

## Cosmic-Ray Studies of Penetrating Showers and Bremsstrahlung at 11 200 Feet by Using Plastic Scintillation Counters\*

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An apparatus was designed to study penetrating showers produced by high-energy cosmic rays. For this purpose, four anthracene-in-polystyrene plastic scintillators of 1 and 2 cm thickness were used with lead plates of thicknesses 2.00 cm, 2.25 cm, and 2.25 cm, respectively, interposed between them. In addition to these scintillation counters, four trays of G-M counters and an arrangement of anticoincidence counters were used. Data on penetrating showers produced in a graphite block and a polystyrene scintillator were obtained. It was found that in the energy range from 2 to 20 Bev, the resulting integral energy spectra of the  $\pi^0$  mesons can be represented by a power law with an exponent equal to  $-1.7 \pm 0.2$  in polystyrene, with no filter for the primaries. Similar results were obtained in graphite and in polystyrene with primaries filtered through 10 cm of lead. The multiplicity of the charged mesons in the penetrating showers produced in graphite can be described by a power law with an exponent  $-4.2 \pm 0.5$ , in general agreement with cloud chamber results. It was found that the energy distribution of bremsstrahlung  $\gamma$  rays and the flux of the  $\mu$  mesons producing this bremsstrahlung in a lead block of 10 cm thickness is in agreement with theory.

### I. INTRODUCTION

A LARGE number of different experimental results have been reported concerning the production of penetrating showers by cosmic-rays. The early work has been summarized by T. G. Walsh and O. Piccioni.<sup>1</sup> The experimental techniques used more recently mainly consisted of the use of photographic emulsions,<sup>2-5</sup> cloud chambers,<sup>6-18</sup> or G-M counters.<sup>19-23</sup> The use of photographic emulsions is particularly suitable for the detailed study of individual events involving charged particles or in some favorable cases, also photons. In the case of using G-M counter arrangements, frequently one finds it difficult to interpret the data unambiguously. This is chiefly due to the fact that the penetrating secondaries are frequently emitted under very narrow angles. In the case of multiplate cloud chamber studies of cascade showers produced by photons originating

from the decay of  $\pi^0$  mesons produced in penetrating showers, it is practically impossible to count the number of secondaries as the shower develops under the lead plates when the energy is considerably higher than a few Bev. In this last respect, a large scintillation counter is more suitable since it is capable of giving a light output proportional to the total number of minimum particles passing through it. Hence it can be used reliably to register the number of electrons in the development of a cascade shower through lead.

Plastic scintillators placed between lead plates are particularly suitable for this purpose. Plastic scintillators can be made in large sizes. Their performance has been found to be reliable and adequate for quantitative measurements.<sup>24</sup> Scintillation counters made of anthracene-in-polystyrene plastic plates have previously been used here for the measurement of cascade showers initiated by high-energy photons and electrons at sea-level.<sup>25</sup> An apparatus utilizing plastic scintillators was constructed for the measurement of penetrating showers produced by cosmic-ray primaries and the cascades in penetrating showers initiated by photons or electrons in the energy range of from 1 Bev to about 50 Bev. The measurements were carried out at Climax (altitude 11 200 ft) from August to October in 1953. Data were obtained on the energy spectrum of  $\pi^0$  mesons and the multiplicity spectrum of charged particles in penetrating showers in a graphite block. Additional measurements on the spectrum of  $\pi^0$  mesons were carried out in polystyrene, which contained equal numbers of carbon and hydrogen molecules. A somewhat modified apparatus was also used to obtain data on bremsstrahlung produced by cosmic-ray  $\mu$  mesons in a lead block above the apparatus.

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<sup>3</sup> E. Pickup and L. Voyvodic, *Phys. Rev.* **84**, 1190 (1951).

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<sup>25</sup> C. N. Chou, *Phys. Rev.* **90**, 473 (1953).

