trating showers produced in the light materials, the effect of heavy meson production is considered small. This is justified from the cloud chamber investigations carried out at approximately the same altitude and in approximately the same energy range.[18]

The results concerning the bremsstrahlung produced by high energy cosmic-ray $\mu$ mesons confirm the essential correctness of the assumptions based upon which the calculations were made.

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# New Formulation of the General Theory of Relativity

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The principle of equivalence is partly abandoned as a basis for general relativity, and cosmic time is introduced as a new field variable. Field equations are obtained which take account of the self-energy of the gravitational field. The central symmetrical solution of the new field equations shows a significant deviation from the well-known Schwarzschild solution. It is free from singularities and gives a slightly smaller value for the perihelion motion of planetary orbits. Other consequences of the new formalism are:

(a) A rigorous definition can be given to the concept of ether.
(b) The energy-stress tensor of gravitational fields can be defined in a satisfactory manner.
(c) The gravitational field energy of a particle is distributed continuously over the space and its integral is equal to the gravitational mass of the particle.
(d) There are proper gravitational waves, generated by oscillating matter and propagating with the velocity of light.
(e) There is a noticable ether drift which tends to increase the gravitational mass of a body of given inertial mass.
(f) The ratio of gravitational and inertial mass of radiating energy is twice the corresponding ratio for neutral static matter.
(g) Hubble's recession constant is equal to the reciprocal of the age of the universe.

An outstanding problem is to determine the coupling constant between the gravitational field and the cosmic time field. The value $\beta=1$ is strongly suggested by cosmological considerations. An experimental determination is possible if the rate of advance of the perihelion of the Mercury orbit is known more accurately.

## 1. INTRODUCTION

THE general theory of relativity rests upon the so-called *principle of equivalence* which states that: (a) It is possible to choose at every point of the space-time continuum a frame of reference which is Galilean at that particular point, i.e., in which special relativity holds in the immediate neighborhood of the point. (b) All frames of reference are equivalent in the sense that there is no general physical property which would distinguish one particular frame (or even a whole class of frames) from among the others.[1]

The two statements have an entirely different standing. Whereas postulate (a) is firmly established and supported by a considerable mass of experimental evidence, postulate (b) rests upon an essentially negative statement which obviously cannot be verified directly. In fact its validity has often been challenged in view of certain conceptual difficulties which arise from it both on the cosmical and local scales.

On the cosmical scale, postulate (b) is clearly in conflict with one of the most important cosmological principles known as *Weyl's postulate*, which necessarily leads to the notion of absolute cosmic time.[2] Although no formulation of Weyl's postulate has ever been given which would reveal that cosmic time has any noticeable physical effects, the conceptual conflict between the two principles can hardly be denied.

On the local scale, it is a well-known weakness of general relativity that it is incapable of defining the energy-stress tensor of the gravitational field in a satisfactory manner. The only known quantity which can be regarded as a substitute for the energy tensor is a pseudo-tensor which, if the principle of equivalence is accepted, can be transformed away in a suitable frame of reference. Closely connected with this is the following observation which was actually the starting point of the present investigations.

[1] A third postulate, often quoted in connection with the principle of equivalence, requires that physical laws should have a form which does not depend on the particular frame which one happens to use. This is not really a physical but an epistemological postulate; it expresses the belief that physical laws can be put in a particular mathematical form, the desirability of which can hardly be disputed.

[2] See H. Bondi, *Cosomlogy* (Cambridge University Press, Cambridge, 1952), p. 70.

In Einstein's law of gravitation,[3]

$$-R_{mn}+\tfrac{1}{2}Rg_{mn}=T_{mn}, \tag{1}$$

the quantity $T_{mn}$ on the right is the total energy-stress tensor which receives contributions from any source of energy that is present. Thus if electromagnetic field alone is present—no neutral matter—then one has to put the Maxwell-Poynting tensor for $T_{mn}$, multiplied, of course, by the constant of gravitation. This expresses the fact that electromagnetic energy, like any other other form of energy, creates gravitation. On the other hand, if gravitational field alone is present, i.e., in "empty space," nothing is put for $T_{mn}$, although it can hardly be denied that gravitational fields contain energy. If we accept the principle that every sort of energy creates gravitation, then $T_{mn}$ should never be zero, not even in empty space, but should be equal to the energy-stress tensor of the gravitational field if other fields are absent.

On the intermediary (planetary) scale there is a further difficulty connected with the principle of equivalence, the problem of inertia. It is well known that apart from local variations of the gravitational field, a frame of reference which does not rotate with respect to distant stars and nebulas is very nearly a Galilean frame. This remarkable fact, first emphasized by E. Mach, remains a most unlikely coincidence when viewed in the light of the principle of equivalence. The usual explanation, based on Mach's principle which makes the distant stars and galaxies themselves responsible for inertia, does not resolve the difficulties completely. What is obviously needed is some sort of physical reality which, propagating in space and time, would determine the frames of inertia at every point.

To resolve these difficulties which seem to be inherent in the principle of equivalence, various modifications of the formalism of general relativity have been proposed. Without striving for completeness, I shall only mention some of the suggestions that have been made more recently. One is an attempt by Hoyle[4] to incorporate Weyl's postulate in the framework of general relativity. Hoyle describes the motion of Weyl's cosmological substratum by a time-like vector field $C_m$ which is supposed to be curl-free (in order to satisfy Weyl's postulate), so that its covariant derivative,

$$C_{mn}=C_{m;n}=C_{m,n}-\left\{\begin{matrix} r \\ mn \end{matrix}\right\}C_r,$$

is a symmetric tensor. Here $C_{m,n}$ denotes the partial derivative $\partial C_m/\partial x_n$. He then proposes a modification of Eq. (1) by adding the term $C_{mn}$ to the right-hand side. The purpose of the term is to replace Einstein's cosmological term and is supposed to be of the same order of smallness as the latter, hence negligible in planetary dimensions. This in any case precludes any measurable effects and extricates the theory from the range of direct experimental verification.

Another modification of general relativity has been proposed by Rosen[5] and more recently by Kohler.[6] These authors introduce a second metric tensor corresponding to a flat space-time which would provide a sort of Galilean background to the gravitational metric. This enables them of course to define the gravitational energy-stress tensor in a quite satisfactory manner. Nevertheless, from the point of view of relativity the procedure must be regarded as a retrograde step, since it renders the introducing of the proper (gravitational) metric tensor rather pointless, or at least somewhat artificial.

The basic idea of the present investigations is similar to Hoyle's: We introduce a scalar field variable $\tau$, called *cosmic time*, and postulate an interaction between the metrical and $\tau$ fields. The interaction, however, is supposed to be quite macroscopic, and certainly not of cosmical smallness like in Hoyle's theory. The field equations are obtained from Hamilton's principle, using a Lagrangian which involves both the curvature scalar and a scalar derived from the $\tau$ field. The field equations are then solved for the central symmetrical case, under the assumption that the field-generating body is at absolute rest, i.e., at rest relatively to the inertial system determined by the gradient of $\tau$. We shall call this inertial system the *ether*. According to this definition, *ether is a state of motion determined uniquely by the gradient of $\tau$ at every point of the space-time continuum.*

The central symmetrical solution has several interesting features. First, it has no Schwarzschild-type singularity but is continuous everywhere including the origin. Consequently, its geodesics are not identical with those in Schwarzschild's solution. Assuming that planetary orbits are geodesics in the metrical field, one obtains a perennial precession of the perihelion which is $[1-(\beta/6(2+\beta))]$ times the Einstein value, where $\beta$ is a certain positive constant. On cosmological grounds there is some reason to believe that the value of the constant $\beta$ is 1. This would give for Mercury 40.5″ per century instead of 43″, a value which is certainly not ruled out by present experimental determinations. For the deflection of light in the sun's gravitational field one obtains the same value as in ordinary relativity, except for terms of second order smallness.

Another feature of the solution is that it is possible to define with the help of $T_{mn}$ and the $\tau$ field an energy-momentum vector density $\rho^n$ in such a manner that the integral of $\rho^0$ over the whole space has a finite value and in fact is equal to the gravitational mass of the central symmetrical body. Thus the energy density of neutral matter is continuously distributed over the space, instead of being concentrated in singularities.

[3] Roman indices go from 0 to 3. The metric ground form in a locally Galilean frame is $ds^2=dx_0^2-dx_1^2-dx_2^2-dx_3^2$.

[4] F. Hoyle, Monthly Notices Roy. Astron. Soc. **108**, 372 (1948) and **109**, 365 (1949).

[5] N. Rosen, Phys. Rev. **57**, 147 (1940).

[6] M. Kohler, Z. Physik **131**, 571 (1952) and **134**, 286 (1953).

