to the detector crystal, constitutes the aperture which defines the scattering solid angle. This beam defining aperture is covered by a vacuum tight window of 0.001-in. aluminum through which the scattered electrons pass.

As long as the region of the foil in which scattering

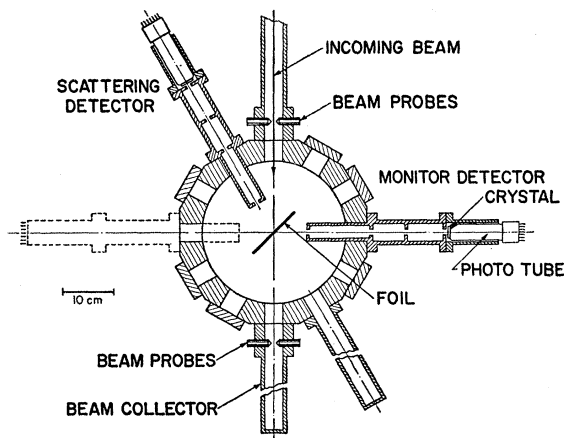


FIG. 1. The scattering chamber.

occurs is smaller than the diameter of the collimating diaphragm holes, these diaphragms will not interfere with the electrons scattered in the solid angle of the defining aperture. However, they do stop a large number of electrons which are backscattered from various parts of the chamber walls, many of which have very nearly the full energy of the initial beam. The diaphragms cannot, of course, stop electrons which are scattered from points directly opposite a collimator, pass undeflected through the foil, and into the detector. The effect was particularly important at scattering angles of  $120^\circ$  and  $150^\circ$  because of the relatively large numbers of electrons originally scattered at  $60^\circ$  and  $30^\circ$  respectively and backscattered into the detectors. This was even more serious with short (six-in.) collimators which were used in the early phases of the experiment. These short collimators were essentially the same as those shown in Fig. 1 except that they did not extend into the scattering chamber and had only three diaphragms. This backscattering was made negligible by (a) making the collimator longer ( $10\frac{1}{2}$  in.), as shown in Fig. 1, so that the area of the chamber wall which could be seen by the detector was reduced, and (b) by locating a long aluminum tube opposite the detector (shown in Fig. 1 at the  $30^\circ$  position). Thus the electrons originally scattered at the small angle do not encounter any material from which to backscatter until they are a considerable distance from the chamber (about four feet) and the probability for backscattering into the detector is reduced considerably. The effect of these changes in geometry on the pulse height distribution can be seen in Fig. 2. Composites of several curves are given for the original geometry at  $150^\circ$  (upper curve), for the improved geometry at  $150^\circ$  (middle curve) and for the improved geometry at angles from  $30^\circ$  to  $120^\circ$ . For the range of pulse height shown, the curves for  $30^\circ$  to  $120^\circ$  are indistinguishable. For pulse heights in the neighborhood of that representing the beam energy, the  $150^\circ$  curve also coincided with the others. For lower pulse heights

<sup>7</sup> R. C. Mobley and R. A. Laubenstein, Phys. Rev. 80, 309 (1950).

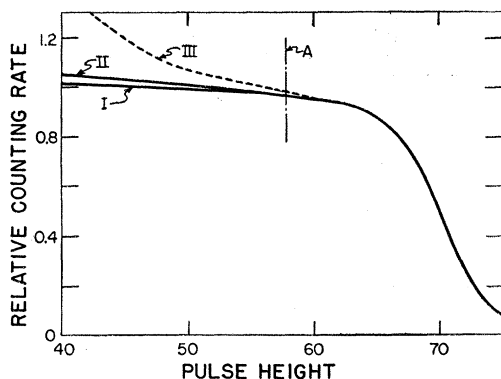


FIG. 2. Typical integral pulse height distributions obtained in relative scattering measurements. Curve I—Composite of characteristics from  $30^\circ$  to  $120^\circ$ , Curve II— $150^\circ$  with improved collimator and chamber geometry, Curve III— $150^\circ$  with original geometry.

there is a slight difference for the improved geometry case, and a considerable difference for the original geometry.

