Flow of Helium II Through Narrow Slits*

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The hydrodynamic properties of helium II have been studied by observation of the flow rate in a narrow slit between two flat glass plates. The isothermal, pressure-induced flow rates have been measured with hydrostatic pressures up to 2300 dynes/cm^2, at temperatures between 1.39 and 2.10°K, and with average slit widths of 2.4 and 4.3 microns. For a fixed pressure head, the observed flow rates have the same temperature dependence as the superfluid concentration in helium II. At pressure heads below 1600 dynes/cm^2 the flow rate is proportional to a power of the pressure. This power is nearly independent of temperature, but may be a function of slit width.

INTRODUCTION

ONE of the most characteristic properties of helium II is the superfluid nature of the flow of the liquid through narrow channels. The flow of helium II can be initiated either by a mechanical potential or by a thermal potential. In the experiments described here only pressure-induced, isothermal flow has been investigated.

A number of investigations[1–5] have shown that in channels wider than 10^{-2} cm the flow properties of helium II are extremely complex, but in some respects are similar to those predicted for a viscous fluid by classical hydrodynamics. As the channel width is decreased below 10^{-2} cm the flow behavior departs from that of a classical viscous fluid. In channels whose width is of the order of one micron or less the flow velocity tends to become independent of the pressure head and the length of the flow channel. Bowers and Mendelssohn[4] have shown that in radial flow through a slit between two annular plates the pressure gradient is practically zero everywhere in the slit except at its narrowest perimeter. The pressure heads employed in their experiments did not exceed six cm of helium. The investigation reported here is an extension of the flow measurements to pressure heads of approximately 15 cm of helium by means of the geometrical arrangement of Bowers and Mendelssohn.

EXPERIMENTAL TECHNIQUE

The apparatus used to study the flow properties of helium II under the influence of a hydrostatic pressure gradient is shown in Fig. 1. It consists of a glass tube (the liquid reservoir) which ends in a polished flange. A flat glass plate is pressed against the flange by means of a clamp and spring. In this way a flow channel whose width is fairly uniform over the area of the slit is formed. The width of the slit is not constant, since after assembly of the apparatus a few broad and slightly curved interference fringes can be seen. From the shape of the fringes and from the shape of the clamping device it is probable that the slit is slightly wider at the inner perimeter than at the outer. In order to measure the hydrostatic pressure at an intermediate point in the flow channel, a one-mm diameter glass tube opens into a small circular groove cut in the flange. The pressure at this point is determined by the height of the liquid column in the tube. The size of the groove is chosen so that the Bernoulli force on the liquid column in the side tube is negligible.

Both the central reservoir and the side tube are provided with approximately 1-mm holes at the top. These holes insure that the temperature of the liquid in the reservoir and in the side tube is the same as the bath temperature. Temperature equalization occurs by evaporation of the liquid or condensation of the vapor, whichever is necessary.[3] If such isothermal conditions were not maintained, the thermomechanical effect would contribute to the flow, and the results would be difficult to interpret. The holes also allow the mobile surface film of helium II to flow into or out of the reservoir. The direction of the film flow is such as to give an apparent increase in flow rate through the slit. To determine the correction to the measured flow rates, an experiment has been made with the slit blocked with glycerine. The rate of film flow was found to be 1×10^{-4} cm^3/sec at a temperature of 1.4°K and to be nearly

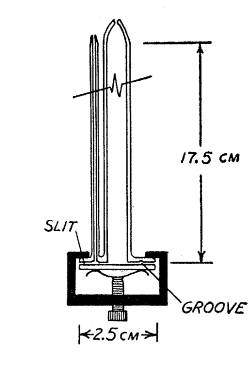

FIG. 1. The experimental apparatus.

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1 J. F. Allen and A. D. Misener, Proc. Roy. Soc. (London) **A172**, 467 (1939).
2 Johns, Wilhelm, and Grayson Smith, Can. J. Research **A17**, 149 (1939).
3 K. R. Atkins, Proc. Phys. Soc. (London) **A64**, 833 (1951).
4 R. Bowers and K. Mendelssohn, Proc. Phys. Soc. (London) **A63**, 178 (1950); Proc. Roy. Soc. (London) **A213**, 158 (1952).
5 G. K. White, Proc. Phys. Soc. (London) **A64**, 554 (1951).

independent of pressure. At all the temperatures to be considered the correction for the film flow amounts to a few percent at very low pressures and is completely negligible at the higher pressures. Since the correction is within the experimental uncertainty in the measured flow rates, it has been ignored in the analysis of the results.

In order to detect the presence of small temperature differences between the reservoir and the bath, a carbon resistance thermometer was placed in the reservoir, and a similar thermometer in the bath. These thermometers are quite sensitive at low temperatures,[6] and temperature changes of a few tenths of a millidegree can be measured. In the case of the resistor in the reservoir the electrical leads were brought out through the hole in the top of the reservoir. They were then dipped into the helium bath before being taken outside the cryostat in order to minimize heat leaks through the wires. As a further precaution these leads were made from No. 40 tantalum wire, which is superconducting at the temperatures under consideration and hence has a greater thermal resistance than normal metals. Any runs in which there was a measurable temperature difference between the bath and the reservoir were discarded. However, the smallest temperature difference which could be measured was about 0.001°K, so that small thermomechanical pressures may have existed, particularly when the reservoir was above the bath level. Several runs were also made with the resistors removed. They yielded the same results as the runs in which the resistors were present.

