THE
PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 97, NO. 2 JANUARY 15, 1955

Departure from Paschen's Law of Breakdown in Gases

W. S. BOYLE AND P. KISLIUK
Bell Telephone Laboratories, Inc., Murray Hill, New Jersey
(Received September 28, 1954)

The failure of Paschen's law of electrical breakdown in gases at both high pressures and extremely small electrode separations is explained by a single breakdown mechanism. The breakdown field in each of these cases is sufficiently great to draw measurable field-emission current from the cathode, which produces a relatively small number of ions. The space-charge field of these ions is great enough to increase the field-emission current appreciably even when the ratio of ion current to electron current is less than one percent. As more ionic space charge is produced, each ion becomes more effective in enhancing the electron current until the breakdown condition is attained. An expression is derived for the yield of electrons per positive ion as a function of the applied field. This expression is shown to be in quantitative agreement with values of γ derived from published data on breakdown voltages at high pressures. It is shown also that the same process explains breakdown at extremely small separations below the "minimum sparking potential."

INTRODUCTION

PASCHEN'S law[1] of electrical breakdown in gases states that the breakdown voltage, V_B, is a function of the pressure, p, (or, more precisely, the density, ρ) and the electrode separation, d, only in the combination pd $[V_B=f(pd)]$. Since the earliest investigations of gas breakdown a considerable literature has accumulated on the failure of Paschen's law, expecially at high pressures.[2–4] Typical behavior showing departure from this law for plane parallel electrodes at high pressures is shown in Fig. 1, where with increasing distance the breakdown voltage increases linearly, while with increasing pressure the voltage starts to increase linearly but the slope falls off at high pressures.

Departure from Paschen's law occurs also for breakdown between closely spaced electrodes at atmospheric pressure with potential differences below the "minimum sparking potential."[5,6] In the case of very small separations, breakdown occurs at potentials for which breakdown does not occur at any pressure if the electrodes are fixed at some larger spacing. The breakdown voltage is thus clearly not a function of the product pd alone. The behavior of the breakdown voltage at small spacings is shown in Fig. 2.

The deviations from Paschen's law occur for just those experimental conditions for which the multiplication of electrons by ionization in the gas is very small. For these conditions the streamer mechanism[7] and the usual Townsend process involving values of the gamma coefficient of the order of magnitude of those usually measured[8] fail to account for the current multiplication necessary to attain breakdown. The theory developed here accounts for breakdown under both of these conditions for which Paschen's law fails. This theory was suggested for low-voltage breakdown by Germer and Haworth[5,9] and for breakdown at high pressures first by Zeier[10] and later by other workers,

[1] F. Paschen, Ann. Phys. u. Chem. **37**, 69 (1889).
[2] A. H. Howell, Trans. Am. Inst. Elec. Engrs. **58**, 193 (1939).
[3] A. Boulloud, Ann. phys. **8**, 909 (1953).
[4] D. R. Young, J. Appl. Phys. **21**, 222 (1950).
[5] L. H. Germer and F. E. Haworth, Phys. Rev. **73**, 1121 (1948).
[6] P. Kisliuk, J. Appl. Phys. **25**, 897 (1954).
[7] L. B. Loeb and J. M. Meek, *The Mechanism of the Electric Spark* (University Press, Stanford, 1941).
[8] M. J. Druyvesteyn and F. M. Penning, Revs. Modern Phys. **12**, 87 (1940).
[9] L. H. Germer and F. E. Haworth, J. Appl. Phys. **20**, 1085 (1949). This mechanism for breakdowns in air at extremely short distances has been considered also in reference 6 and by M. M. Atalla, Bell System Tech. J. (to be published).
[10] O. Zeier, Ann. Physik **14**, 415 (1932). Several authors have subsequently suggested this mechanism for breakdown at high pressures, notably Trump, Cloud, Mann, and Hanson, Elec. Eng. **69**, 961 (1950); F. L. Jones and C. G. Morgan, Phys. Rev. **82**, 920 (1951); A. Boulloud, Compt. rend. **233**, 932 (1951) and reference 4. None of these authors develops the ideas further, and some of them suggest alternatives. These alternatives are largely discredited in reference 3, where the author finds support for this mechanism by a process of elimination.

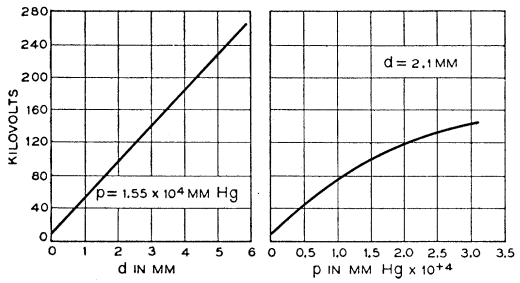

FIG. 1. Breakdown voltage plotted against distance for constant pressure, and against pressure at constant distance, showing the failure of Paschen's law at high pressure (data from reference 2).

but none of these developed the consequences of the theory beyond its initial conception.