The foils were mounted on an oval aluminum frame with inside dimensions of 1.25 in.  $\times$  2 in. The frames were mounted on a shaft which passed through one of the scattering chamber side plates and which could be rotated from outside the chamber. The side plate was accurately positioned so that the axis of the shaft coincided with the center of the chamber. A special side plate with provision for positioning either of two foil frames in the center of the chamber was used to determine the background from x-rays, etc. In these measurements, one of the frames had a foil and the other was blank.

The aluminum foils used were commercially prepared and had a thickness of about  $0.2 \text{ mg/cm}^2$ . The measurements on aluminum at 1.0 Mev and those for  $30^\circ$  and  $60^\circ$  at 0.6 Mev were made with a foil consisting of a foil support of  $\sim 60 \text{ } \mu\text{g/cm}^2$  Parlodion supporting a  $\frac{3}{8}$ -in. diameter spot of the  $0.2\text{-mg/cm}^2$  aluminum in the center. The original purpose of providing the scatterer only in the center was to eliminate any scattering outside the  $\frac{3}{8}$ -in. diameter spot which could be seen by the detector. Since subsequent measurements showed that a negligible amount of the beam fell outside a  $\frac{3}{8}$ -in. circle, the remaining measurements of aluminum scattering at 0.6 Mev and those at 1.7 Mev were made with an unbacked foil across the entire frame. The gold scatterer consisted of a spot of gold  $\frac{3}{8}$  in. in diameter evaporated on the center of a Parlodion foil support. The gold was thin enough to transmit visible light. The thickness determined from the absolute scattering at  $60^\circ$  was  $\sim 16 \text{ } \mu\text{g/cm}^2$ , while that estimated by assuming isotropic evaporation and 100 percent sticking of the gold atoms on the Parlodion was  $\sim 20 \text{ } \mu\text{g/cm}^2$ , or about 25 percent higher. It is interesting in this connection to note that Chase and Cox<sup>8</sup> obtained an estimate of the thickness of

an evaporated aluminum foil by assuming isotropic evaporation which was 19 percent higher than the value they obtained by microchemical analysis of the same foil.

The detectors were scintillation counters each consisting of a  $\frac{1}{4}$ -in.-thick  $\times$   $1\frac{1}{2}$ -in.-diameter anthracene crystal and a 2-in.-diameter type 6292 photomultiplier. The pulses from the photomultipliers were fed into linear amplifiers and pulse height discriminators and then into scale-of-1000 scalers. The scalers were switched on and off simultaneously so that the counts in the two channels were recorded over the same time interval. The pulse height resolution, as determined with a single-channel pulse height analyzer, was of the order of 10 percent for all detectors.

The alignment of the beam was accomplished by measuring the beam position with probes and moving the chamber with respect to the Van de Graaff generator until the beam passed centrally through the entry and exit ports. The probes are shown schematically in Fig. 1 and consist of two sets of four coplanar rods with conical points. One set is located at the beam entry port and the other at the beam exit port. The probes are symmetrically located around the nominal beam position, with their pointed tips equidistant from the centerline of the port. The probes are retracted during scattering measurements so that they do not interfere with the beam. To align the beam, the currents to the four probes in the entry port are measured simultaneously and the chamber is moved laterally until the currents are equal. Then the currents to the four exit port probes are measured and the chamber is tilted until these currents are equal. The mechanism for the tilting adjustment (not shown in Fig. 1) consists of provisions for motion about a pair of axes which are perpendicular to each other and to the beam. These axes are located close to the entry port beam probes so as not to disturb the lateral alignment.

### III. EXPERIMENTAL PROCEDURE

The measurements of relative scattering cross sections were made by determining the ratio of scattering at a given angle to that at  $90^\circ$ . The procedure is as follows. The scattering detector is placed at the angle for which the scattering is to be measured and the monitor detector is placed at the  $90^\circ$  position on the opposite side of the scattering chamber. The apparatus as shown in Fig. 1 is in the configuration used to measure scattering at  $150^\circ$ . The ratio of scattering into these two detectors is determined by counting simultaneously for a given time and recording the total number of counts. The ratio of electrons scattered into the same two detectors is then determined again with the scattering detector moved to the  $90^\circ$  position, directly opposite the monitor detector. By dividing the first ratio by the second, the ratio of scattering at the given angle to that at  $90^\circ$  is determined. This procedure is employed rather than the simpler one of merely taking the first ratio de-

<sup>8</sup> C. T. Chase and R. T. Cox, Phys. Rev. 58, 243 (1940).