The apparatus was suspended from a brass holder, which could be raised out of the helium bath or lowered into it. The technique of measurement was as follows. The position of the reservoir relative to the bath level was quickly changed. Then readings were taken of the heights of the levels in the reservoir and in the side tube, relative to the bath level, as a function of time, as the liquid flowed either into or out of the reservoir. The level heights were determined by observation of a scale, graduated in millimeters and 17.5 cm in length, etched onto the side of the glass reservoir. Time measurements were made in two ways. The first measurements were made by concurrent visual observation of the reservoir level and a stop watch. These measurements were later checked with an Esterline-Angus recording milliammeter as a time indicator. A graphic record of the time intervals was made by energizing the meter with a hand switch while visually observing the reservoir. The roll speed was approximately 3 in./min, so that time measurements could be made to better than $\frac{1}{4}$-sec accuracy. The illumination necessary for these measurements was provided by six two-watt neon bulbs, mounted in a vertical column and located one meter from the cryostat. During a measurement the temperature of the bath was kept constant to within four thousandths of a degree by means of a differential oil manometer and a needle valve in the pumping line.

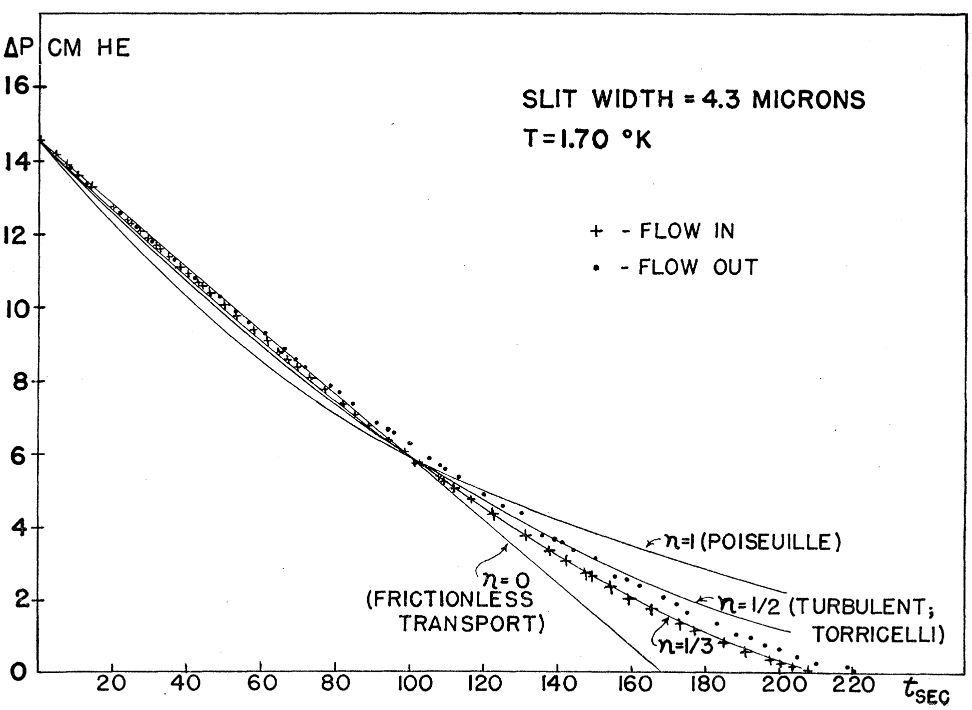

FIG. 2. Experimental observations of liquid height *vs* time. The points are the experimental points of several different measurements normalized at high pressures. The solid lines are obtained from $\dot{V} \propto (\Delta p)^n$.

[6] Boorse, Zemansky, and Brown, Phys. Rev. 84, 1050 (1951).

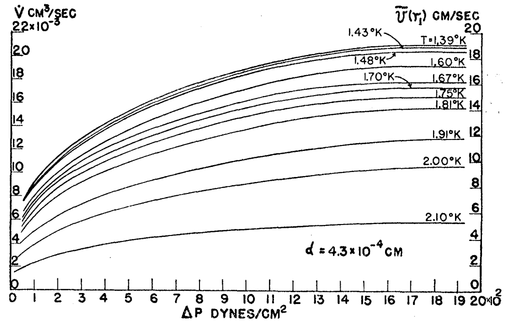

FIG. 3. Volume and linear flow rates *vs* pressure at constant temperature. Slit width=4.3μ.

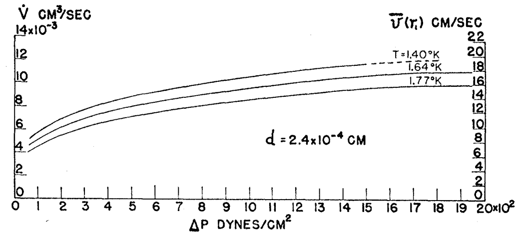

FIG. 4. Volume and linear flow rates *vs* pressure at constant temperature. Slit width=2.4μ.