The explicit expression for $\rho^n$ is

$$\rho^n = \frac{1}{8\pi\kappa}(C_k C^k)^{-1}(2C^s \mathfrak{T}_s{}^n - C^n \mathfrak{T}_s{}^s), \qquad (2)$$

where $\kappa$ is the gravitational constant. In the case of an ideal fluid with rest density $\mu$ and isotropic pressure $p$, resting in the ether,[7] this gives

$$\rho = \rho^0 = \mu + 3p, \quad \rho^1 = \rho^2 = \rho^3 = 0. \qquad (3)$$

If the fluid is moving with uniform velocity $v$ relatively to the ether[8] in the direction of the positive $x_1$ axis and the pressure is negligible, then in a frame moving with the fluid we get

$$\rho^0 = (1-v^2)^{-\frac{1}{2}}\mu, \quad \rho^1 = v(1-v^2)^{-\frac{1}{2}}\mu, \quad \rho^2 = \rho^3 = 0. \quad (4)$$

Thus the gravitational mass density is $(1-v^2)^{-\frac{1}{2}}\mu$, as opposed to the inertial mass density which is $\mu$. This is a typical ether-drift effect which, if it exists, cannot be reconciled with the principle of equivalence.

If $\alpha$ is the angular distance from the south pole of the ecliptic of the sun's direction of absolute motion, $v_s$ its absolute velocity, and $v_e$ the orbital velocity of the earth relatively to the sun, then the square of the absolute velocity of the earth is between the values $v_s^2 + v_e^2 \pm 2v_e v_s \sin\alpha$, and the maximum seasonal variation of the factor $(1-v^2)^{-\frac{1}{2}}$ is

$$|(1-v_s^2-v_e^2-2v_e v_s \sin\alpha)^{-\frac{1}{2}} - (1-v_s^2-v_e^2+2v_e v_s \sin\alpha)^{-\frac{1}{2}}| \sim 2v_e v_s |\sin\alpha|.$$

A seasonal variation of the same magnitude can therefore be predicted for the gravitational acceleration at any point of the earth's surface. To estimate the magnitude of the effect, we have to know the direction and magnitude of the absolute velocity of the solar system. This was determined by Miller from ether-drift experiments of the Michelson-Morley type,[9] the positive outcome of which is yet unexplained. According to Miller $\alpha = 7°$, $v_s = 0.7 \times 10^{-3}$, and since $v_e = 10^{-4}$, the relative magnitude of the above effect is expected to be $1.7 \times 10^{-8}$. This is perhaps just within the limits of observability with present-day experimental techniques.

In a frame which rests in the ether, the components of the vector density (4) become

$$\rho^0 = \frac{1+v^2}{1-v^2}\mu, \quad \rho^1 = \frac{2v}{1-v^2}\mu, \quad \rho^2 = \rho^3 = 0. \qquad (5)$$

Hence the gravitational mass of a high-velocity particle with rest mass $m$ is not $(1-v^2)^{-\frac{1}{2}}m$ but $(1+v^2)(1-v^2)^{-\frac{1}{2}}m$. In the limiting case of photons $v=1$, and therefore the gravitational mass is double the inertial mass, i.e., *the apparent gravitational constant of radiating energy is twice the gravitational constant of static matter.* The same result can be obtained from formula (2) directly, if one puts for $T_n{}^m$ the energy-stress tensor of isotropic radiation. The result is quite unexpected and it throws a new light on the fact that Einstein's light deflection value is twice the Newtonian value.

In Sec. 7 it will be shown that the scalar quantity $\Psi = \Box\tau$ propagates in waves generated by nonstatic matter. Thus inertial frames are affected by every change in the matter distribution of distant parts of the universe, the disturbance being propagated with the velocity of light. So our new formulation of general relativity removes the most important conceptual difficulties which arise from the principle of equivalence, without upsetting the logical structure of the theory.

The new formalism may also help in clearing up some difficulties which have arisen in connection with recent experimental findings of Finlay-Freundlich. In the early days of relativity Einstein predicted that the Fraunhofer lines in light coming from heavy stars must show a shift towards the red, in the proportion $q = (g_{00})^{\frac{1}{2}}$. Recent measurements of the sun by Finlay-Freundlich[10] seem to indicate that at the central portions of the disk, the observed red shift is considerably smaller than the predicted value, but increases rapidly towards the limb, eventually exceeding the Einstein value. It has been suggested[11] that part of the observed effect is nongravitational and is due to photon-photon collisions. There is, however, a small residual red shift which may be due to gravitational effects and which is about one-fifth of the Einstein value, a result which is also confirmed by observations on Sirius B.

At the end of Sec. 6 it will be shown that if one makes the perfectly plausible assumption that in Bohr's frequency relation,

$$E_2 - E_1 = h\nu,$$

$\nu$ is to be measured not in metric but in cosmic time units, then the higher metric frequency of light emitted by an atom on the sun just compensates the red shift to be expected and the observed gravitational red shift should be zero. It appears therefore that the actual (experimental) red shift of a photon is not 1/5 but 6/5 times the theoretical value. It is interesting to note in this connection that Freundlich's deflection value for photons in the sun's gravitational field also exceeds the predicted value, by about the same amount. It is not unlikely that the two discrepancies have a common origin.

There are various cosmological models compatible with the new field equations. One of them, corresponding to $\beta = 1$, is a particularly simple one. It has a linear expansion law and has the constant energy density 3/2 when measured in absolute cosmical units.

[7] The velocity of light is taken to be 1.

[8] It is assumed in the calculations that there is no significant ether drag by the moving body.

[9] D. C. Miller, Revs. Modern Phys. **5**, 203 (1933). Miller found for optical phenomena an ether drag coefficient 0.0514.

[10] E. Finlay-Freundlich, Nachr. Akad. Wiss. Göttingen Math.-physik. Kl., **1954a**, No. 7 (1954).

[11] E. Finlay-Freundlich, Proc. Phys. Soc. (London) **A67**, 192 (1954) and M. Born, Proc. Phys. Soc. (London) **A67, 193 (1954)**.

## 2. THE FIELD EQUATIONS

The fundamental assumption underlying these investigations is that cosmic time $\tau$ is not a mere coordinate but a scalar field variable which has the property that its gradient,

$$C_m = \tau_{,m} = \partial\tau/\partial x_m, \tag{6}$$

is a time-like vector, $g^{mn}C_mC_n > 0$. Since we want to derive the field equations from Hamilton's principle, the fundamental problem is to find a suitable expression for the world Lagrangian density in which the action of the $\tau$ field has been incorporated. In postulating a Lagrangian we cannot very well reply on empirical considerations since nothing definite is known about the interaction of the $g_{mn}$ and $\tau$ fields. In fact physicists have never found it necessary (or even desirable) to postulate the existence of a $\tau$ field at all. Hence we must reply on certain principles which are partly aesthetic (e.g., formal simplicity from the mathematical point of view) and partly motivated by the results we expect to find.

Let $R_{mrsn}$ be the covariant Riemannian curvature tensor and

$$R = g^{mn}g^{rs}R_{mrsn} \tag{7}$$

the curvature scalar. The corresponding density is written

$$\mathfrak{R} = \Delta^{\frac{1}{2}}R, \quad \Delta = -\det g_{mn}. \tag{8}$$

Now put

$$C_{mn} = C_{m;n} = C_{m,n} - C_r\left\{ {r \atop mn} \right\}, \tag{9}$$

$$C_{mrsn} = C_{mr}C_{sn}, \tag{10}$$

$$C = g^{mn}g^{rs}C_{mrsn}, \quad \mathfrak{C} = \Delta^{\frac{1}{2}}C. \tag{11}$$

Clearly $C_{mn}$ is a symmetrical tensor,

$$C_{mn} = \tau_{,m,n} - \left\{ {r \atop mn} \right\}\tau_{,r} = C_{nm}. \tag{12}$$

With these notations, the following expression suggests itself as a suitable Lagrangian density:

$$\mathfrak{L} = \mathfrak{R} + \tfrac{1}{2}\beta\mathfrak{C}, \tag{13}$$

where $\beta$ is a suitable constant. The numerical value of the coupling constant $\beta$ depends on the normalization of $\tau$ and is left undetermined for the time being.

From the mathematical-aesthetic point of view it is quite natural to combine the scalars $R$ and $C$. Expressions (7) and (11) have a very similar form, and a glance at (10) shows that $C_{mnrs}$ forms a kind of symmetrical counterpart to the tensor $R_{mnrs}$. In fact $C_{mnrs}$ has the symmetries

$$C_{mnrs} = C_{nmrs} = C_{mnsr} = C_{rsmn},$$

whereas $R_{mnrs}$ has the well-known symmetries

$$R_{mnrs} = -R_{nmrs} = -R_{mnsr} = R_{rsmn},$$

$$R_{mnrs} + R_{mrsn} + R_{msnr} = 0.$$

Note also that $C_{mnrs}$ does not involve higher than second-order derivatives of $\tau$. Of course, a final justification of postulate (13) must be postponed until we have explored some of its physical consequences.