scribed, in order to eliminate errors due to differences in collimators and detectors. This procedure also minimizes possible errors due to misalignment of the chamber with respect to the beam, since the signs of such errors would be the same for the two angles being compared. Motion of the scattering chamber with respect to the beam of as little as 0.1 mm in translation could be detected. An error of the order of 0.06 percent would result from a misalignment of this order of magnitude. It is felt that the maximum misalignment due to all causes was probably less than ten times the minimum detectable value given, resulting in a maximum probable error due to translational chamber misalignment of about 0.6 percent. The error due to possible rotational misalignment is estimated to be of the same order of magnitude as that from translational misalignment.

Each measurement consisted of three to five runs of from 20 000 to 100 000 counts for the slower scaler. To cancel out errors due to mounting the foil slightly off the axis of the foil mounting shaft, a second measurement was made for each configuration with the foil turned through  $180^\circ$ , and the mean of the two values was taken to be the measured scattering. In the majority of cases the two measurements did not differ by more than the statistical uncertainty of a few tenths of a percent, indicating that the foils were being centered to within a fraction of a millimeter. In all cases the foil was placed at  $45^\circ$  with respect to the beam.

The measurements at  $30^\circ$  and  $60^\circ$  were made with the foil in the "transmission" position, i.e., with the scattered electrons and the beam emerging from the same side of the foil, while at  $120^\circ$  and  $150^\circ$  the foil was in the "reflection" position. The measurements at  $120^\circ$  and  $150^\circ$  had to be adjusted to account for the asymmetry in the transmission-reflection scattering. This asymmetry was measured for each foil and at each energy by taking the ratio of scattering at  $90^\circ$  to that at  $270^\circ$  with the foil in the four possible positions obtainable by rotation through  $90^\circ$ . The square root of the quotient of the average ratios for scattering with the  $90^\circ$  detector measuring reflected electrons to that with the same detector measuring transmitted electrons gives the transmission-reflection asymmetry at  $90^\circ$ . The values obtained ranged from 3 percent for aluminum at 0.6 Mev to 0.2 percent for gold at 1.7 Mev. This measured scattering asymmetry was used to adjust the data for  $120^\circ$  and  $150^\circ$  in a manner which is discussed in the section on the treatment of data.

The pulse height discriminator setting was checked before each count was taken. As may be seen from the curves in Fig. 2, the counting rate changes very slowly with discriminator setting over a wide range of settings. The discriminator was set at a given fraction of the value at which the counting rate was half of the plateau value. Usually this fraction was 0.82 although the exact value is not important as long as the same value is used throughout a given measurement. This is true because the shape of the curve (Fig. 2) is independent

of angle for pulse heights greater than about 80 percent of that representing the beam energy.

An effort was made in all scattering measurements to keep the counting rates in the two detectors within certain bounds in order to avoid large errors due to the finite resolving time of the scaling circuits and to the background from radioactivity and cosmic rays. In order to keep the corrections small, an attempt was made to have the counting rates in the two detectors differ by not much more than a factor of ten. Since the ratio of scattering at  $30^\circ$  to that at  $90^\circ$  was about 90 for aluminum, a special collimator with a solid angle about one-tenth that of the standard collimator was used at  $30^\circ$  in order that the counting rates would differ by no more than a factor of ten. The ratio of scattering at  $150^\circ$  to that at  $90^\circ$  was a factor of twelve and the ratios for  $60^\circ$  and  $120^\circ$  were less than this, so that standard collimators were used for all measurements except those at  $30^\circ$ . At 1.7 Mev, the  $30^\circ$  measurement was made using standard collimators by placing the monitor at the  $60^\circ$  position rather than at  $270^\circ$ .

The measurements of absolute scattering were made by comparing the number of electrons scattered at  $60^\circ$  with the integrated beam current. The beam charge was collected on a Plasticon condenser of 5.0- $\mu\text{f}$  capacity. The potential to which the condenser was charged was measured using a vibrating reed electrometer. The scattering chamber had a long tube or cup on the bottom which acted as a Faraday cage. Approximately 90 percent of the total beam charge was collected in this cup, the remainder being collected in the scattering chamber itself, which was insulated from the Van de Graaff generator. Actual scattering measurements were made with the Faraday cup and the scattering chamber connected electrically.