The vapor pressures were converted to temperatures on the 1949 temperature scale.[7]

In order to determine the flow velocities, the slope of the curves of height h *versus* time t must be obtained. (See Fig. 2.) This analysis has been done in two ways. The flow rates were first obtained by differentiation of the expression $h=h_0+a_1t+a_2t^2+a_3t^3$, where the constants h_0, a_1, a_2, and a_3 were obtained from the experimental data by the method of least squares. From five to twenty measurements were made at one temperature, and the resulting values of h *versus* t plotted on a single graph. The least-squares fit was made to a smooth curve drawn through these points.

The second method of obtaining the slopes of the h *versus* t curves utilizes the information obtained by the first method. The first method revealed that at least for low pressure heads, the discharge rate was proportional to a power of the pressure head $\dot{h}\propto(\Delta p)^n$ with $n\simeq\frac{1}{3}$. Since the bath level is practically constant during a measurement, the flow rate is proportional to $\dot{h}$ and the pressure head proportional to h, where h is the difference between the bath level and the reservoir level. Integration of the relation $\dot{h}=-\alpha h^{\frac{1}{3}}$ yields an expression of the form $h=h_0(1-t/t_0)^{\frac{3}{2}}$, which contains two adjustable parameters and can be fitted to the experimental points. One of the solid curves in Fig. 2 is a plot of this function. Both of the above methods of analysis yield analytic curves that fit the data within the experimental error and give the same results at low pressure heads. They yield somewhat different results at the highest pressure heads.

The volume rate of flow of liquid through the slit, $\dot{V}$, at a given pressure head Δp is the derivative dh/dt, at a given value of h, multiplied by the cross-sectional area of the reservoir, which is 0.196 cm². The pressure is given by ρgh, where ρ is the density of liquid helium at the temperature under consideration, g the acceleration of gravity, and h the height. To determine the linear velocities of flow, the width of the slit must be measured. This was done by observation of the rate of flow of helium I through the slit.[4] The average linear velocity of flow in the slit is given by $\bar{v}(r)=\dot{V}/(2\pi rd)$, where r is the distance from the axis of the reservoir to the point in the slit at which $\bar{v}$ is desired, and d is the average or effective width of the slit. In the curves to follow, the average linear velocity is calculated for r equal to the inner radius of the slit, which is 0.41 cm. This is the maximum velocity, since the velocity decreases as r^{-1} as one proceeds to the outer perimeter of the slit.

[7] H. van Dijk and D. Shoenberg, Nature **164**, 151 (1949).

EXPERIMENTAL RESULTS

By the methods described in the preceding section the volume rate of flow of liquid helium through the slit may be calculated as a function of the hydrostatic pressure head, at a constant temperature. Experiments have been made using two different slit widths, and the results will be classified into two groups, corresponding to the two widths. The first group, with a slit width of 4.3×10^{-4} cm, covers the temperature range between 1.39°K and 2.10°K in steps of at most 0.11°K. The pressures involved extend to 2300 dynes/cm², i.e., that given by about 16 cm of helium. This range of temperatures and pressures represents the greatest variation which can be obtained in a simple way. At temperatures close to the λ point the flow rate is so small that the normal fluid flow and the film flow become an appreciable fraction of the total flow. On the other hand, although temperatures below 1.39°K can be obtained, the quantity of liquid which remains is too small to allow the use of large pressure heads. The other group involves measurements at three temperatures made with a slit width of 2.4×10^{-4} cm. From the results of the two groups it is possible to estimate the influence of the slit width on the flow rate.

The pressure dependence of the flow rate for the two groups is shown in Figs. 3 and 4. The quantity $\dot{V}$ is the volume rate of flow of liquid in cm³/sec. The quantity $\bar{v}(r_1)$ is the average linear velocity of flow in cm/sec through the slit at its inner perimeter (r_1). The curves give the results obtained by the first method of analysis. The second method gives values for the flow rates which are the same as those of the first method at low pressure heads but which are about 15 percent larger at the highest pressure heads. In order to determine if a relation of the form $\dot{V}=(\text{constant})(\Delta p)^n$ is valid for these results, logarithmic plots of flow rate *versus* pressure head are given in Figs. 5 and 6. The solid lines indicate the results of the first method of

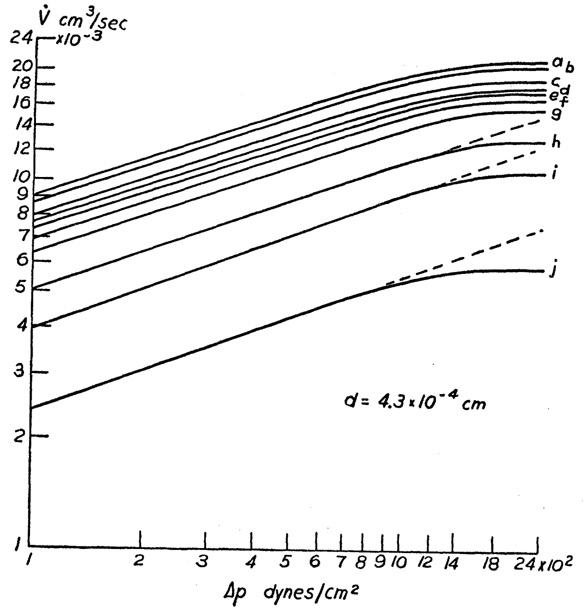

FIG. 5. Logarithmic plot of volume flow rate *vs* pressure at constant temperature for the 4.3-μ slit. Curves *a–j* represent temperatures of 1.39, 1.48, 1.60, 1.67, 1.70, 1.75, 1.81, 1.91, 2.00, and 2.10°K respectively. The solid curves are obtained from the 1st method of analysis and the dashed curves from the 2nd method.