Hamilton's principle requires that

$$\delta\int(\mathfrak{R} + \tfrac{1}{2}\beta\mathfrak{C})d^4x = 0 \tag{14}$$

for any variation of $g_{mn}$ and $\tau$ which vanishes at the boundary of the domain of integration. Variation of $g_{mn}$ gives

$$\delta\mathfrak{R} = (\mathfrak{R}_{mn} - \tfrac{1}{2}\mathfrak{R}g_{mn})\delta g^{mn},$$

where

$$R_{mn} = g^{rs}R_{rmns}.$$

$\delta C$ is obtained from (10), (11), and (12). We have

$$C_{mr} = C_{m,r} - \tfrac{1}{2}g^{ns}(g_{sm,r} + g_{sr,m} - g_{rm,s})C_n,$$

and a routine calculation gives, using partial integration and the relation $\delta g_{mn} = -g_{mr}g_{ns}\delta g^{rs}$,

$$\delta\mathfrak{C} = \{-C_m\mathfrak{C}_n{}^r{}_{;r} - C_n\mathfrak{C}_m{}^r{}_{;r} + C_{mn}\mathfrak{C}_r{}^r + C^r\mathfrak{C}_{mn;r} - \tfrac{1}{2}g_{mn}\mathfrak{C}\}\delta g^{mn}.$$

Hence the gravitational field equations are

$$R_{mn} - \tfrac{1}{2}Rg_{mn} + \beta T_{mn} = 0, \tag{15}$$

where

$$T_{mn} = -\tfrac{1}{2}C_mC_n{}^r{}_{;r} - \tfrac{1}{2}C_nC_m{}^r{}_{;r} + \tfrac{1}{2}C_{mn}C_r{}^r + \tfrac{1}{2}C^rC_{mn;r} - \tfrac{1}{4}g_{mn}C. \tag{16}$$

$T_{mn}$ is the gravitational field energy-stress tensor, it satisfies the conservation law

$$T_n{}^m{}_{;m} = 0. \tag{17}$$

This follows immediately from (9). (17) is a "weak" conservation law, i.e., not an identity but valid only in consequence of the field equations.

Leaving $g_{mn}$ unchanged and varying $\tau$ one obtains the supplementary field equation,

$$C^{mn}{}_{;n;m} = 0. \tag{18}$$

Since the term $\frac{1}{2}\beta C$ in (13) is macroscopic, of the same order of magnitude as $R$, it cannot replace the cosmological term, like in Hoyle's theory. Therefore, when dealing with the cosmological situation we shall have to add another term of cosmical smallness to the Lagrangian. Instead of the usual $-\gamma\Delta^{\frac{1}{2}}$, where $\gamma$ is a small constant of dimension $(\text{length})^{-2}$, we shall add the term $-\gamma\tau^{-2}\Delta^{\frac{1}{2}}$ to the right-hand side of (13), where $\gamma$ is an absolute constant provided that the unit of cosmic time is adjusted to the unit of metric length. It is assumed here that $\tau$ is measured from an absolute origin so that at the present epoch it has a very large value (in ordinary units). The final form of the Lagrangian is therefore

$$\mathfrak{L} = \mathfrak{R} + \tfrac{1}{2}\beta\mathfrak{C} - \gamma\tau^{-2}\Delta^{\frac{1}{2}}. \tag{19}$$

Hamilton's principle gives the cosmological field equations

$$R_{mn}-\tfrac{1}{2}Rg_{mn}+\beta T_{mn}+\tfrac{1}{2}\gamma\tau^{-2}g_{mn}=0, \qquad (20)$$

where $T_{mn}$ is defined by (16), and

$$\beta C^{mn}{}_{;n;m}+2\gamma\tau^{-3}=0. \qquad (21)$$

In the next section it will be shown that there exist simple cosmological solutions of these equations and a particularly simple model is obtained if $\beta=1$ and $\gamma$ has the value $\frac{3}{2}$.

### 3. COSMOLOGICAL MODELS

In order to obtain some information regarding the constants $\beta$ and $\gamma$ we begin with the discussion of some simple cosmological models which are compatible with the field equations. Our standpoint is that a cosmological model is "empty," i.e., *it derives all its energy from its gravitational field*. Later on we shall verify the correctness of this view in the case of a central symmetrical neutral body.

In all what follows, Greek suffixes shall run from 1 to 3 only. For reasons of uniformity it is desirable that in a cosmological model the line element shall have the form[12]

$$g_{00}=1,\quad g_{0\mu}=0,\quad g_{\mu\nu}=-\left(\delta_{\mu\nu}+\frac{K}{1-r^2K}x_\mu x_\nu\right)F,$$

where $F$ is a function of $t=x_4$ alone, $r^2=x_1^2+x_2^2+x_3^2$, and $K$ is a constant. In addition we require that also $\tau$ shall be a function of $t$ alone. In a strictly uniform universe we may even expect to find $K=0$ and $d\tau/dt=\alpha=\text{const}$. I am going to show now that the field equations have in fact solutions of this simple form. By adjusting the unit of cosmic time to the unit of metric time (i.e., by using a suitable normalizing factor for $\tau$) we can make $\alpha=1$, and hence

$$\tau=t. \qquad (22)$$

For the metric tensor we put

$$g_{00}=1,\quad g_{0\mu}=0,\quad g_{\mu\nu}=-\delta_{\mu\nu}F(t). \qquad (23)$$

The only nonvanishing Christoffel symbols are

$$\begin{Bmatrix}0\\ \mu\nu\end{Bmatrix}=\tfrac{1}{2}F'\delta_{\mu\nu},\quad \begin{Bmatrix}\mu\\ 0\nu\end{Bmatrix}=\tfrac{1}{2}(F'/F)\delta_{\mu\nu} \qquad (24)$$

and they give

$$R_{00}=\tfrac{3}{2}(F''/F)-\tfrac{3}{4}(F'/F)^2,\quad R_{0\mu}=0,$$
$$R_{\mu\nu}=[\tfrac{1}{2}(F''/F)+\tfrac{1}{4}(F'/F)^2]g_{\mu\nu},$$
$$R=3F''/F,$$

hence

$$R_{00}-\tfrac{1}{2}g_{00}R=-\tfrac{3}{4}(F'/F)^2, \qquad (25)$$

$$R_{0\mu}-\tfrac{1}{2}g_{0\mu}R=0, \qquad (25')$$

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=[-(F''/F)+\tfrac{1}{4}(F'/F)^2]g_{\mu\nu}. \qquad (25'')$$

[12] Reference 2, p. 102.

Similarly, we obtain from (9), (11), (22), (23), and (24):

$$C_0=1,\quad C_\mu=0,\quad C_{00}=C_{0\mu}=0,\quad C_{\mu\nu}=-\tfrac{1}{2}F'\delta_{\mu\nu},$$
$$C_r{}^r=\tfrac{3}{2}(F'/F),\quad C=\tfrac{3}{4}(F'/F)^2,$$
$$C_0{}^r{}_{;r}=-\tfrac{3}{4}(F'/F)^2,\quad C_\mu{}^r{}_{;r}=0,\quad C_{00;0}=C_{0\mu;0}=0,$$
$$C_{\mu\nu;0}=\tfrac{1}{2}[(F''/F)-(F'/F)^2]g_{\mu\nu};$$

hence by (16),

$$T_{00}=\tfrac{9}{16}(F'/F)^2,\quad T_{0\mu}=0, \qquad (26)$$

$$T_{\mu\nu}=\tfrac{1}{4}[(F''/F)-\tfrac{1}{4}(F'/F)^2]g_{\mu\nu}. \qquad (26')$$

These, together with (20) and (25)–(25″), give

$$(\tfrac{9}{16}\beta-\tfrac{3}{4})(F'/F)^2+\tfrac{1}{2}\gamma t^{-2}=0, \qquad (27)$$

$$(\tfrac{1}{4}\beta-1)(F''/F)+(\tfrac{1}{4}-\tfrac{1}{16}\beta)(F'/F)^2+\tfrac{1}{2}\gamma t^{-2}=0. \qquad (28)$$

Finally, (21) gives

$$-\tfrac{3}{2}\beta\frac{F'}{F}\frac{F''}{F}+\tfrac{3}{8}\beta\left(\frac{F'}{F}\right)^3+2\gamma t^{-3}=0. \qquad (29)$$

Equation (27) shows that $F$ must have the form[13]

$$F=t^\alpha, \qquad (30)$$

and the equations give the relations

$$(\tfrac{9}{16}\beta-\tfrac{3}{4})\alpha^2+\tfrac{1}{2}\gamma=0,$$
$$(\tfrac{1}{4}\beta-1)(\alpha^2-\alpha)+(\tfrac{1}{4}-\tfrac{1}{16}\beta)\alpha^2+\tfrac{1}{2}\gamma=0,$$
$$-\tfrac{3}{2}\beta\alpha(\alpha^2-\alpha)+\tfrac{3}{8}\beta\alpha^3+2\gamma=0.$$

Eliminating $\gamma$ from the first two equations, we get, assuming that $\alpha\neq 0$,

$$\beta=8/(3\alpha+2); \qquad (31)$$

hence, by the first equation,

$$\gamma=\tfrac{3}{2}\alpha^2(3\alpha-4)/(3\alpha+2). \qquad (32)$$

The third equation is also satisfied by these values, and we get a one-parameter family of solutions.