II. EXPERIMENTAL ARRANGEMENT

The geometry of the experimental arrangement drawn to scale is shown in Fig. 1.  $S_1, S_2, S_3,$  and  $S_4$  are four anthracene-in-polystyrene plastic scintillators.  $S_3$  has the cross-sectional area of  $18 \times 9 \text{ cm}^2$  and is 1.8 in thickness,  $S_2$  and  $S_4$  are correspondingly  $18 \times 18 \times 0.9$ , and  $S_1$   $18 \times 18 \times 1.8$ . As shown in the figure, each scintillator was coupled through a Lucite light-pipe to a magnetically shielded RCA 5819 photomultiplier tube. Between the scintillators,  $S_2, S_3, S_4,$  and  $S_1$  lead plates of thicknesses 2.00, 2.25, and 2.25 cm respectively are interposed.  $A, C,$  and  $D$  are parallel trays of metal G-M counters, and  $B$  is a cross tray to  $A$ . Each G-M counter is one inch in diameter. The counters  $E$  are in anticoincidence and their purpose is to protect the scintillators from registering particles from the sides. Hence practically all events registered were due to particles from above except for very energetic neutrons. During part of the experiment, above the counter tray  $A$  the lead block  $PB$  was placed. Between the bottom scintillator  $S_1$  and the G-M counter tray  $C$  a lead block of 6 cm thickness is placed on top of which an aluminum plate of 2-cm thickness is located in order to cut off the effect of the low energy backward shower particles produced in the 6-cm lead block. For the same purpose another aluminium plate of 1-cm thickness was placed under the G-M counter tray  $B$ . Between the counter trays  $C$  and  $D$  12 cm of lead was interposed.  $G$  is a graphite block in which penetrating showers were generated and which was in the position indicated in Fig. 1 during part of the experiment. All the scintillators and the G-M trays  $C$  and  $D$  are shielded with lead block and trays of lead shot as shown in the figure. The overall shielding corresponded to at least 8 cm of lead. Within the solid angle defined by the G-M counter trays  $A$  and  $B$  and scintillator  $S_3$ , a cosmic-ray particle of sufficient energy coming vertically downward could produce nuclear events in the scintillators or the lead plates interposed between them, and also in the graphite block  $G$  or the lead block  $PB$  when they were in position. At the same time, cascade showers initiated by the

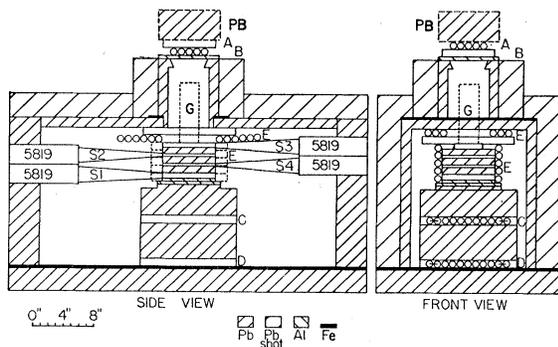


FIG. 1. Experimental arrangement.  $G$  denotes graphite block.  $PB$  denotes lead block.  $A, B, C,$  and  $D$  denote G-M counter trays.  $E$  denotes anticoincidence side G-M counters.  $S_1, S_2, S_3,$  and  $S_4$  denote plastic scintillators with Lucite light pipes.

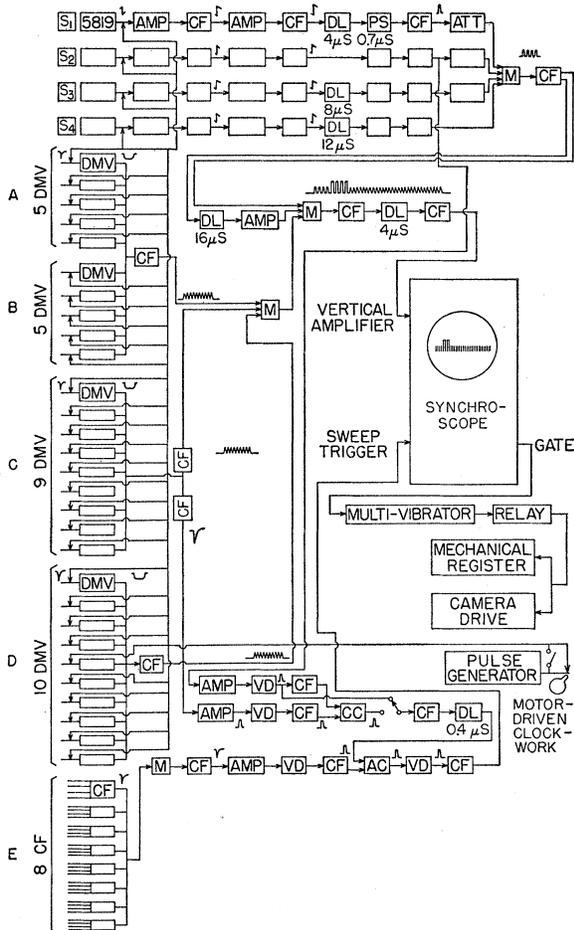


FIG. 2. Block diagram of the recording circuits. The abbreviations have the following meanings:  $S_1, S_2, S_3,$  and  $S_4$ , scintillator inputs;  $AMP$ , amplifier;  $CF$ , cathode follower;  $DL$ , delay line unit;  $PS$ , pulse shaping unit;  $ATT$ , attenuator;  $M$ , mixing unit;  $DMV$ , delaying multi-vibrator;  $VD$ , voltage discriminator;  $CC$ , coincident circuit;  $AC$ , anti-coincident circuit. The approximate wave-forms at various points are also given. The numbers below  $PS$  and  $DL$  give the width of the pulse and the time of delay, respectively, in  $\mu\text{sec}$ .