Absolute scattering measurements were made using the same collimators as for the relative measurements. However the crystal was not mounted directly behind the diaphragm which defines the solid angle, as was the case in the relative measurements. The reason for this is that there was evidence to show that a significant number of electrons were being scattered from the inner edge of the defining aperture and into the detecting crystal. The crystal was mounted behind a stripping diaphragm located  $1\frac{3}{4}$  in. behind the diaphragm which defines the solid angle. This stripping diaphragm had a hole which was just large enough that electrons scattered directly from the foil would not be intercepted. However those which were scattered from the edges of the beam defining diaphragm could be intercepted, since, in general, the direction of such electrons would have been changed in the process. Analysis of the results of the first absolute scattering experiment gave evidence that the precautions described were not adequate to reduce the contribution from diaphragm scattering to a negligible amount. Consequently the crystal was moved to a position 6 in. behind the beam defining diaphragm and two stripping diaphragms were

used, one next to the crystal and the other at an intermediate position.

The scatterer used in the first measurements was an aluminum foil positioned perpendicular to the beam and intersecting it at the center of the scattering chamber. The foil was mounted in such a way that it could be shifted parallel to itself between measurements without disturbing the chamber alignment. Four positions were chosen for the foil mount, corresponding to four spots in the foil for which scattering measurements could be made. These same four positions were used for the measurements at the three energies. After the scattering measurements were made, the foil was cut into two rectangles approximately  $1.8 \text{ cm} \times 2.4 \text{ cm}$ , each containing two of the scattering spots. The dimensions and the masses of the rectangular foils were measured to determine their average thicknesses.

Another determination of absolute scattering was made at 1.7 Mev using the two stripping diaphragms previously described. Four separate foils were used in this measurement. After the absolute scattering was determined, pieces approximately 1.6-cm-square were cut from the foils with the scattering spot in the center. The thicknesses of these squares were then determined in the same way as in the first experiment.

#### IV. TREATMENT OF DATA

In analyzing the data, account was taken of background counting rate, counting losses due to the finite resolving time of the scaling circuits, multiple and plural scattering, and scattering in the Parlodion foil support. The maximum contribution for each of these was of the order of a percent and was in most cases less than this.

The background counting rate due to cosmic rays and other natural sources was determined for each set of data by taking counts for 300 to 500 sec with the Van de Graaff generator off. The variations in the back-

ground counting rate at a given energy were no more than expected from statistical fluctuations. The average background counting rate was about 1.5 counts/sec for 0.6 Mev, 0.3 counts/sec for 1 Mev, and 0.1 counts/sec for 1.7 Mev. The background counting rate was never more than about one percent of the total counting rate.

The background due to x-rays and to stray electrons not originally scattered by the foil was determined by placing a blank foil holder in the scattering chamber. This was done by mounting two foil frames, one with a foil and one without, on a rotating plate arranged so that either frame could be positioned at the center of the chamber. The apparatus was first set up using the foil, and then the blank frame was placed in the scattering position. Measurements were made at 1 Mev using an aluminum foil. The counting rate with the blank foil frame in place represents the sum of the natural background and the background due to x-rays and stray electrons, while the counting rate with the Van de Graaff off represents only the natural background. The maximum contribution to the background from the Van de Graaff generator itself amounted to about 0.01 percent of a typical scattering measurement, and was therefore neglected in analyzing the data.

The measured counting rates were adjusted for losses due to the finite resolving time of the scaling circuit. For low enough counting rates this loss is directly proportional to the counting rate. The resolving time of the circuit was measured by determining the difference in counts recorded at different counting rates for a given integrated beam current. The value determined in this way,  $5.6 \pm 0.5 \mu\text{sec}$ , compares with the value of  $\sim 6 \mu\text{sec}$  determined with a double pulse generator.