Discarding $\alpha=0$ which gives $\gamma=0$ and leads to a flat space, the most promising solution is the one corresponding to $\alpha=2$ which leads to a linearly expanding model. It gives

$$\beta=1,\quad \gamma=\tfrac{3}{2}, \qquad (33)$$

and the line element becomes

$$ds^2=dt^2-t^2\sum dx_\mu{}^2.$$

Properties of the more general model,

$$g_{00}=1,\quad g_{0\mu}=0,\quad g_{\mu\nu}=-t^\alpha\delta_{\mu\nu},\quad \alpha>0 \qquad (34)$$

will be investigated in the last section. For later reference we note the following formulas which follow easily

[13] Apart from a constant factor, which however can be made 1 by a trivial transformation of the space coordinates.

from (24), (26), (26′), (30), (31), (32), and (34):

$$\left\{ {0 \atop \mu\nu} \right\} = \tfrac{1}{2}\alpha t^{\alpha-1}\delta_{\mu\nu}, \quad \left\{ {\mu \atop 0\nu} \right\} = \tfrac{1}{2}\alpha t^{-1}\delta_{\mu\nu}, \tag{35}$$

$$T_0{}^0 = \tfrac{9}{16}\alpha^2 t^{-2}, \quad T^\mu{}_\nu = \tfrac{1}{16}\alpha(3\alpha-4)t^{-2}\delta_{\mu\nu}, \tag{36}$$
$$T = T^r{}_r = \tfrac{3}{8}\alpha(3\alpha-2)t^{-2}.$$

### 4. THE GRAVITATIONAL FIELD OF A CENTRAL SYMMETRICAL BODY

We proceed now to integrate the field equations under the assumption that the cosmological term can be neglected and that the metric tensor has the form

$$g_{00} = g, \quad g_{0\mu} = 0, \quad g_{\mu\nu} = -[\delta_{\mu\nu} + (f-1)\xi_\mu\xi_\nu], \tag{37}$$

where for abbreviation we have put

$$\xi_\mu = x_\mu / r,$$

and $f$ and $g$ are functions of $r = (x_1{}^2 + x_2{}^2 + x_3{}^2)^{\frac{1}{2}}$ alone. We also assume that $\tau$ depends only on $t$, or in other words that the body is at rest relatively to the ether.

From (37) we obtain

$$g^{00} = g^{-1}, \quad g^{0\mu} = 0, \quad g^{\mu\nu} = -\delta_{\mu\nu} + (1-f^{-1})\xi_\mu\xi_\nu, \tag{38}$$
$$\Delta = fg,$$

and the nonvanishing Christoffel symbols are

$$\left\{ {\mu \atop 00} \right\} = \frac{1}{2}\frac{g'}{f}\xi_\mu, \quad \left\{ {0 \atop \mu 0} \right\} = \frac{1}{2}\frac{g'}{g}\xi_\mu,$$

$$\left\{ {\mu \atop \nu\rho} \right\} = \frac{1}{2}\frac{f'}{f}\xi_\mu\xi_\nu\xi_\rho + \frac{1}{r}(1-f^{-1})\xi_\mu(\delta_{\nu\rho} - \xi_\nu\xi_\rho), \tag{39}$$

$$\left\{ {\rho \atop \mu\rho} \right\} = \frac{1}{2}\frac{f'}{f}\xi_\mu.$$

They give the following expressions:[14]

$$R_{00} - \tfrac{1}{2}g_{00}R = \frac{1}{r^2}g\left\{1 - f^{-1} + r\frac{f'}{f^2}\right\}, \tag{40}$$

$$R_{0\mu} - \tfrac{1}{2}g_{0\mu}R = 0, \tag{40′}$$

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = \left[-\frac{1}{r}\frac{g'}{g} + \frac{1}{r^2}(f-1)\right]\xi_\mu\xi_\nu + f^{-1}\left[-\frac{1}{2}\frac{g''}{g} + \frac{1}{4}\left(\frac{g'}{g}\right)^2 + \frac{1}{4}\frac{f'g'}{fg} - \frac{1}{2r}\left(\frac{g'}{g} - \frac{f'}{f}\right)\right](\delta_{\mu\nu} - \xi_\mu\xi_\nu). \tag{40″}$$

[14] See P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, Inc., New York, 1942), p. 201.

Since $\tau$ only depends on $t$, we obtain by (6), (9), (11), (38), (39)

$$C_0 = \tau', \quad C_\mu = 0, \quad C^0 = g^{-1}\tau', \quad C^\mu = 0, \quad C_{00} = \tau'',$$

$$C_0{}^0 = g^{-1}\tau'', \quad C_{\mu\nu} = C_\nu{}^\mu = 0, \quad C_{0\mu} = -\frac{1}{2}\frac{g'}{g}\tau'\xi_\mu,$$

$$C_0{}^\mu = \frac{1}{2}\frac{g'}{fg}\tau'\xi_\mu, \quad C_\mu{}^0 = -\frac{1}{2}\frac{g'}{g^2}\tau'\xi_\mu,$$

$$C_r{}^r = g^{-1}\tau'', \quad C = g^{-2}(\tau'')^2 - \tfrac{1}{2}(g')^2 f^{-1}g^{-3}(\tau')^2,$$

$$C_0{}^r{}_{;r} = \frac{1}{f}\left[\frac{1}{2}\frac{g''}{g} - \frac{1}{4}\left(\frac{g'}{g}\right)^2 - \frac{1}{4}\frac{f'g'}{fg} + \frac{1}{r}\frac{g'}{g}\right]\tau' + \frac{1}{g}\tau'',$$

$$C_\mu{}^r{}_{;r} = -\frac{g'}{g^2}\tau''\xi_\mu,$$

$$C_{00;0} = \tau''' + \frac{1}{2}\frac{(g')^2}{fg}\tau', \quad C_{0\mu;0} = -\frac{g'}{g}\tau''\xi_\mu,$$

$$C_{\mu\nu;0} = \frac{1}{2}\left(\frac{g'}{g}\right)^2\tau'\xi_\mu\xi_\nu,$$

$$C^{\mu r}{}_{;r;\mu} = \frac{1}{fg}\left(\frac{g''}{g} - 2\left(\frac{g'}{g}\right)^2 - \frac{1}{2}\frac{f'g'}{fg} + \frac{2}{r}\frac{g'}{g}\right)\tau'',$$

$$C^{0r}{}_{;r;0} = \frac{1}{fg}\left(\frac{1}{2}\frac{g''}{g} + \frac{1}{4}\left(\frac{g'}{g}\right)^2 - \frac{1}{4}\frac{f'g'}{fg} + \frac{1}{r}\frac{g'}{g}\right)\tau'' + \frac{1}{g^2}\tau'''.$$

With these values one obtains

$$T_{00} = \frac{1}{f}\left[-\frac{1}{2}\frac{g''}{g} + \frac{5}{8}\left(\frac{g'}{g}\right)^2 + \frac{1}{4}\frac{f'g'}{fg} - \frac{1}{r}\frac{g'}{g}\right](\tau')^2 + \tfrac{1}{4}g^{-1}(\tau'')^2 - \tfrac{1}{2}g^{-1}\tau'\tau'', \tag{41}$$

$$T_{0\mu} = -\tfrac{1}{4}(g'/g^2)\tau'\tau''\xi_\mu, \tag{41′}$$

$$T_{\mu\nu} = -\frac{1}{g}\left[\frac{1}{8}\left(\frac{g'}{g}\tau'\right)^2 - \frac{1}{4}\frac{f}{g}(\tau'')^2\right]\xi_\mu\xi_\nu - \frac{1}{fg}\left[\frac{1}{8}\left(\frac{g'}{g}\tau'\right)^2 - \frac{1}{4}\frac{f}{g}(\tau'')^2\right](\delta_{\mu\nu} - \xi_\mu\xi_\nu), \tag{41″}$$

$$C^{rs}{}_{;s;r} = \frac{1}{fg}\left[\frac{3}{2}\frac{g''}{g} - \frac{7}{4}\left(\frac{g'}{g}\right)^2 - \frac{3}{4}\frac{f'g'}{fg} + \frac{3}{r}\frac{g'}{g}\right]\tau'' + \frac{1}{g^2}\tau'''. \tag{42}$$