In brief, the theory is as follows. When gas multiplication is small, the electric field becomes sufficiently large before breakdown to draw field-emission currents from small irregularities on the cathode. The field-emission electrons produce a small number of ions which, by their space charge, increase the field at the cathode. Because of the steep increase in current with field, this process is able to build up to breakdown even when the probability of any particular electron having an ionizing collision is extremely small.

Paschen's law is based on the observation that, over a large range of pressures and electrode separations, the probability of ionization per collision in the gas and the probability of the production of electrons by ions by a secondary process are both dependent on the average kinetic energy of the electrons and ions, and therefore[11] on E/p. The proposed process of electron emission due to the combined applied and ionic-space-charge fields depends primarily on E rather than E/p, leading directly to deviations from Paschen's law.

It is shown below that the process, whereby field emission is enhanced by the increased field due to approaching positive ions, will ultimately become unstable and breakdown will occur. In this paper this theory is developed in quantitative form and an explicit expression is obtained for γ', the effective yield of electrons per ion at breakdown. This expression may be compared with experimental data obtained from the literature on electrical breakdown in air at high pressures using the published values of η, the ionization coefficient per volt along an electron path.[12] The experimental values of γ' are found to vary with applied field at breakdown in the way predicted by theory, confirming the proposed mechanism. The theory accounts also for low-voltage breakdown across very short gaps in air at atmospheric pressure and in vacuum, although in these cases experimental data are not available for a quantitative check.

[11] L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939), pp. 410–412.
[12] F. H. Sanders, Phys. Rev. 44, 1020 (1933).

THEORY

Consider the empirical form of the field emission equation:

$$j_E = A \exp(-B/E), \tag{1}$$

where j_E is the electron current density, E is the field at the cathode, and A and B are constants. Let $E = E_A + E^+$, where E_A is the applied field and E^+ the field due to the positive-ion space charge. Because of the steepness of the dependence of current density on field, $E^+ \ll E_A$ and by expansion we find

$$j_E \approx A \exp(-B/E_A) \exp(BE^+/E_A^2) = j_0 \exp(BE^+/E_A^2), \tag{2}$$

where $j_0 = A \exp(-B/E_A)$ is the current which would flow due to the applied field in the absence of space charge. Furthermore, to a good approximation,

$$E^+ = Cj^+, \tag{3}$$

where C is a constant involving the ionic mobility and the electrode microgeometry and j^+ is the positive-ion current density. Let

$$j^+ = Gj_E{}^n, \tag{4}$$

where G is a constant involving the electron and ion trajectories and n is to be evaluated in the later discussion. Then, $E^+ = CGj_E{}^n$, and Eq. (2) becomes

$$j_E = j_0 \exp(BCGj_E{}^n/E_A^2) = j_0 \exp(Mj_E{}^n), \tag{5}$$

where, since E_A is a very weak function of j_E, the coefficients of $j_E{}^n$ have been included in a single constant, $M = BCG/E_A^2$.

The breakdown condition for this process can be determined by examining Eq. (5). Figure 3 is a plot of the left- and right-hand sides of Eq. (5) for the particular case $n=2$. For small j_0 there is a stable solution, whereas for large j_0 there is no solution and the breakdown condition has been exceeded; breakdown

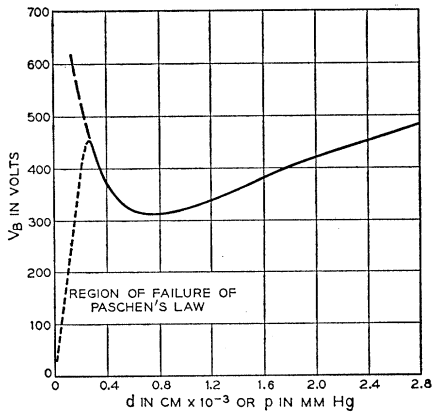

FIG. 2. Breakdown voltage plotted against distance at a constant pressure of 1 mm Hg (– – – –) and against pressure at a constant distance of 1 cm (— — — —), to show the failure of Paschen's law at short distances. In the solid portion (——) the two curves coincide and Paschen's law is valid.

occurs when the curves are tangent. Thus, the critical condition for breakdown is $(\partial/\partial j_E)[j_0 \exp(Mj_E{}^n)]=1$, which with Eq. (5) can be written $M=1/nj_E{}^n$, or

$$j_E=j_0 \exp(1/n). \quad (6)$$

Expressed in words, Eq. (6) states that breakdown occurs when the applied field is such that the field-emission current is increased by space charge by just the factor $\exp(1/n)$.