In taking account of multiple scattering, an equation derived by Chase and Cox<sup>8</sup> was used. This equation is based on the assumption that the electrons are deflected through a large angle  $\theta'$  in a single scattering event and through a small angle  $\epsilon$  by multiple scattering in such a way that the resultant is the experimental

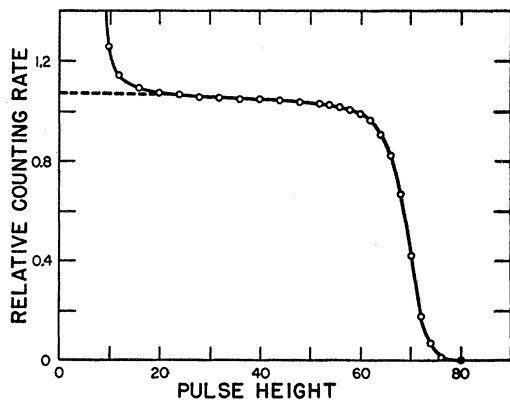


FIG. 3. Integral pulse height distribution obtained in absolute scattering measurement. Dashed portion of curve shows extrapolation of plateau.

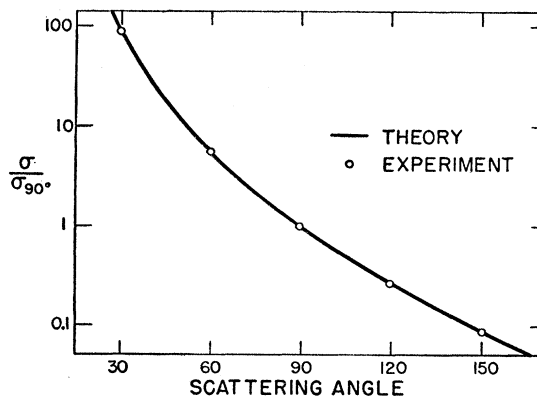


FIG. 4. Differential scattering cross section of aluminum for 1-Mev electrons. Measurements are normalized at  $90^\circ$ .

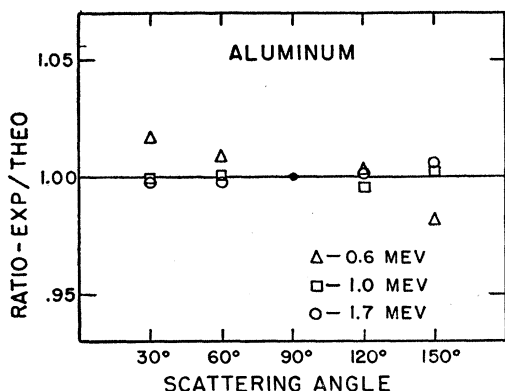


FIG. 5. Differential scattering cross sections of aluminum for 0.6-, 1.0-, and 1.7-Mev electrons, expressed as the ratio of experimental to theoretical values. Measurements are normalized at 90°.

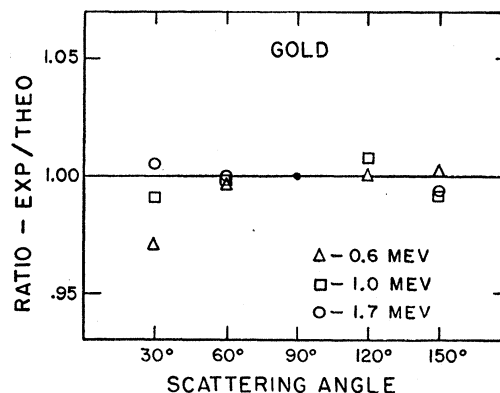


FIG. 6. Differential scattering cross sections of gold for 0.6-, 1.0- and 1.7-Mev electrons, expressed as the ratio of experimental to theoretical values. Measurements are normalized at 90°.

angle  $\theta$  of interest. Their expression is

$$\frac{P'(\theta) - P(\theta)}{P(\theta)} = \epsilon^2 \left( \frac{\theta}{2} - \frac{1}{2} \right),$$

where  $P(\theta)$  is the probability of a single scattering through an angle  $\theta$  and  $P'(\theta)$  is the probability of a single scattering through  $\theta'$  and multiple scattering through  $\epsilon$  resulting in a net scattering through  $\theta$ , averaged over all possible values of  $\theta'$ . For the foil thicknesses used this correction was negligible at all angles except 30°, where it was of the order of one to three percent. The mean square multiple scattering angle  $\langle \epsilon^2 \rangle_{Av}$  was obtained by assuming that the angular distribution of multiple scattered electrons was gaussian with a distribution  $\exp(-\epsilon^2/2\langle \epsilon^2 \rangle_{Av})$  with  $\langle \epsilon^2 \rangle_{Av}$  given by