analysis. The second method of analysis yields straight lines that are extrapolations of the lower pressure curves obtained from the first method. Some of these lines are indicated with dashed lines. From Figs. 5 and 6 it is evident that for the lower values of the pressure head the flow rate is proportional to a power of the pressure. In agreement with other workers,[1,4,5] the value of the index n is nearly independent of temperature, although there may be a slight increase in n at temperatures near the λ point. The following table gives n as a function of temperature for the two slit widths:

(a) $d=4.3\mu$, $\Delta p=800$ dynes/cm²:

$T=$1.39 1.48 1.60 1.67 1.70 1.75 1.81 1.91 2.00 2.10

$n=$0.33 0.34 0.33 0.33 0.33 0.33 0.33 0.35 0.37 0.36;

(b) $d=2.4\mu$, $\Delta p=1400$ dynes/cm²:

$T=$1.40 1.64 1.77

$n=$0.27 0.27 0.28.

Figures 5 and 6 show that the first method of analysis yields flow rates that approach saturation values at the highest pressures. Since these saturation values do not appear in the second method of analysis, we conclude that they may well be due to a systematic error inherent in the first method of analysis. Since the difference between a pressure independent flow rate and a flow rate depending on the $\frac{1}{3}$ power of the pressure cannot be distinguished at these pressures, we must wait for measurements at higher pressure heads to settle this question. These measurements are in progress at the present time.

The side tube behavior is the same as that reported by Bowers and Mendelssohn.[4] For either inflow or outflow, the side tube follows the bath level within the capillary rise of 1 mm. During one set of three runs with substantially the same average slit width as in previous runs, the side tube registered approximately $\frac{1}{10}$ of the reservoir pressure. This may have been caused by a chance confluence of small irregularities in the glass which did not affect the flow of helium I, from which the slit width was determined, but which did affect the flow of helium II. However, the reservoir level behaved in the usual way in these runs, and we have not been able to reproduce the unusual side tube behavior.

The linear velocities of flow through the slit have also been determined in these studies. Since the determination of these velocities requires a knowledge of the slit width, they are subject to a greater uncertainty than are the volume flow rates. The uncertainty in the measured width of the slit amounts to about 15 percent. In view of this it is not possible to make a precise comparison between the results for the two slit widths. However, it is evident from the curves of Figs. 3 and 4 that the average linear velocity of flow does not decrease when the slit is made narrower. It is likely that the average linear velocity actually increases as the slit width is decreased, as has been found to be the case by Allen and Misener[1] and by Bowers and Mendelssohn.[4]

Figure 7 shows the temperature variation of the volume flow rate at various values of the pressure head, for the 4.3-micron slit. In Fig. 8 the temperature dependence of the flow rate is compared with that of the superfluid concentration ρ_s/ρ. The values of ρ_s/ρ are taken from Andronikashvili.[8] The comparison is made at pressures of 200, 700, and 1800 dynes/cm². The values of $\dot{V}$ are normalized to agree with the ρ_s/ρ curve at 1.81°K. It is evident that the temperature dependence of the flow is essentially the same as that of the superfluid concentration.

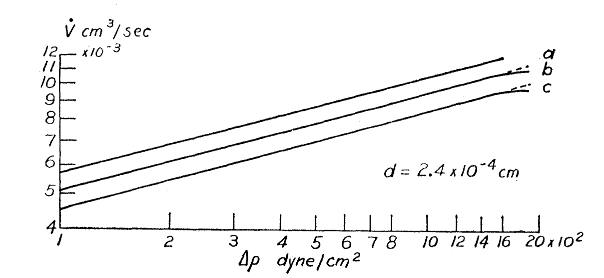

FIG. 6. Logarithmic plot of volume flow rate *vs* pressure at constant temperature for the 2.4-μ slit. Curves *a*, *b*, and *c* represent temperatures of 1.40, 1.64, and 1.77°K respectively. The solid curves are obtained from the first method of analysis and the dashed curves from the second method.

[8] E. L. Andronikashvili, J. Exptl. Theoret. Phys. (U.S.S.R.) 18, 424 (1948).