Hence, using the field equations (15) and (18), we obtain from (40)–(40″), (41)–(41″), and (42)

$$\frac{1}{r^2}g\left(1-f^{-1}+r\frac{f'}{f^2}\right)+\beta f^{-1}\left[\frac{1}{2}\frac{g''}{g}-\frac{5}{8}\left(\frac{g'}{g}\right)^2-\frac{1}{4}\frac{f'g'}{fg}+\frac{1}{r}\frac{g'}{g}\right](\tau')^2+\tfrac{1}{4}\beta g^{-1}(\tau'')^2-\tfrac{1}{2}\beta g^{-1}\tau'\tau''=0, \quad (43)$$

$$(g'/g^2)\tau'\tau''=0, \quad (44)$$

$$-\frac{1}{r}\frac{g'}{g}+\frac{1}{r^2}(f-1)+\beta g^{-1}\left[\frac{1}{8}\left(\frac{g'}{g}\tau'\right)^2-\frac{1}{4}\frac{f}{g}(\tau'')^2\right]=0, \quad (45)$$

$$f^{-1}\left[\frac{1}{2}\frac{g''}{g}-\frac{1}{4}\left(\frac{g'}{g}\right)^2-\frac{1}{4}\frac{f'g'}{fg}+\frac{1}{2r}\left(\frac{g'}{g}-\frac{f'}{f}\right)\right]+\tfrac{1}{8}\beta(g')^2f^{-1}g^{-3}(\tau')^2-\tfrac{1}{4}\beta g^{-2}(\tau'')^2=0, \quad (46)$$

$$\left[\frac{3}{2}\frac{g''}{g}-\frac{7}{4}\left(\frac{g'}{g}\right)^2-\frac{3}{4}\frac{f'g'}{fg}+\frac{3}{r}\frac{g'}{g}\right]\tau''+\frac{f}{g}\tau'''=0. \quad (47)$$

These equations can be satisfied with $g'=0$, but then (43), (45), and (46) give $f=1$, $\tau''=0$, i.e., a flat space. Discarding this possibility we assume $g'\neq 0$; hence, by (44), $\tau''=0$ and $\tau'=\text{const}$. Using a suitable scale factor for $\tau$, we may put $\tau'=1$, and the field equations (43), (45), and (46) become

$$\frac{1}{r^2}\left(f-1+r\frac{f'}{f}\right)+\beta g^{-1}\left[\frac{1}{2}\frac{g''}{g}-\frac{5}{8}\left(\frac{g'}{g}\right)^2-\frac{1}{4}\frac{f'g'}{fg}+\frac{1}{r}\frac{g'}{g}\right]=0, \quad (48)$$

$$f=1+r\frac{g'}{g}-\tfrac{1}{8}\beta r^2\frac{(g')^2}{g^3}, \quad (49)$$

$$\frac{f'}{f}=\left[r\frac{g''}{g}-\tfrac{1}{2}r\left(\frac{g'}{g}\right)^2+\frac{g'}{g}+\tfrac{1}{4}r\beta\frac{(g')^2}{g^3}\right]\left(1+\tfrac{1}{2}r\frac{g'}{g}\right)^{-1}. \quad (50)$$

These are three equations to be satisfied by the functions $f$ and $g$. Substituting $f$ and $f'/f$ from (49) and (50) into (48), we obtain

$$\left(1+\frac{1}{2}\frac{\beta}{g}\right)\left[\frac{g''}{g}+\frac{2}{r}\frac{g'}{g}-\left(\frac{g'}{g}\right)^2\right]+\left(\frac{g'}{g}\right)^2\left(1+\frac{1}{4}\frac{\beta}{g}\right)\left(1-\tfrac{1}{4}\beta r\frac{g'}{g^2}\right)=0. \quad (51)$$

The same relation is obtained if $f'/f$ is expressed from (49) and substituted into (50). This shows that Eqs. (48)–(50) are compatible.

Equation (51) can be satisfied by a power series which is regular at $r=\infty$. If the unit of time is chosen so that $g(\infty)=1$, then the power series begins with the terms

$$g=1-\frac{2m}{r}+\frac{\beta}{2+\beta}\frac{m^2}{r^2}+O\left(\frac{m^3}{r^3}\right),$$
$$g^{-1}=1+\frac{2m}{r}+\frac{8+3\beta}{2+\beta}\frac{m^2}{r^2}+O\left(\frac{m^3}{r^3}\right), \quad (52)$$

where $m$ is a constant. From (49) we get

$$f=1+\frac{2m}{r}+\frac{16+2\beta-\beta^2}{4+2\beta}\frac{m^2}{r^2}+O\left(\frac{m^3}{r^3}\right),$$
$$f^{-1}=1-\frac{2m}{r}=\frac{6\beta+\beta^2}{4+2\beta}\frac{m^2}{r^2}+O\left(\frac{m^3}{r^3}\right). \quad (53)$$

We shall see presently that $m$ is the gravitational radius of the attracting body.

Equation (51) can be brought to a more manageable form by making the substitution $u=1/r$ and regarding $u$ as a function of $g$. We obtain

$$\frac{dg}{dr}=-\frac{u^2}{u'},\quad \frac{d^2g}{dr^2}=\frac{2u^3}{u'}-u^4\frac{u''}{(u')^3}, \quad (54)$$

where $u'=du/dg$ and $u''=d^2u/dg^2$. This transforms (51) into the linear equation

$$g(g+\tfrac{1}{2}\beta)u''+\tfrac{1}{4}\beta u'-\tfrac{1}{4}(\beta/g^2)(g+\tfrac{1}{4}\beta)u=0. \quad (55)$$

The initial conditions are, by (52),

$$u(1)=0,\quad u'(1)=-1/2m. \quad (56)$$

The general solution of (55) is

$$u=c_1\exp\left(1+\frac{1}{2}\frac{\beta}{g}\right)^{\frac{1}{2}}\left[\left(1+\frac{1}{2}\frac{\beta}{g}\right)^{\frac{1}{2}}+1\right]^{-1}+c_2\exp\left[-\left(1+\frac{1}{2}\frac{\beta}{g}\right)^{\frac{1}{2}}\right]\left[\left(1+\frac{1}{2}\frac{\beta}{g}\right)^{\frac{1}{2}}-1\right]^{-1}.$$

Noting the boundary conditions (56) we obtain

$$u=\frac{1}{2m}\left\{[1+(1+\tfrac{1}{2}\beta)^{-\frac{1}{2}}]\left[\left(1+\frac{1}{2}\frac{\beta}{g}\right)^{\frac{1}{2}}+1\right]^{-1}\times\exp\left[\left(1+\frac{1}{2}\frac{\beta}{g}\right)^{\frac{1}{2}}-(1+\tfrac{1}{2}\beta)^{\frac{1}{2}}\right]-[1-(1+\tfrac{1}{2}\beta)^{-\frac{1}{2}}]\left[\left(1+\frac{1}{2}\frac{\beta}{g}\right)^{\frac{1}{2}}-1\right]^{-1}\times\exp[(1+\tfrac{1}{2}\beta)^{\frac{1}{2}}-(1+\tfrac{1}{2}\beta g^{-1})^{\frac{1}{2}}]\right\}. \quad (57)$$

Hence $u(g)$ increases monotonically from 0 to $+\infty$ when $g$ decreases from 1 to 0. In particular $g(r)>0$ for $r>0$ and *there is no Schwarzschild-type singularity of the metric tensor* at some finite distance from the origin.

We proceed now to work out some of the geometrical and physical consequences of the solution.