Let us now define the effective γ as the ratio of the field-emission electron current density to the incoming positive-ion current density,

$$\gamma=j_E/j^+=1/Gj_E{}^{n-1}=BC/E_A{}^2Mj_E{}^{n-1}, \quad (7)$$

by Eq. (4). From Eq. (6) this becomes at breakdown,

$$\gamma'=BCnj_E'/E_A{}'^2 = \{BCn[\exp(1/n)]/E_A{}'^2\}A \exp(-B/E_A'), \quad (8)$$

where γ', j_E', j_0', and E_A' are the values of these variables just at breakdown. Since the exponential dependence is strong enough to mask the variation in $1/E_A{}'^2$, γ' is given to a good approximation by,

$$\gamma'=K \exp(-B/E_A'), \quad (9)$$

where K is a constant and B the same constant that appears in the empirical expression for field emission, Eq. (1). Note that while K depends on n, the form of Eq. (9) is independent of n. This theoretical relation is a direct consequence of the theory of field emission breakdown. Confirmation of the theory will be obtained if there is a satisfactory agreement between Eq. (9) and published experimental data on breakdown at high pressures.

The usual formulation of the breakdown condition in terms of ionization by electrons and a secondary process of electron production is

$$\gamma'(e^{\eta' V'}-1)=1, \quad (10)$$

where $(e^{\eta' V'}-1)$ is the number of ionizing collisions resulting from the emission from the cathode of one electron, V' is the breakdown voltage, and η' is the number of ionizing collisions per volt of potential difference at breakdown. The quantity η is a strong function of E/p and values of η as a function of E/p are available in the literature for some gases. Thus if the pressure, p, the breakdown voltage, V', and the separation are known, γ' can be computed from Eq. (10). If breakdown data for a range of pressures are available, the experimental dependence of γ' on E_A' will be determined.

The comparison between theory and experiment can be made reliably only for the case of breakdown at high pressure. For breakdown at low voltage the material evaporated by pre-breakdown field emission currents may seriously affect the pressure which is therefore not sufficiently well known to compute γ'. Experimental data for three different conditions are considered below: breakdown at high pressure, low-voltage breakdown at atmospheric pressure, and low-voltage breakdown in high vacuum.

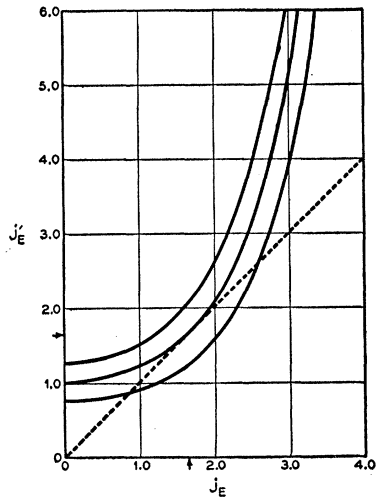

FIG. 3. Plot of $j_E'=j_E$(- - -), and $j_E'=j_0 \exp(Mj_E{}^2)$(——) for various values of j_0, in arbitrary units selected so that $j_0=1$ at breakdown.

Breakdown at High Pressure

At high pressure the exponent n in Eq. (4) is unity. The coefficients C and G are not strictly constants but their variations are second-order effects which leave the exponential form of Eq. (9) essentially unaffected. The coefficient B of Eqs. (1) and (9) can be obtained in terms of the apparent applied field from the Fowler Nordheim equation.[13] The expression is

$$B=6.85\times 10^7 \Phi^{\frac{3}{2}}/\beta, \quad (11)$$

where Φ is the work function and β the maximum field multiplication due to the surface irregularity giving rise to the emission. $\Phi^{\frac{3}{2}}$ is of the order of magnitude of 10, while β is of the order of 50, whence $B\approx 10^7$ volts/cm.

Plots of $\log\gamma'$ against $1/E_A'$ from experimental data taken in air are given in Fig. 4. It can be seen that the points fall on satisfactory straight lines. In Table I are values of B obtained from these plots. These are of the right order of magnitude.

It would be desirable to find an experimental check for gases in addition to air. There are very few gases, however, for which data are available for both of the required quantities, namely η at small values of E/p and the breakdown voltage between plane parallel electrodes at high pressure. Only N_2, H_2, CO_2, and air appear to be possibilities at present. Except for air, the agreement with Eq. (9) is poor.