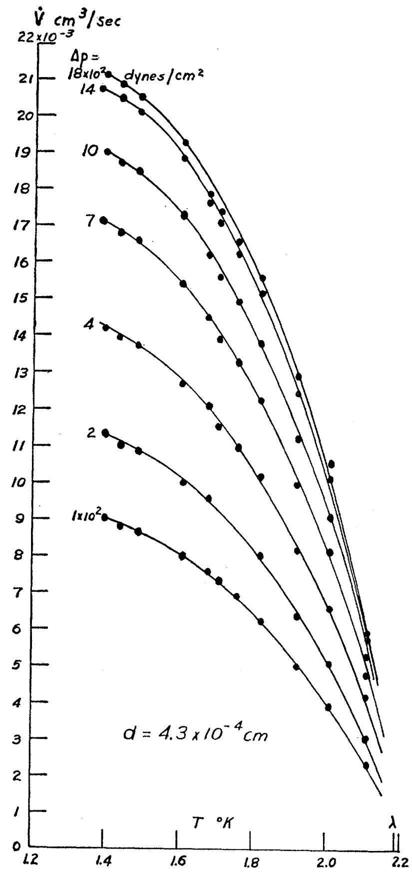

FIG. 7. Volume flow rate *vs* temperature at constant pressure. Slit width = 4.3 μ (first method of analysis).

DISCUSSION OF THE RESULTS

(A) Accuracy of the Flow Rates

The maximum uncertainty in the experimental observations amounts to 0.05 cm in the height of the liquid column and 0.25 second in the time. This leads to a maximum uncertainty in flow rate of 15 percent at high rates and 10 percent at very low flow rates, or at high pressures and low pressures respectively. The associated uncertainty in the pressure varies from a negligible quantity at high pressures to about 10 percent at low pressures. These uncertainties are greater than the errors of measurement of the geometrical constants of the apparatus by a factor of 2 or more, so that the latter errors were neglected. The power n in the relation $\dot{V} \propto (\Delta p)^n$ which best fits the experimental points over the whole range of pressure heads can be determined within approximately 10 percent. It cannot be accurately determined at high pressure heads, as evidenced by the different results of the two methods of analysis.

Several mechanisms exist which can give rise to errors in the observed flow rates. These errors will vary with the pressure head and can influence the measured pressure dependence of the flow rates. If isothermal conditions are not maintained in the flow process, the resulting thermomechanical pressure gradient will be opposite in sign to the hydrostatic pressure gradient. Thus the net pressure on the liquid in the reservoir will be smaller than the measured pressure. Another possible source of error, which will have the same effect on the flow rates as the above situation, is the absorption of radiant energy by the liquid in the reservoir when the latter is raised above the bath level. The contribution of this effect to the flow velocities should be greatest at low pressures, i.e., when the volume of liquid in the reservoir is small. Excessive evaporation losses can also cause high apparent flow rates if the reservoir is too far above the bath level.

Experimental verification of these errors has been obtained. Upon plotting the experimental results of height *versus* time, nearly all the points corresponding to measurements made with the reservoir below the bath level lie on a smooth curve. On the other hand, the points corresponding to the reservoir above the bath level show much more scatter and deviate slightly from the points obtained with the reservoir below the bath (see Fig. 2). In view of these observations, and also in view of the divergence of the streamlines during outflow, the flow rates have been calculated from only those measurements made with the reservoir below the bath level. It is felt that in this way the errors introduced by the thermomechanical effect and excess evaporation are minimized, because temperature equalization will take place more rapidly when the reservoir is below the bath level than when it is above the bath.

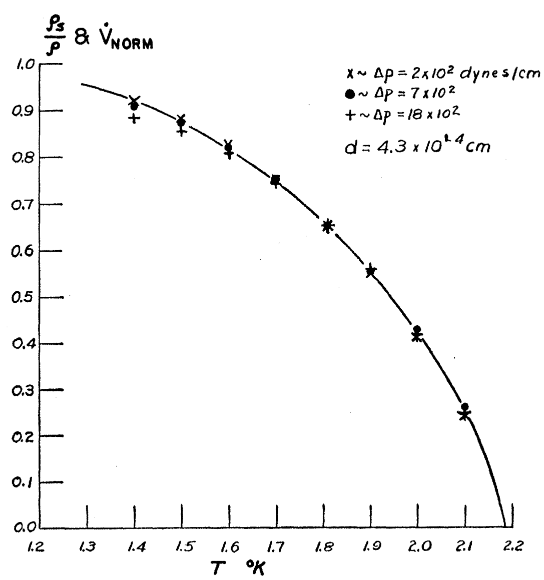

FIG. 8. Comparison of the temperature dependence of the volume flow rate with that of the superfluid concentration. The solid curve represents ρ_s/ρ. The flow rate is normalized to ρ_s/ρ at 1.81°K. Slit width = 4.3 μ.

A further possibility of error arises from the presence of the side tube. When the position of the reservoir and side tube is changed relative to the bath level, the liquid level in the side tube adjusts itself rapidly to approximate coincidence with the bath level. During the time of equalization of levels, the flow in the side tube will decrease the flow in the reservoir. Such an error is avoided by delaying the start of a measurement until the level in the side tube is static. The time required for this is from 6 to 10 seconds, depending on the temperature.

Contamination by solidified gases is a frequent source of difficulty in flow measurements. This is especially true in the case of the helium film. Little is known of the effect of contamination on flow in narrow channels. We have taken no special care to outgas the flow equipment, other than to carefully evacuate the apparatus and flush it with helium. Elaborate vacuum techniques[9] are required to obtain a surface free from any adsorbed gases. Extremely high temperatures and low pressures (10^{-10} to 10^{-11} mm Hg) are necessary to remove the last layer. It is likely that no flow measurements have been made on surfaces not contaminated by at least one layer of adsorbed gas. A test was made to determine the effect of allowing air to come in contact with the flow apparatus. At the end of two of the runs, a small amount of air was let into the system. No observable difference could be detected in the flow rates or side tube behavior. It seems likely that contamination does not easily occur unless the flow surfaces are directly exposed to the gas, as is the case in film experiments.