### 5. PLANETARY ORBITS

The equations of geodesics are, by (39),

$$\frac{d^2x_\mu}{ds^2}+\left(\frac{1}{2}\frac{f'}{f}+\frac{1}{rf}-\frac{1}{r}\right)\xi_\mu\left(\frac{dr}{ds}\right)^2+\frac{1}{r}\left(1-\frac{1}{f}\right)\xi_\mu\sum_\nu\left(\frac{dx_\nu}{ds}\right)^2+\frac{1}{2}\frac{g'}{f}\xi_\mu\left(\frac{dt}{ds}\right)^2=0, \quad (58)$$

$$\frac{d^2t}{ds^2}+\frac{g'}{g}\frac{dt}{ds}\frac{dr}{ds}=0. \quad (59)$$

If we note that

$$\frac{dx_\mu}{ds}=\frac{dx_\mu}{dt}\frac{dt}{ds},$$

$$\frac{d^2x_\mu}{ds^2}=\frac{d^2x_\mu}{dt^2}\left(\frac{dt}{ds}\right)^2+\frac{dx_\mu}{dt}\frac{d^2t}{ds^2}=\left(\frac{d^2x_\mu}{dt^2}-\frac{g'}{g}\frac{dx_\mu}{dt}\frac{dr}{dt}\right)\left(\frac{dt}{ds}\right)^2$$

by (59), we obtain from (58), after cancelling by $(dt/ds)^2$ and writing $v^2=\sum_\nu(dx_\nu/dt)^2$,

$$\frac{d^2x_\mu}{dt^2}+\left(\frac{1}{2}\frac{f'}{f}+\frac{1}{rf}-\frac{1}{r}\right)\xi_\mu\left(\frac{dr}{dt}\right)^2+\frac{1}{r}\left(1-\frac{1}{f}\right)\xi_\mu v^2+\frac{1}{2}\frac{g'}{f}\xi_\mu-\frac{g'}{g}\frac{dx_\mu}{dt}\frac{dr}{dt}=0. \quad (58')$$

If we assume that $m/r$ is small, this gives by (52) and (53):

$$\frac{d^2x_\mu}{dt^2}+\frac{m}{r^2}\xi_\mu=\left\{3\frac{m}{r^2}\left(\frac{dr}{dt}\right)^2\left[1+O\left(\frac{m}{r}\right)\right]-\frac{2m}{r^2}v^2\left[1+O\left(\frac{m}{r}\right)\right]+\left(2+\frac{\beta}{2+\beta}\right)\frac{m^2}{r^3}\times\left[1+O\left(\frac{m}{r}\right)\right]\right\}\xi_\mu+\frac{2m}{r^2}\left[1+O\left(\frac{m}{r}\right)\right]\frac{dr}{dt}\frac{dx_\mu}{dt}. \quad (60)$$

The equation shows that $m$ is the gravitational radius of the attracting body, i.e.,

$$m=\kappa M, \quad (61)$$

where $M$ is the mass in ordinary units. The terms on the right-hand side of (60) represent radial and tangential disturbing accelerations. They are identical with those in general relativity with the exception of the term $[\beta m^2/(2+\beta)r^3]\xi_\mu$ which represents a radial outwards disturbing accelerations whose magnitude is

$$F=\beta m^2/(2+\beta)r^3. \quad (62)$$

The effect of the disturbance can most conveniently be computed by Herschel's method. Let $e$ be the eccentricity, $a$ the length of the the major semiaxis. $\omega$ the longitude of the major axis of the instantaneous orbit, and $\theta$ the true anomaly. Then we have

$$r=a(1-e^2)(1+e\cos\theta)^{-1}$$

and

$$d\theta/dt=h/r^2,$$

where $h^2=ma(1-e^2)$. By (62), the radial disturbing acceleration is

$$F=\frac{m^3}{r^2h^2}\frac{\beta}{2+\beta}(1+e\cos\theta),$$

which gives[15]

$$\frac{d\omega}{dt}=-\frac{F}{e}\frac{h}{m}\cos\theta=-\frac{m^2}{r^2he}\frac{\beta}{2+\beta}\cos\theta(1+e\cos\theta),$$

hence

$$\frac{d\omega}{d\theta}=\frac{d\omega}{dt}\frac{r^2}{h}=-\frac{m^2}{h^2e}\frac{\beta}{2+\beta}\cos\theta(1+e\cos\theta).$$

For a complete revolution

$$\Delta\omega=-\frac{\beta}{2+\beta}\frac{m^2}{h^2}\pi,$$

and the rate of advance of the perihelion due to the disturbance is

$$\frac{1}{2\pi}\Delta\omega=-\frac{\beta}{4+2\beta}\frac{m^2}{h^2}.$$

This represents a perennial motion of the perihelion in a sense opposite to the motion of the planet. Therefore it must be subtracted from the Einstein value $3m^2/h^2$, and the total rate of advance is

$$\frac{1}{2\pi}\Delta\omega=\left(3-\frac{\beta}{4+2\beta}\right)\frac{m^2}{h^2}. \quad (63)$$

Thus, *by measuring the rate of advance of the perihelion of planets or asteroids it is possible to determine the value of the coupling constant* $\beta$ *experimentally.*

For geodesic null lines, $v$ is very nearly 1 and (60) becomes

$$\frac{d^2x_\mu}{dt^2}+\frac{3m}{r^2}\left\{1-\left(\frac{dr}{dt}\right)^2\left[1+O\left(\frac{m}{r}\right)\right]\right\}\xi_\mu-\frac{2m}{r^2}\frac{dr}{dt}\frac{dx_\mu}{dt}\left[1+O\left(\frac{m}{r}\right)\right]=0. \quad (60')$$

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[15] See W. H. Besant, *Dynamics* (G. Bell and Sons, London, 1902), p. 207.

Here the term which depends on $\beta$ has disappeared and the equation gives the same result for the deflection of light in the sun's gravitational field as the general theory of relativity.

### 6. GRAVITATIONAL ENERGY

(41), (41″), (52), and (53) give the following expression for the components of the gravitational field energy-stress tensor:

$$T_{00}=\frac{6+\beta}{4+2\beta}\frac{m^2}{r^4}+O\left(\frac{m^3}{r^5}\right), \tag{64}$$

$$T_{0\mu}=0, \tag{64'}$$

$$T_{\mu\nu}=\frac{m^2}{r^4}(\xi_\mu\xi_\nu-\tfrac12\delta_{\mu\nu})+O\left(\frac{m^3}{r^5}\right). \tag{64''}$$

Apart from numerical factors they have the same form as Maxwell's energy-stress, tensor in Coulomb fields which lends a strong support to the correctness of postulate (13).

Consider now the vector density

$$\rho^n=(\beta/8\pi\kappa)(C_kC^k)^{-1}(2C^r\mathfrak{T}_r{}^n-C^n\mathfrak{T}), \tag{65}$$

where $T=T_r{}^r$. Using the frame of Sec. 4 in which $C_0=1$, $C_\mu=0$, we get

$$\rho=\rho^0=(\beta/8\pi\kappa)(2\mathfrak{T}_0{}^0-\mathfrak{T}),\quad \rho^\mu=0. \tag{66}$$

We shall show now that

$$4\pi\int_0^\infty \rho r^2dr=M=\frac{1}{\kappa}m. \tag{67}$$

By (37), (41), (41″), (49), and (50),

$$T=\frac{1}{fg}\left(-\frac12\frac{g''}{g}+\frac34\left(\frac{g'}{g}\right)^2+\frac14\frac{f'g'}{fg}-\frac1r\frac{g'}{g}\right),$$

$$2\mathfrak{T}_0{}^0-\mathfrak{T}=(fg)^{-\frac12}\left(-\frac12\frac{g''}{g}+\frac12\left(\frac{g'}{g}\right)^2+\frac14\frac{f'g'}{fg}-\frac1r\frac{g'}{g}\right)$$

$$=\left[g+rg'-\tfrac18\beta r^2\left(\frac{g'}{g}\right)^2\right]^{-\frac12}(1+\tfrac12rg')^{-1}$$

$$\times\left(-\frac12\frac{g''}{g}+\frac14\left(\frac{g'}{g}\right)^2-\frac1r\frac{g'}{g}+\tfrac18r\left(\frac{g'}{g}\right)^3+\tfrac1{16}r\beta\frac{(g')^3}{g^4}\right);$$

hence, introducing the variable $u=1/r$,

$$\int_0^\infty(2\mathfrak{T}_0{}^0-\mathfrak{T})r^2dr$$

$$=-\int_0^1u'\left[g-\frac{u}{u'}-\tfrac18\beta\left(\frac{u}{u'g}\right)^2\right]^{-\frac12}\left(1-\frac12\frac{u}{u'g}\right)^{-1}$$

$$\times\left[\frac12\frac{u''}{(u')^3g}+\frac14\frac{1}{(u'g)^2}-\frac18\frac{u}{(u'g)^3}-\frac1{16}\frac{\beta}{g}\frac{u}{(u'g)^3}\right]dg$$

$$=\int_0^1[g^3(u')^2-g^2uu'-\tfrac18\beta u^2]^{-\frac12}\left(u'-\frac12\frac{u}{g}\right)^{-1}$$

$$\times\left(\frac12u''+\frac14\frac{u'}{g}-\frac18\frac{u}{g^2}-\frac1{16}\beta\frac{u}{g^3}\right)dg. \tag{68}$$