This may be due, in part, to the effect of small concentrations of impurities which become important at high pressures, especially for gases of high ionization

[13] R. H. Fowler and L. Nordheim, Proc. Roy. Soc. (London) **A119**, 173 (1928).

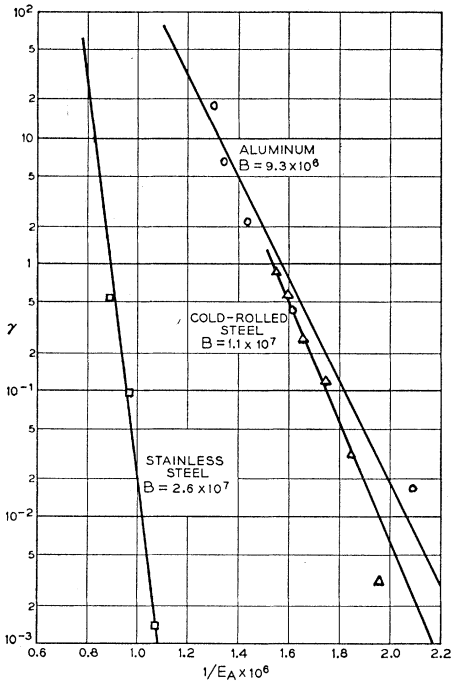

FIG. 4. γ computed from breakdown voltages in air plotted against the inverse of the apparent breakdown field $(1/E_A)$, from the data of reference 3 (stainless steel and aluminum) and reference 2 (cold rolled steel), using η values from reference 12.

potential. In other words, data on η taken for nearly pure gases at a particular value of E/p at low pressure may not be applicable to the same gases at the same value of E/p at much higher pressure. Furthermore there is direct evidence for CO_2 that η may not be a function of E/p alone at pressures approaching the condensation point.[4] Thus an experimental check of Eq. (9) is probably reliable only for a mixture including a considerable concentration of impurities, and air is the only such mixture for which we have data.

The proposed mechanism seems to be justified by the satisfactory fit of the straight lines in Fig. 4 and by the agreement of the numbers in Table I with the estimated value of B. A complete check, including computation of the constant K of Eq. (9), may be quite difficult, especially if there are complications due to electron attachment, ionization by metastables, etc., but fortunately knowledge of this constant is not necessary. It would be useful, however, in computing maximum pre-breakdown current densities and accurate breakdown fields.

A number of experimental observations can be explained by the proposed mechanism. The breakdown field between plane parallel electrodes at high pressure is expected to depend strongly on the work function and roughness of the cathode surface. This is justified by a great many investigations.[2-4] An observation by Howell[2] that anode roughness also affects the breakdown voltage may be discounted because his treatment of the anode must have left loose particles which, in the presence of the high field, will be exchanged between electrodes.

Young[3] has observed that deviations from Paschen's law occur at lower pressure for smaller electrode spacings. This is to be expected because the decreased gas multiplication at smaller spacings allow the breakdown fields to increase to the values necessary for field emission at lower pressures.

Low Voltage Breakdown in Air

Assume for the moment, that the pressure between the electrodes is unknown, either because of material evaporated by pre-breakdown currents or because, in the case of closing electrodes, air may be compressed between them. The ionization coefficient, η, cannot have a value more than its maximum which is ≈ 0.015 for air.[8] Since breakdowns have been observed as low as 30 volts,[7] breakdown is possible when $(e^{\eta V}-1)$ is less than 0.65, whence, for breakdown, [by Eq. (15)] $\gamma > 1.5$. This is very large, typical values observed in glow discharges[8] being of the order of 0.01. If the pressure is atmospheric, E/p is much greater than the value corresponding to a maximum η for the observed breakdown fields, so that η is well below its maximum, and γ may well be in excess of four electrons/ion. This is consistent with the observed very short times of current buildup in these breakdowns.[14,6]

The observed breakdown fields are of the order necessary to draw measurable field emission currents when the same electrodes are placed in vacuum. Thus the situation is proper for a small ionic space charge to increase the current greatly as described previously.

Experiments have been conducted to observe the pre-breakdown currents when electrodes are closed in air with potentials less than the minimum sparking potential ($\sim$300 v). When the potential difference was above 180 volts, these currents were too small to be observed on the equipment used (<3 μa). For the lowest potential differences at which breakdown could be observed ($\sim$30 volts), the pre-breakdown currents increased to about 20 microamperes. This increase in pre-breakdown current is undoubtedly a result of the higher breakdown field needed, according to Eq. (9), to obtain the required high value of γ.