$$\langle \epsilon^2 \rangle_{Av} = \frac{4\pi N x e^4 Z(Z+1)}{p^2 v^2} G.$$

Here  $e$ ,  $p$ , and  $v$  are the charge, momentum, and velocity of the electron,  $N$  the number of nuclei per unit volume,  $Z$  the atomic number of the scatterer,  $x$  the foil thickness, and  $G$  a very slowly varying function of  $Z$  and  $p$ .<sup>9</sup> The value of  $G$  was measured experimentally by Kinzinger and Bothe<sup>3</sup> for the case of 245-keV electrons scattered in aluminum. Their value of  $G=2.3$  was used in adjusting the data of all measurements, since  $G$  is for practical purposes a constant. The contributions from multiple scattering were small enough that the choice of  $G$  was relatively unimportant. The value of  $\langle \epsilon^2 \rangle_{Av}$  was of the order of three degrees or smaller for all foils and energies used.

The effects of the divergence of the focused beam and of the spread in angle of the measured electrons due to the finite size of the collimator aperture are more important at 30° than at any of the other angles for which measurements were made. The divergence of the beam is largest for 0.6 MeV and is estimated to be no

larger than 0.6° (from nominal beam position to the electron path of maximum deviation). If the divergence were actually this large, the scattering at 30° would be only 0.15 percent greater than for a truly parallel beam. For other angles and energies the effect would be even less, so this effect was neglected.

The effect on the measured values of scattering due to the fact that the measurements extended over a small angular range centered about the angle of interest, is most pronounced at small scattering angles. However at 30° for 0.6 and 1.0 MeV, a collimator was used which had a considerably smaller angular spread than the standard collimator. At 1.7 MeV, where the standard collimator was used at 30°, the adjustment required was only 0.1 percent. This adjustment was the only one made since for all other measurements they were considerably smaller than 0.1 percent.

Plural scattering, wherein a small number of scattering events results in an electron being deflected through the angle of interest, is quite small for the foil thicknesses used in the experiment. The most probable event of this type is one in which an electron is first scattered into the plane of the foil and is then scattered out of the foil in the direction of interest. The path length of the electron through the foil is much larger in this case than for the single scattering, and the probability for this double scattering is correspondingly higher than would be expected for a simple double scattering in the foil. For the foils used in this experiment the effect was measured by placing detectors at the 90° and the 270° positions and comparing the scattering from the "reflection" side of the foil with that from the "transmission" side. To eliminate differences in detectors, measurements were made with the foil oriented so that first one detector faces the transmission side of the foil and then the other. The asymmetry thus obtained was considered to be due entirely to scattering twice through 45° to give 90°. Adjustments were made in the scattering measurements in which the electrons were scattered from the incident or reflection side of the foil, namely

<sup>9</sup> Groetzinger, Berger, and Ribe, Phys. Rev. **77**, 584 (1950).

TABLE I. Relative differential scattering cross section.

Angle	0.6 Mev			Aluminum 1.0 Mev			1.7 Mev		
	Experiment <sup>a</sup>	Theory	Deviation percent <sup>a</sup>	Experiment <sup>a</sup>	Theory	Deviation percent <sup>a</sup>	Experiment <sup>a</sup>	Theory	Deviation percent <sup>a</sup>
30°	84.67 ±0.23	83.31	+1.6±0.3	89.06 ±0.27	89.08	-0.0±0.3	92.99 ±0.27	93.15	-0.2±0.3
60°	5.291 ±0.011	5.244	+0.9±0.2	5.504 ±0.010	5.496	+0.1±0.2	5.662 ±0.017	5.674	-0.2±0.3
120°	0.2969±0.0007	0.2960	+0.3±0.3	0.2654±0.0007	0.2665	-0.4±0.3	0.2462 ±0.0005	0.2457	+0.2±0.2
150°	0.1166±0.0003	0.1188	-1.8±0.3	0.0857±0.0003	0.0854	+0.3±0.3	0.06221±0.00018	0.06187	+0.6±0.3
Gold									
30°	37.66 ±0.10	38.79	-2.9±0.3	39.50 ±0.12	39.87	-0.9±0.3	40.71 ±0.12	40.49	+0.5±0.3
60°	3.767 ±0.010	3.781	-0.4±0.2	3.898 ±0.009	3.889	-0.2±0.2	3.950 ±0.011	3.954	-0.1±0.3
120°	0.3403±0.0010	0.3404	-0.0±0.2	0.3204±0.0009	0.3178	+0.8±0.3		0.3036	
150°	0.1296±0.0004	0.1294	+0.2±0.3	0.0961±0.0004	0.0969	-0.8±0.4	0.07612±0.00022	0.07656	-0.6±0.3