These mechanisms are believed to be the only important ones which could falsify the observed pressure dependence of the flow velocity. The measurements are made in such a way as to minimize these errors.

(B) Theoretical Considerations

To summarize the experimental results, the dependence of volume flow rate on pressure head indicates the following facts. For pressure heads below approximately 10 cm of helium, the flow rate is proportional to a power n of the pressure head. The constant of proportionality depends on the temperature as the superfluid concentration, ρ_s/ρ. The index n is nearly independent of temperature, but may decrease with decreasing slit width. Its value is approximately $\frac{1}{3}$. For the highest pressure heads, there is some indication that the flow rates become less dependent on the pressure, although this behavior can only be established by measurements at higher pressures. The flow rates can be expressed as:

$$\dot{V}=B(\rho_s/\rho)(\Delta p)^{\frac{1}{3}}\ \text{cm}^3/\text{sec},$$

with Δp the pressure head in dynes/cm^2 and $B=2.1\times10^{-3}$ cgs units for the 4.3-μ slit.

An attempt has been made to correlate the experimental observations with the existing theories of the hydrodynamical properties of helium II. In the absence of a temperature gradient, the equations of motion as given by Gorter and Mellink[10] may be written as:

$$\rho_s\frac{d\mathbf{v}_s}{dt}=-\frac{\rho_s}{\rho}\nabla p-\mathbf{F}_{sn}-\mathbf{F}_s,$$

$$\rho_n\frac{d\mathbf{v}_n}{dt}=-\frac{\rho_n}{\rho}\nabla p+\mathbf{F}_{sn}+\eta_n[-\nabla\times\nabla\times\mathbf{v}_n+4/3\nabla(\nabla\cdot\mathbf{v}_n)]-\mathbf{F}_n.$$

The flow of the normal fluid in the 4.3-μ slit is a small fraction of the total flow, except at temperatures very close to the λ point. Measurements in helium I indicate that the normal fluid flow is of the order of 1/100 of the superfluid flow. Under these circumstances, we may disregard the second equation and put $v_n=0$. The pressure distribution and flow characteristics will then be determined by the properties of the superfluid. The first equation becomes:

$$\rho\frac{d\mathbf{v}_s}{dt}=-\nabla p-\frac{\rho}{\rho_s}\mathbf{F}_{sn},$$

where $\mathbf{F}_{sn}$ is the force between the superfluid, and the normal fluid and the walls. We consider several different possibilities.

(a) *Perfect Fluid:* $\mathbf{F}_{sn}=0$

For this case the equation of motion integrates to give Bernoulli's equation. If we assume streamline flow filling the slit, the following behavior would be expected:

1. The index n in the relation for the pressure dependence of the flow rate should be $=\frac{1}{2}$ (Torricelli's theorem). This value is not observed. In addition, the pressure drop necessary to produce the average velocities of approximately 20 cm/sec is much smaller than the measured drop:[11]

$$h=(v^2/2g)^{\frac{1}{2}}\simeq2\ \text{mm of He}.$$

2. Because of the increase of the cross section of the flow channel with increase in r, the flow rates for flow out should be greater than those for flow in. However, the streamlines are diverging for flow out, and the flow pattern is extremely unstable.[12] It is probable that streamline flow would break down near the narrowest section of the slit and the liquid would break away from one wall. In this case the two rates would be nearly equal. This consideration suggests that the flow ge-

[9] Homer D. Hagstrum, Rev. Sci. Instr. **24**, 1122 (1953).

[10] C. J. Gorter and J. H. Mellink, Physica **15**, 285 (1949).

[11] This drop would be increased by the assumption of an effective mass for the acceleration of the superfluid in the slit greater than the gravitational mass.

[12] L. Prandtl and O. G. Tietjens, *Applied Hydro- and Aeromechanics* (McGraw-Hill Book Company, Inc., New York, 1934), p. 52.

ometry is probably not a good one to use to determine in what ways He II behaves as a perfect fluid.

3. The side tube behavior depends on the conditions that exist in the groove. If there is no dissipation (streamline flow from the inner radius to the outer), the side tube should follow the reservoir for flow out and the bath for flow in. If the kinetic energy of the fluid flowing into the groove is completely dissipated, then the side tube pressure should be $\frac{1}{5}$ of the total pressure head for flow out and $\frac{1}{4}$ the total head for flow in. On the other hand, if the flow breaks down as described under 2, the side tube would follow the bath for either flow in or flow out.

The model of a perfect fluid is thus not very satisfactory, since it gives the wrong magnitude for the flow rate and also a wrong power dependence. Qualitatively, the behavior of the side tube and the equality of inflow and outflow rates can be explained if we assume that the flow does not fill the channel except in the narrowest constriction.