The value of n in Eq. (5) is uncertain for low-voltage air breakdown because it is not known whether

TABLE I. Experimental values of the slope of the logγ *versus* $1/E$ curves.

	B(volts/cm)
Cold rolled steel[a]	1.1×10^7
Stainless steel[b]	2.6×10^7
Aluminum[b]	9.3×10^6

[a] Data from reference 2.
[b] Data from reference 3.

[14] L. H. Germer and J. L. Smith, J. Appl. Phys. **23**, 553 (1952).

the pressure between the electrodes depends on the pre-breakdown current. It is, in any case, between 1 and 3.

Low-Voltage Breakdown Between Closely Spaced Electrodes in High Vacuum

In this case the pre-breakdown currents may become much larger than in air, amounting to several milliamperes for breakdown voltages of the order of one kilovolt. The power dissipated on the anode is then sufficient to evaporate metal, breakdown occurring in the resulting vapor.[15] The electron current density, in addition to being the primary means of ionization, affects the positive ion current density by producing the vapor, the pressure of which determines η and the ionic mobility. The power of j_E occurring in Eq. (5) is thus between two and three depending on whether the breakdown pressure is sufficient to affect the velocity of the ions.

Examination of pulsed field-emission currents with uniformly increasing applied voltage up to the point of breakdown fail to show any observable evidence of enhancement of the current directly by ionization in the gas.[15] This is consistent with the field-emission mechanism of breakdown because a very small fraction of ionic current is sufficient to increase the field-emission current to the point where Eq. (6) is satisfied.

CONCLUSION

A quantitative treatment of the role played by field emission in the electrical breakdown of gases has been given. In particular, a new criterion for electrical breakdown has been formulated which appears to have wide application. The regenerative field-emission breakdown process applies whenever large fields sufficient to draw field-emission currents appear at the cathode. This situation arises when, either because of high pressure or close electrode separation, the probability of an electron making an ionizing collision is small.

The deviation from Paschen's law which occurs for all gases at sufficiently high pressure is satisfactorily explained. In the particular case of air, where the ionization coefficient is known over the appropriate range of E/p, the theory is in very good quantitative agreement with the form of the breakdown curve at high pressures.

For the case of closely spaced electrodes in air, it is known that breakdown may occur at a potential difference far below the minimum sparking potential determined by a Townsend mechanism involving the usual measured values of γ. It has been shown that this limitation does not apply for the field-emission process, however, and that breakdown will always occur as long as the potential difference between the electrodes is sufficient to give at least some ionization of the gas and the electrode separation is sufficiently small to lead to a certain critical field emission current.

It should be noted finally that this breakdown criterion should apply equally well to liquids of high dielectric strength where once again electrical breakdown is preceded by the flow of field-emission currents.

[15] Boyle, Kisliuk, and Germer, J. Appl. Phys. (to be published).

PHYSICAL REVIEW VOLUME 97, NUMBER 2 JANUARY 15, 1955

Nuclear Susceptibility of Liquid and Solid He^3

P. J. PRICE
I. B. M. Watson Scientific Computing Laboratory, Columbia University, New York, New York
(Received September 1, 1954)

An interpretation of the observed deviation from Curie's law of the nuclear paramagnetic susceptibility of liquid helium He^3, based on a "cell" type of model of the liquid, is presented. The theory is able to account qualitatively, but not precisely, for the deviation. The reasons for its imprecision are discussed. The bearing of the point of view of the theory on the paramagnetism of the solid is considered: it is concluded that there should be no deviation from Curie's law at the lower temperatures, but that a small deviation near the rising part of the melting curve is quite possible.

LIQUID helium He^3 has a magnetic susceptibility due partly to the diamagnetism of the atomic electron shells and partly to the paramagnetism of the orientation distribution of nuclear magnetic moments. The latter contribution has recently been measured separately, by the method of resonance absorption, by Fairbank *et al.*[1,2] This paramagnetic susceptibility (which we shall denote by χ, since there will be no need of a symbol for the diamagnetism) is of interest in the theory of quantum liquids, because its magnitude is a direct measure of the influence of quantum statistics. The actual magnetic interaction energy of neighboring atoms is very small compared with kT at any attainable temperature; and hence, if Boltzmann statistics applied, the value of χ would be

$$\chi_B = N\mu^2/kT \text{ per mole}, \quad (1)$$

[1] Fairbank, Ard, Dehmelt, Gordy, and Williams, Phys. Rev. 92, 208 (1953).
[2] Fairbank, Ard, and Walters, Phys. Rev. 95, 566 (1954).