Now we obtain for

$$U=g^3(u')^2-g^2uu'-\tfrac18\beta u^2,$$

by differentiating with respect to $g$ and substituting for $u''$ from (55),

$$dU/dg=2g^3u'u''-g^2uu''+2g^2(u')^2-(2g+\tfrac14\beta)uu'$$

$$=\frac{4g+\beta}{2g+\beta}\left[g^2(u')^2-uu'-\frac18\frac{\beta}{g}u^2\right]$$

$$=\frac{4g+\beta}{g(2g+\beta)}U.$$

Hence, observing the boundary conditions (56),

$$U=g^3(u')^2-g^2uu'-\tfrac18\beta u^2=(4m^2)^{-1}(1+\tfrac12\beta)^{-1}g(g+\tfrac12\beta). \tag{69}$$

Similarly, we get from (55):

$$\frac12u''+\frac14\frac{u'}{g}-\frac18\frac{u}{g^2}-\frac1{16}\frac{u}{g^3}\beta=\frac14\frac{u'}{g+\frac12\beta}-\frac18\frac{u}{g(g+\frac12\beta)}. \tag{70}$$

Substituting (69) and (70) into (68) we get

$$\int_0^\infty(2\mathfrak{T}_0{}^0-\mathfrak{T})r^2dr=\tfrac12m(1+\tfrac12\beta)^{\frac12}\int_0^1g^{-\frac12}(g+\tfrac12\beta)^{-\frac32}dg$$

$$=2m/\beta,$$

which together with (66) gives the desired result. It can be shown similarly that any other linear combination of $\mathfrak{T}_0{}^0$ and $\mathfrak{T}$ in (66) would give an infinite value for the integral (67).

Equation (67) shows that if we *define* $\rho^n$ to be the gravitational energy-momentum density vector then the integral of $\rho^0$ over the whole space is equal to the gravitational mass of the body. This is an expression of the fact that *the total mass of neutral matter comes from the energy of its gravitational field.*

To compare our result with the Newtonian theory of gravitation, consider the energy-stress tensor of a perfect fluid with rest density $\mu$ and isotropic pressure $p$. We have, in absolute gravitational units,

$$\beta T_{mn}=8\pi\kappa[\mu u_m u_n+p(u_m u_n-g_{mn})]$$

where $u_m$ is the world direction of the fluid.[16] It follows that in a Galilean frame with respect to which the fluid is at rest,

$$\frac{\beta}{8\pi\kappa}\mathfrak{T}_0{}^0=\mu,\quad \frac{\beta}{8\pi\kappa}\mathfrak{T}_\sigma{}^\rho=-p\delta_\sigma{}^\rho,\quad \frac{\beta}{8\pi\kappa}\mathfrak{T}=\mu-3p.$$

Hence assuming that the fluid is at absolute rest,

$$\rho=\frac{\beta}{8\pi\kappa}(2\mathfrak{T}_0{}^0-\mathfrak{T})=\mu+3p, \tag{71}$$

which verifies formula (3) of the introduction. It shows that not $\mu$ but $\mu+3p$ is the gravitational field-producing density from the point of view of Newtonian attraction. In the case of isotropic radiation of density $\mu$ one has $p=\frac{1}{3}\mu$ which gives $\rho=2\mu$, i.e., twice the inertial mass density.

If the fluid is moving with velocity $v$ relatively to the ether, say in the positive $x_1$ direction, then

$$C^0=(1-v^2)^{-\frac{1}{2}},\quad C^1=-v(1-v^2)^{-\frac{1}{2}},\quad C^2=C^3=0,$$

which by (65) gives

$$\rho^0=(1-v^2)^{-\frac{1}{2}}(\mu+3p),\quad \rho'=v(1-v^2)^{-\frac{1}{2}}(\mu-p),\quad \rho^2=\rho^3=0. \tag{71'}$$

From this, formula (4) follows immediately. The formula shows the curious fact that from the point of view of gravitational attraction particles behave roughly as if they were moving with velocity $2v$ instead of $v$ relatively to the ether. This explains the appearance of the term $3p$ in formula (71). To the (inertial) rest mass density one has to add a gravitational mass correction due to the kinetic energy of the particles of the fluid relatively to the ether.

Finally, consider the quantity

$$q=(C_rC^r)^{-\frac{1}{2}}. \tag{72}$$

In the central symmetrical field it has the value

$$q=g^{\frac{1}{2}}=1-(m/r)+O(m^2/r^2),$$

when $r$ is large, and it can be regarded as an expression for the gravitational potential. Thus in the present theory it is possible to define the Newtonian potential in an invariant way. For small $r$, we have from (57):

$$q\sim(\tfrac{1}{2}\beta)^{\frac{1}{2}}[\log(m/r)]^{-1},$$

which shows roughly the behavior of the gravitational potential near the origin.

The geometrical significance of $q$ is that it gives the ratio of the units of local "metric" or "proper" time and "cosmic" time. If therefore we make the assumption, as suggested at the end of the introduction, that in Bohr's frequency relation

$$E_1-E_2=h\nu,$$

$\nu$ is expressed not in metric but in cosmic time units, then measured in metric time units the relation takes the form

$$E_1-E_2=hq\nu. \tag{73}$$

From this it follows immediately that the observed Einstein shift is zero, provided that photons behave like particles with rest mass zero and velocity 1.

## 7. GRAVITATIONAL WAVES

Let us assume that the field is weak and the frame very nearly Galilean and resting in ether, i.e.,

$$g_{mn}=\bar{g}_{mn}+\gamma_{mn},\quad \tau=t+\varphi, \tag{74}$$

where $\bar{g}_{mn}$ is the metric tensor corresponding to the line element $dt^2-(dx_1{}^2+dx_2{}^2+dx_3{}^2)$, and the quantities $\gamma_{mn}$, $\varphi$ are sufficiently small so that products can be neglected. Using the notations

$$\gamma=\gamma_r{}^r,\quad \gamma_m=\gamma^r{}_{m,r},$$

we get

$$R_{mn}=\tfrac{1}{2}\Box\gamma_{mn}-\tfrac{1}{2}(\gamma_{m,n}+\gamma_{n,m})+\tfrac{1}{2}\gamma_{,m,n}, \tag{75}$$

and

$$R=\Box\gamma-\gamma^m{}_{,m}, \tag{74'}$$

where

$$\Box=g^{mn}\partial^2/\partial x_m\partial x_n=\partial^2/\partial t^2-(\partial^2/\partial x_1{}^2+\partial^2/\partial x_2{}^2+\partial^2/\partial x_3{}^2).$$

We also have, from (74),

$$C_m=\bar{C}_m+\varphi_{,m},$$

where $\bar{C}_0=1$, $\bar{C}_\mu=0$ for $\mu\neq 0$,

$$C_{mn}=C_{m,n}-C_r\left\{\begin{matrix} r \\ mn\end{matrix}\right\}$$
$$=\varphi_{,m,n}-\tfrac{1}{2}(\gamma_{0m,n}+\gamma_{0n,m}-\gamma_{mn,0}),$$
$$C^{mn}{}_{;n;m}=\Box\Box\varphi-\Box\gamma_0+\tfrac{1}{2}\gamma^m{}_{,m,0}.$$

Hence, writing

$$\Psi=C_m{}^m=\Box\varphi-\gamma_0+\tfrac{1}{2}\gamma_{,0} \tag{76}$$

for the covariant d'Alembertian of $\varphi$ and using (75'), field equation (18) takes the form

$$\Box\Psi=\tfrac{1}{2}R_{,0}. \tag{77}$$

A remarkable feature of this wave equation is that, unlike the gravitational wave equations of general relativity, it holds in an arbitrary frame of the form (74) and there is no need to impose further auxiliary conditions upon the coordinates.[17] It should be noted

[16] H. Weyl, *Space-Time-Matter* (Dover Publications, New York, 1922), p. 205.

[17] For a criticism of such axuailiary conditions see F. A. Kaempffer, Can. J. Phys. 31, 501 (1953).

in this connection that (77) is the outcome of the secondary field equation (18) which has no counterpart in ordinary relativity. The approximate equations corresponding to the primary field equations (15) can be obtained from (75) and

$$T_{mn}=\tfrac{1}{2}\varphi_{,m,n,0}-\tfrac{1}{2}(\delta_{m0}\Box\varphi_{,n}+\delta_{n0}\Box\varphi_{,m})$$
$$+\tfrac{1}{4}\delta_{m0}(\Box\gamma_{0n}+\gamma_{0,n}-\gamma_{n,0})$$
$$+\tfrac{1}{4}(\Box\gamma_{0m}+\gamma_{0,m}-\gamma_{m,0})$$
$$-\tfrac{1}{4}(\gamma_{0m,n,0}+\gamma_{0n,m,0}-\gamma_{mm,0,0}). \quad (78)$$

They determine the change of the local gravitational field under the influence of the incoming $\Psi$ waves.