<sup>a</sup> The standard deviation due to counting statistics is given with the experimental values and with the deviations.

those at 120° and 150°. The asymmetries at these two angles were determined by assuming that electrons were scattered into the plane of the foil (45°) and then through 75° and 105° to give a total of 120° and 150° respectively. The experimentally measured 45°-45° asymmetry was multiplied by the theoretical ratios of scattering at 75°/45° and 105°/45° to give asymmetries for 45°-75° and 45°-105° double scattering.

The scattering measurements in gold were adjusted for the scattering in the Parlodion foil supports by measuring directly the contribution of the Parlodion at 60°. These measurements were made by determining the absolute scattering from the foil support before and after the gold was evaporated on. The contributions at angles other than 60° were calculated by normalizing at the measured 60° value and using the theoretical cross sections for Parlodion. To check the validity of using the theoretical cross sections for Parlodion, a measurement of the scattering from a Parlodion foil was made which agreed with the theory within the experimental error. Since the Parlodion consists mostly of hydrogen and carbon, and since the relative scattering is an insensitive function of  $Z$  for small  $Z$ , the form of the relative scattering in the foil support was essentially the same as in the aluminum. Consequently the effect of the Parlodion backing was insignificant in the case of aluminum and no adjustments were made.

The integral bias curve for the absolute scattering measurements, shown in Fig. 3, does not have a perfectly flat "plateau" but the counting rate increases slowly as the pulse height decreases. These data were taken using the improved detector geometry. The rapid rise at small pulse heights is caused mostly by x-rays from the beam electrons, which are stopped in the Faraday cage and in the chamber. The slope of the plateau was considered to be due principally to electrons having been backscattered out of the anthracene crystal before dissipating all of their energy. The energy distribution of electrons backscattered from carbon<sup>10</sup> was used in extrapolating the plateau back to zero pulse height. In this extrapolation, the energy distri-

bution was normalized by assuming that the slope of the plateau between 40 and 60 percent of the beam energy was due entirely to backscattering from the crystal.

## V. RESULTS

The derivation given by Mott<sup>1</sup> for the coulomb scattering of electrons is based on Dirac's relativistic quantum mechanics. The resulting differential scattering cross section is

$$d\sigma/d\Omega = q^2(1-\beta^2)FF^* \csc^2(\theta/2) + GG^* \sec^2(\theta/2),$$

where

$$q = \alpha/\beta = Ze^2/\beta\hbar c.$$

Here  $\beta$  is the electron velocity in units of the velocity of light, and  $F$  and  $G$  are infinite series in the Legendre polynomials of  $\cos\theta$ . The cross section is in units of  $(\hbar/p)^2$  where  $p$  is the momentum of the electron.

The differential cross section for the scattering of electrons by Hg ( $Z=80$ ) has been evaluated numerically by Bartlett and Watson.<sup>11</sup> They give the results for various energies up to  $\sim 1.7$  Mev and for several scattering angles from 15° to 180°. McKinley and Feshbach<sup>12</sup> have expanded Mott's series for  $F$  and  $G$  as a power series in  $\alpha (=Ze^2/\hbar c \cong Z/137)$  and  $\alpha/\beta$  and have calculated the coefficients of terms to fourth order in  $\alpha$ . Assuming that the coefficients of the terms in  $\alpha^5$  and higher are not  $\gg 1$ , the errors in the scattering calculations for aluminum caused by neglect of terms in  $\alpha^5$  should be considerably smaller than the experimental errors, since  $\alpha^5$  for aluminum is  $8 \times 10^{-6}$ . The assumption that the coefficients of the higher order terms are not  $\gg 1$  is plausible because of the fact that the calculated coefficients are all of the order of unity or smaller. In contrast with the case for aluminum, this  $\alpha^4$  approximation for gold is not expected to yield accurate results since  $\alpha^5 = 0.06$ . The experimental measurements in gold were compared with values obtained from those given by Bartlett and Watson. The cross sections for gold

<sup>11</sup> J. H. Bartlett and R. E. Watson, Proc. Am. Acad. Arts Sci. 74, 53 (1940).