(b) *Mutual Friction:* $\mathbf{F}_{sn}=A\rho_n\rho_s\mathbf{v}_s^3$

This expression for $\mathbf{F}_{sn}$, due to Gorter and Mellink,[10] can be used in the equation for the superfluid motion and a solution obtained for the slit geometry of these experiments. We assume a constant velocity profile and take from the equation of continuity:

$$\mathbf{v}_s(r)=v_s(r_i)[r_i/r]\mathbf{i}_r,$$

where $v_s(r_i)$ is the (maximum) velocity at the inner radius of the slit and $\mathbf{i}_r$ is the unit vector in the r direction. We must solve:

$$\rho\frac{d\mathbf{v}_s}{dt}=\rho\left[\frac{\partial\mathbf{v}_s}{\partial t}+(\mathbf{v}_s\cdot\nabla)\mathbf{v}_s\right]=-\nabla p-A\rho\rho_n\mathbf{v}_s^3.$$

Now,

$$(\mathbf{v}_s\cdot\nabla)\mathbf{v}_s=\tfrac{1}{2}\nabla\mathbf{v}_s^2-\mathbf{v}_s\times\nabla\times\mathbf{v}_s.$$

The first term on the right (the Bernoulli term) is negligible, as shown in (a), and we take $\nabla\times\mathbf{v}_s=0$ (irrotational flow). Also, for steady flow $\partial\mathbf{v}_s/\partial t=0$. Then we obtain

$$\nabla p=-A\rho\rho_n\mathbf{v}_s^3,$$

or, since $\nabla p=(\partial p/\partial r)\mathbf{i}_r$,

$$p_r-p_{r0}=\tfrac{1}{2}A\rho\rho_n\mathbf{v}_s^3(r_i)r_i^3\left[\frac{1}{r^2}-\frac{1}{r_0^2}\right],$$

with r_0= outer radius of flow channel. The following results are obtained:

1. For the total pressure drop, we obtain

$$\Delta p=0.026A\rho_n v_s^3(r_i),\text{ or } v_s(r_i)=38(A\rho_n)^{-\frac{1}{3}}(\Delta p)^{\frac{1}{3}}.$$

The dependence of the average velocity on pressure head is correctly given. However, the temperature dependence of the velocity at constant pressure is wrong, if we take A as a constant. Also, the pressure heads required to produce the observed average velocities of approximately 20 cm/sec are too low. If we take $A\rho_n=2$ cgs units, in accordance with the work of Hung, Hunt, and Winkel,[13] then $\Delta p=3$ cm of He is the required pressure head. This is too small by almost an order of magnitude.

2. Side tube behavior. The observed behavior is a strong argument against the presence of any dissipative force, such as mutual friction, in the slit. The side tube pressure should be $\frac{1}{6}$ of the total pressure head, or at least $\frac{1}{6}$ of 3 cm of helium, according to the mutual friction theory. It has been suggested by Heaps[14] that the dissipation could occur in a wider part of the flow channel as the fluid enters the slit, since the mutual friction seems operative in wide channels. However, it is likely that this success of the mutual friction theory is associated with the presence of normal fluid flow, and in any event there should be a similar dissipation as the fluid leaves the slit.

The mutual friction theory does not seem adequate to explain the observed behavior. Although the correct pressure dependence of the flow rates is given, the temperature dependence and the side tube behavior are in disagreement with the theory. This conclusion does not mean that the mutual friction theory is not of importance for higher velocities and wider slits. In fact, much of the most recent work[13] indicates that it is not applicable to the low velocities and narrow slits encountered in our experiments.

(c) *Critical Velocity Theory*

Mendelssohn and his co-workers[4] have advanced a theory for the flow of helium II involving the existence of a critical velocity. The theory has not been formalized to the extent of the Gorter-Mellink theory. The liquid flow is described as a frictionless transport under zero pressure gradient if the flow velocity is less than a critical velocity of the order of 20–50 cm/sec. The entire pressure drop is supposed to occur across the part of the flow channel having the greatest resistance to flow, and in particular across the narrowest constriction. For velocities greater than critical, frictional forces come into consideration which in some cases can be described by the Gorter-Mellink equations.

Since many of the elements of the critical velocity theory were inferred from experiments similar to those presented here, the results obtained are in qualitative agreement with the theory. However, the theory is not quantitative, and there are many unclarified points. The pressure dependence of the flow for velocities less than critical is not given, and in fact no reason is given for any pressure drop at all. There is no positive evidence of a critical velocity in our experiments, although sufficiently high pressure heads may not have been used.

Thus, the critical velocity theory is capable of explaining some of the qualitative features of the results.

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[13] Hung, Hunt, and Winkel, Physica **18**, 629 (1952).

[14] W. V. Houston and C. W. Heaps (private communication).

It fails in providing any quantitative relations for the flow behavior. In the next section we suggest a way of providing some of these relations by combining some of the characteristics of the critical velocity theory with the properties of a perfect fluid.

(d) Modification of the Velocity Profile

It has been suggested by Houston[14] that the anomalous pressure dependence of the flow could be brought into a theoretical frame-work by modifying the velocity profile. Prandtl[15] has shown how a velocity profile can be chosen to yield any given pressure dependence for the average velocity. However, his arguments apply only to turbulent flow, and imply a dissipative interaction with the walls. The behavior of the side tube rules out such dissipation, and Prandtl's theory is not directly applicable.