Equation (77) shows that the $\Psi$ waves propagate with the velocity of light and are generated by oscillating matter. In fact, if tension-free matter with rest density $\mu$ is present, then $R=8\pi\kappa\mu$, and (77) becomes

$$\Box\Psi=4\pi\kappa\partial\mu/\partial t. \quad (79)$$

In an arbitrary nearly Galilean frame which is not resting in ether, it takes the form

$$\Box\Psi=4\pi\kappa C^m\partial\mu/\partial x_m. \quad (79')$$

The wave equation (77) has an interesting corollary. We have seen in Sec. 6 that the gravitational mass density $\rho^0$ is not strictly proportional to the inertial mass density, as postulated by the principle of equivalence, but the ratio of the two densities depends on the state of motion of matter. Since inertial mass is conserved, we cannot expect the same to be true for the gravitational mass. The rate of creation of gravitational mass density is given by the expression $\rho^n{}_{,n}$. Using a frame which rests in ether we obtain

$$\rho^n=\frac{1}{8\pi\kappa}\beta(C_kC^k)^{-1}(2C^r\mathfrak{T}_r{}^n-C^n\mathfrak{T})=-\frac{1}{4\pi\kappa}R_0{}^n;$$

hence, by (75) and (75′),

$$\rho^n{}_{,n}=-\frac{1}{4\pi\kappa}R_0{}^n{}_{,n}=-\frac{1}{8\pi\kappa}(\Box\gamma_{,0}-\gamma^n{}_{,n,0})$$
$$=-\frac{1}{8\pi\kappa}R_{,0}. \quad (80)$$

Combining (77) and (80), we find

$$\Box\Psi=-4\pi\kappa\rho^n{}_{,n}. \quad (81)$$

The equation shows that the creation rate of gravitational mass can be regarded as the "charge" which is responsible for the generation of $\Psi$ waves.

If matter of density $\mu$ is present, then Eq. (80) takes the form

$$\rho^n{}_{,n}=-\partial\mu/\partial t. \quad (82)$$

One can give the following interpretation of this peculiar relation.

When a particle moves in the ether, then gravitational mass is continuously annihilated in the front half and created in the back half of the field of the particle.[18] In other words, the ether drift transforms particles into a dipole with respect to the creation charge so that the ether drift vector forms the axis of polarization. In order to account for Eq. (82), the strength of the dipole ought to be equal to $mv$, where $m$ is the gravitational rest mass of the particle and $v$ its absolute velocity.

## 8. PROPERTIES OF THE COSMOLOGICAL MODEL

We shall investigate now some properties of the cosmological model found in Sec. 3. Its line element is

$$dt^2-t^\alpha(dx_1^2+dx_2^2+dx_3^2), \quad (83)$$

for some positive constant $\alpha$. The equations of a geodesics passing through the origin in any direction are, by formula (35) of Sec. 3,

$$\frac{d^2r}{ds^2}+\frac{\alpha}{t}\frac{dr}{ds}\frac{dt}{ds}=0, \quad (84)$$

$$\frac{d^2t}{ds^2}+\tfrac{1}{2}\alpha t^{\alpha-1}\left(\frac{dr}{ds}\right)^2=0. \quad (84')$$

An integral of this is

$$\left(\frac{dt}{ds}\right)^2-t^\alpha\left(\frac{dr}{ds}\right)^2=1, \quad (85)$$

which, with (84′), gives

$$t\frac{d^2t}{ds^2}+\tfrac{1}{2}\alpha\left(\frac{dt}{ds}\right)^2=\tfrac{1}{2}\alpha.$$

This gives, on integration,

$$dt/ds=[1+(b/t)^\alpha]^{\frac{1}{2}},$$

where $b$ is a constant. Hence, by (85),

$$dr/ds=b^{\frac{1}{2}\alpha}t^{-\alpha},$$
$$dr/dt=(b/t)^{\frac{1}{2}\alpha}(b^\alpha+t^\alpha)^{-\frac{1}{2}}.$$

Since the velocity of light is $c=t^{-\frac{1}{2}\alpha}$, the velocity of the particle with respect to the frame (83) is, at the time $t$,

$$v=[1+(t/b)^\alpha]^{-\frac{1}{2}}. \quad (86)$$

The particle slows down and at $t=\infty$ comes to a rest. If at time $t_0$ the velocity was $v_0$, then by (86)

$$b^\alpha=t_0{}^\alpha v_0{}^2(1-v_0{}^2)^{-1},$$

hence

$$v=v_0[v_0{}^2+(1-v_0{}^2)(t/t_0)^\alpha]^{-\frac{1}{2}}. \quad (87)$$

[18] Preliminary calculations made on the field of central bodies in absolute motion confirm the existence of such an effect. There seems to be a continuous inwards flux of gravitational mass in the front half and an equivalent outwards flux in the trailing half of the particle.

This shows that if the particle was at rest in the frame (83) then it will always remain so. Therefore (83) can be regarded as a suitable frame to describe cosmic phenomena. But the (metric) distance between two points with fixed coordinates increases with time, hence the model (83) represents an expanding universe.

For a particle with rest mass $m$, we obtain from (87)

$$m(1-v^2)^{-\frac{1}{2}}=m(1-v_0^2)^{-\frac{1}{2}}[1-v_0^2+v_0^2(t_0/t)^{\alpha}]^{\frac{1}{2}};$$

or, if $E$ is the energy of the particle,

$$E=E_0[1-v_0^2+v_0^2(t/t_0)^{\alpha}]^{\frac{1}{2}}. \tag{88}$$

In the limiting case of photons, $v_0=1$; hence[19]

$$\nu=\nu_0(t_0/t)^{\frac{1}{2}\alpha}. \tag{89}$$

This expresses Hubble's law and shows that the value of the recession constant is $\frac{1}{2}\alpha/t$. If $\alpha=2$, the recession constant is simply $1/t$.

The model (83) represents a universe with uniform matter distribution. The density of gravitational energy at some epoch $t$ and in gravitational units is

$$8\pi\kappa\rho=\beta(2\mathfrak{T}_0^0-\mathfrak{T})=\frac{6\alpha}{3\alpha+2}t^{\frac{3}{2}\alpha-2}, \tag{90}$$

by formulas (31) and (36). If the gravitational radius is measured not in metric length but in units of the cosmologically more satisfactory space coordinates $x_\nu$, then we have to multiply the quantity (90) by $t^{-\frac{1}{2}\alpha}$:

$$8\pi\kappa t^{-\frac{1}{2}\alpha}\rho=\frac{6\alpha}{3\alpha+2}t^{\alpha-2}. \tag{90'}$$

This gives the density in "cosmic units." If, in particular, $\alpha=2$, then the gravitational mass measured in cosmic units is a pure number and the density has the constant value $\frac{3}{2}$. The simplicity of this result lends a strong support in favor of the value $\alpha=2$.

To compare (90) with experimental values, it is more convenient to use the conventional coordinates $y_\mu=t^{\frac{1}{2}\alpha}x_\mu$ at the fixed epoch $t$. The density in these coordinates and in ordinary units is

$$\rho=\frac{1}{4\pi k}\frac{3\alpha}{3\alpha+2}t^{-2}, \tag{91}$$

or

$$\rho=\frac{3}{16\pi\kappa}t^{-2} \tag{91'}$$

if $\alpha=2$. This is half of the value found by Hoyle. In Hoyle's corresponding formula,

$$\rho=\frac{3}{8\pi\kappa}T^{-2},$$

$T$ is a constant and so is the density in ordinary units. Because of the expansion of the universe, this can only be true if matter is created continuously, which is indeed a basic conclusion of Hoyle's theory. From the present theory it appears that "continuous creation" is largely a matter of interpretation and depends on the units in which energy is measured at various epochs. In fact, if measured in absolute cosmic units, as explained above, then the total energy of a fixed region of the universe remains constant, apart from statistical fluctuations. This of course does not imply that the number of protons and electrons in a fixed region must remain constant. This would only be true if the gravitational radius of a proton, measured in cosmic units, would remain a constant, which is very doubtful, to say the least. The size of elementary particles, however, is a question which by its very nature lies outside the scope of a purely gravitational theory.

[19] The method used here for calculating the red shift is based on the particle picture of photons. It has the advantage that its applicability is not restricted to any particular type of frame.