<sup>12</sup> W. A. McKinley, Jr. and H. Feshbach, Phys. Rev. 74, 1759 (1948).

<sup>10</sup> W. Bothe, Z. Naturforsch. 4a, 542 (1949).

were obtained by multiplying these values for mercury by the ratio of scattering in gold to that in mercury given by the  $\alpha^4$  approximation. This procedure may be expected to give results accurate to within one percent.

The results for the relative scattering in gold and aluminum at the three energies are shown in Figs. 4 to 6 and in Table I. The data in Fig. 4 are for aluminum at 1 Mev, plotted on a semi-logarithmic scale. Since the diameters of the dots representing the experimental points are about ten times as large as the actual experimental deviations, it is impossible to show the extent of the deviations on such a plot. Consequently, in Fig. 5 the same data are plotted in the form of the ratio of the experimental to the theoretical values. The data for aluminum at 0.6 Mev and at 1.7 Mev are also shown in Fig. 5. The experimental values for 1.7 Mev agree with the theory as shown. The results at 0.6 Mev do not agree quite as well as those at 1.7 Mev, but the maximum deviation is only 1.8 percent.

The results for gold are shown in Fig. 6. Here the results deviate from the theory a little more than was the case for aluminum. However, there is no evidence for a consistent trend in the deviation. The measured value at  $30^\circ$  and 0.6 Mev is 2.9 percent less than the theoretical value. The cause of this deviation is not known but because of the consistent agreement of other measurements it is quite likely to be an instrumental error. Before adjustments were made in this datum for scattering in the Parlodion foil support and for multiple scattering, the measured value was 0.7 percent greater than the theoretical value. The measurement of scattering at  $120^\circ$  and 1.7 Mev was not completed because of Van de Graaff generator failure.

The results of the measurements on absolute scattering are given in Table II. The results at 0.6 Mev and 1.0 Mev agree with the theory within the standard deviations of the four determinations made at each energy. The first measurement at 1.7 Mev gives results which are somewhat higher than the theoretical value. It can be seen that the initial measurements (the first

TABLE II. Absolute differential scattering cross section for aluminum at  $60^\circ$ .

Energy Mev	Experiment $10^{-24}$ cm <sup>2</sup> a	Theory $10^{-24}$ cm <sup>2</sup>	Deviation percent <sup>a</sup>
0.6 <sup>b</sup>	$15.95 \pm 0.27^b$	16.08	$-0.8 \pm 1.7^b$
1.0 <sup>b</sup>	$6.84 \pm 0.10^b$	6.76	$+1.2 \pm 1.5^b$
1.7 <sup>b</sup>	$2.84 \pm 0.041^b$	2.705	$+4.8 \pm 1.4^b$
1.7 <sup>c</sup>	$2.80 \pm 0.15^c$	2.705	$+3.4 \pm 5.4^c$

<sup>a</sup> The standard deviation of the four measurements made is given with the experimental value and with the deviation.

<sup>b</sup> Measurements made with original slit-edge-scattering stripping geometry, 1  $\frac{1}{4}$  in. long with a single stripping diaphragm.

<sup>c</sup> Measurement made with improved slit-edge-scattering stripping geometry, 6 in. long with two stripping diaphragms.

three tabulated in Table II) show an increase in experimental cross section with increasing energy, when compared with the theory. An analysis of the results indicated that the stripping diaphragm was probably not intercepting a majority of the electrons at the high energies. Consequently another experiment was performed at 1.7 Mev with the improved stripping diaphragm geometry described previously. The slope of the plateau of the integral bias curve for this second experiment was reduced to 54 percent of the previous value. This indicates that the first stripping diaphragm geometry had indeed not been completely effective. The results of this second measurement at 1.7 Mev are given in Table II. The larger standard deviation given for the second experiment probably results from the fact that the foil pieces were smaller and hence their thicknesses were more difficult to determine than was the case in the first experiment. Despite the larger standard deviation, the agreement with theory is better for the second determination than for the first, and is well within this larger standard deviation.

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