Most interpretations of the flow velocities suffer from two defects: the velocity of flow is taken as a constant across the flow channel, and the average flow velocity is equated to the superfluid velocity. There is considerable evidence[16] to indicate that the velocity profile is not a straight line, but that the transfer in narrow channels is a surface effect. Mott[17] has presented arguments to show that a surface flow of the superfluid around a stationary core of liquid can be stable under certain conditions. Kapitza[18] suggested the same picture on the basis of the results of his experiments on heat conduction in capillaries. We tentatively adopt such a picture and choose a velocity profile as in Fig. 9. The experimental results can be summarized as:

$$\Delta p=\left[\frac{\rho}{\rho_s}\frac{\dot{V}}{B}\right]^3,\quad \dot{V}=2\pi r_i d\langle(\rho_s/\rho)v_s(r_i)\rangle_{\text{Av}}.$$

(The average flow velocity $\bar{v}$ is set equal to $\langle(\rho_s/\rho)v_s\rangle_{\text{Av}}$.) If we adopt a modified Bernoulli theorem, then $\Delta p=\frac{1}{2}\langle\rho v_s^2\rangle_{\text{Av}}$ gives the pressure drop at the entrance to the slit. For the velocity profile of Fig. 9,

$$\langle v_s\rangle_{\text{Av}}=\frac{2}{d}\int_0^{d/2}v_s(z)dz=2v_0t/d,$$

$$\langle v_s^2\rangle_{\text{Av}}=2v_0^2t/d,$$

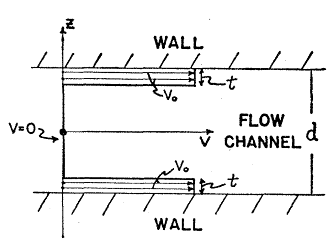

FIG. 9. Velocity profile for flow of the superfluid through a narrow slit: $v=0$ for $|z|<(d/2-t)$; $v=v_0$ for $(d/2-t)<|z|<(d/2)$.

[15] L. Prandtl and O. G. Tietjens, reference 12, p. 70; L. Prandtl, Z. angew. Math. u. Mech. **5**, 136 (1925).
[16] J. G. Daunt and R. S. Smith, Revs. Modern Phys. **26**, 177 (1954).
[17] N. F. Mott, Phil. Mag. **40**, 61 (1949).
[18] P. L. Kapitza, J. Phys. (U.S.S.R.) **4**, 181 (1941).

where v_0 is the average velocity of the superfluid in a layer of thickness t. To satisfy the Bernoulli equation, we must have

$$\tfrac{1}{2}\langle\rho v_s^2\rangle_{\text{Av}}=\left(\frac{\rho/\rho_s}{B}\right)^3\dot{V}^3$$

or

$$\tfrac{1}{2}\rho\langle v_s^2\rangle_{\text{Av}}=\left(\frac{\rho}{\rho_s}\right)^3\left(\frac{2\pi r_i d}{B}\right)^3\left(\frac{\rho_s}{\rho}\right)^3\langle v_s(r_i)\rangle_{\text{Av}}^3.$$

Putting in the values of $\langle v_s\rangle_{\text{Av}}$ and $\langle v_s^2\rangle_{\text{Av}}$, we obtain

$$v_0t^2=C,\quad C=\frac{\rho}{d}\left(\frac{B}{4\pi r_i}\right)^3.$$

The quantity C is a constant and can be evaluated from the experimental results: $C=23\times10^{-9}$ cm^3/sec. The product v_0t is proportional to the flow rate: $(\rho_s/\rho)v_0t=\dot{V}/4\pi r_i$. Therefore, as the discharge rate increases, v_0 must increase and t decrease. Values of v_0 and t can be obtained from the experimental results: $v_0=7.2(\Delta p)^{\frac{2}{3}}$ cm/sec, $t=5.7\times10^{-5}(\Delta p)^{-\frac{1}{3}}$ cm. For $\Delta p=2100$ dynes/cm^2 the values are $v_0=12$ m/sec and $t=4.5\times10^{-6}$ cm. These figures are in some respects reasonable. v_0 is of the order of the critical velocities calculated on the basis of Landau's theory,[19] and t is of the order of the film thickness.

The temperature dependence of the flow is explained on the basis of the two-fluid theory. If we assume thermal equilibrium in the slit, only a fraction ρ_s/ρ of the slit cross section will be available for flow of the superfluid. At a given pressure head, the velocity of the superfluid is determined. Therefore, the flow rate will have the same temperature dependence as the "effective" slit cross section, or as ρ_s/ρ. Also, the average velocity of the superfluid is, on this picture, not equal to $\bar{v}$. Instead, $\bar{v}_s=\bar{v}\rho/\rho_s$ gives the average superfluid velocity.[20] $\bar{v}_s$ is, of course, the average velocity measured at sufficiently low temperatures.

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[19] R. B. Dingle, Advance in Physics **1**, 111 (1952).
[20] J. G. Daunt and R. S. Smith, Revs. Modern Phys. **26**, 217 (1954).