electromagnetic components of an event could develop through the whole counter arrangement. These developments were recorded by the scintillators and the G-M counters. However, large air showers were excluded by the anticoincidence counters  $E$ . Figure 2 shows a block diagram of the recording circuits. The pulse shape is indicated at various points. The pulses from the four photo-multiplier tubes which viewed the four plastic scintillators  $S_1, S_2, S_3,$  and  $S_4$  were delayed by  $4 \mu\text{sec}, 0 \mu\text{sec}, 8 \mu\text{sec},$  and  $12 \mu\text{sec}$ , respectively. They were then shaped to a square pulse of a width of  $0.7 \mu\text{sec}$  each. This was done by using shorted delay-lines. The four pulses  $P_1$  were adjusted to the same size and then mixed. These same pulses  $P_1$  were amplified about 10 times in amplitude and delayed  $16 \mu\text{sec}$ . These will be called  $P_2$ . The differentiated outputs of the delaying

multivibrators from the G-M counter pulses of trays  $AB$ ,  $C$ , and  $D$  will be called  $P_3$ . The pulses  $P_1$ ,  $P_2$ , and  $P_3$  were then mixed. The whole length of the sweep was about 160  $\mu$ sec.

Part of the output pulse of scintillator  $S_2$  (Fig. 1 and Fig. 2) was used for starting the sweep of the synchroscope (Tektronix Type 513D). A  $10^{-5}$  sec coincidence of part of the output pulse of  $S_2$  and any of the G-M counter pulses from trays  $A$  and  $B$  could also start the sweep. The sweep was actuated by one of these two triggering arrangements provided that there was no anticoincidence pulse present in any of the G-M counters  $E$ . The actual triggering used was as follows:

(1)  $S_2^m$ : A pulse was required in scintillator  $S_2$  corresponding to 3 or 4 times the maximum pulse height in the Landau distribution corresponding to a singly charged minimum ionization particle.

(2)  $AB-S_2^s$ : G-M coincidence between any one counter from trays  $A$  or  $B$  and a pulse in  $S_2$ . Any pulse in  $S_2$  corresponding to about one fourth of the maximum pulse height of a singly charged minimum ionization particle was used for this purpose.

(3)  $AB-S_2^m$ : Coincidence of any one G-M counter pulse from trays  $A$  or  $B$  and a pulse in  $S_2$  corresponding to 3 or 4 times the maximum pulse height of a singly charged minimum ionization particle.

When the sweep was triggered the two sets of scintillator pulse outputs  $P_1$  and  $P_2$ , and the outputs  $P_3$  of all the G-M counters in trays  $AB$ ,  $C$ , and  $D$ , delayed by various time intervals by the respective multivibrators, were fed into a synchroscope. A photographic record was made and a gate pulse from the scope then actuated the mechanical register and the camera drive to advance the film forward by one frame and to wait for the next event. A pulse generator produced pulses similar to the four pulses due to  $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$ . This same pulse generator produced pulses similar to pulses from G-M trays  $AB$ ,  $C$ , and  $D$ . The pulses from the pulse generator could be fed into the input of the vacuum tube circuit. A motor-driven clock work actuated these calibration signals once every hour.

The apparatus was installed at Climax and kept running continuously from August to October 1953. The analysis of several thousand events which were registered is included in this paper.

### III. EXPERIMENTAL PROCEDURE

Throughout the experiment, runs were made with triggering  $AB-S_2^s$  (with or without the lead block  $PB$ ). Out of these runs those events were selected in which only one single G-M counter in the trays  $C$  and  $D$  was triggered. It will be explained later that these must be due to singly charged minimum ionization particles. As explained above, both nuclear events and electromagnetic cascade showers could develop through the whole counter arrangement. A procedure of analysis was devised to separate and analyze these events. The

pulse sizes observed in scintillators  $S_3$ ,  $S_2$ ,  $S_4$ , and  $S_1$  were first converted to equivalent number of singly charged minimum ionization particles from the results obtained above based on  $AB-S_2^s$  events. In our analysis only events in which none of the G-M counters  $E$  on the side was fired were included. It was also assumed that the observed events were all "usual" events amenable to the statistics of averages. The following types of events were selected by this apparatus:

#### (1) Singly Charged Penetrating Particles

The lead block  $PB$  was in position. In these events one and only one G-M counter in trays  $A$ ,  $B$ , and  $C$  was actuated and not accompanied by a pulse in  $E$ , and the scintillators  $S_3$ ,  $S_2$ ,  $S_4$ , and  $S_1$  showed pulses for single minimum-ionization particles. These events were assumed to be due to high-energy  $\mu$  mesons. A small fraction, however, could be due to high-energy protons or  $\pi$  mesons, and corrections could be made from the known rate of nuclear interactions as described in a later subsection. The effect has been found to be too small (2-3 percent of nuclear component) to be of any significance. The probability of an electron traversing the whole thickness of matter without producing secondaries to be detected was entirely negligible. The actually observed pulse size distribution in all the scintillators empirically observed supported this interpretation. These singly charged minimum ionization particles were mainly used for calibration purposes.

#### (2) Bremsstrahlung Produced in the Lead Block $PB$ by $\mu$ Mesons

One and only one G-M counter in trays  $A$ ,  $B$ , and  $D$  was fired. One or more than one G-M counter in tray  $C$  was fired. The photon produced in the process of bremsstrahlung in  $PB$  developed into cascade showers passing through the scintillators  $S_3$ ,  $S_2$ ,  $S_4$ , and  $S_1$ . Their energy was determined from cascade theory.<sup>25,26</sup> Graphs for photons of different energies were plotted showing the number of electrons expected from shower theory passing through the four scintillators after traversing the lead plates interposed between them. In the analysis of this subsection and subsections (3) and (4) we analyzed only events which gave a pulse in scintillator  $S_2$  of a height equal to or greater than that corresponding to 16 minimum ionization particles. This requirement of large number of particles made the average statistics very good. Then a best fit among the four scintillator readings was adjusted to determine the energy of the photon. These events could be produced by knock-on electrons in lead block  $PB$  accompanying the primary  $\mu$  meson which initiated it. This effect is found to be small, especially in the case of high-energy secondaries, which varies as  $E'^{-2}$  for knock-on electrons and as  $E'^{-1}$  for bremsstrahlung, where  $E'$  denotes the energy of the secondary.

<sup>26</sup> K. Ott, Z. Naturforsch. **9a**, 488 (1954).

### (3) Penetrating Showers Produced in Graphite Block G

The lead block  $PB$  was in position. The triggering arrangement  $AB-S_2^m$  was used. A proton having traversed the lead block  $PB$  could produce a penetrating shower in the graphite block  $G$ . The few charged  $\pi$  mesons and the primary proton were registered in the top scintillator  $S_3$ . The total energy of the  $\pi^0$  mesons was measured by that of photon-initiated cascade showers as described in subsection (2) above.<sup>25,26</sup> These events could also be interpreted as due to a small nuclear star in the top scintillator  $S_3$  and not in the graphite block  $G$ . However, this type of events were eliminated by applying the criterion described in the following subsection. In the present analysis we measured the maximum number of charged  $\pi$  mesons produced in the graphite block  $G$  up to 10 only, because of the background effect of nuclear stars.

### (4) Penetrating Showers Produced in Scintillator $S_3$

We identified these events from triggering arrangements  $S_2^m$ ,  $AB-S_2^s$ , and  $AB-S_2^m$ . In separating nuclear interactions from electromagnetic cascade showers results of photographic emulsion investigations were used as a partial guide.<sup>4</sup> Additional information was obtained from the registration of G-M counters in trays  $C$  and  $D$ .

Thus, the nuclear events occurred in scintillator  $S_3$  and the subsequent development of the photons from the decay of the  $\pi^0$  mesons were identified and the energy of photons determined. Using similar criterions, we rejected events which could have been confused with events discussed in earlier subsections.

## IV. DISCUSSION OF RESULTS

First the range of validity of the energy measurements of the cascade showers will be briefly discussed here. The apparatus could yield reliable results for the measurement of cascade showers initiated by photons or electrons of total energies in the range of 1 Bev to 30 Bev. However, in the energy range less than 2 Bev, the sensitivity of identifying a cascade shower event dropped off. This is because of the fact that in the analysis procedure adopted, allowance was made for the fluctuation in the shower theory. This range of allowance fell below the minimum pulse required in the present analysis when the cascade theory was less than 2 Bev.

### (a) Penetrating Showers from Graphite and Polystyrene

The procedure of analysis has been explained in subsections III (3) and (4) above. For the study of the nuclear interactions produced in the graphite block  $G$ , the triggering arrangement used was that of  $AB-S_2^m$  with the lead block  $PB$  in position. The effect of the

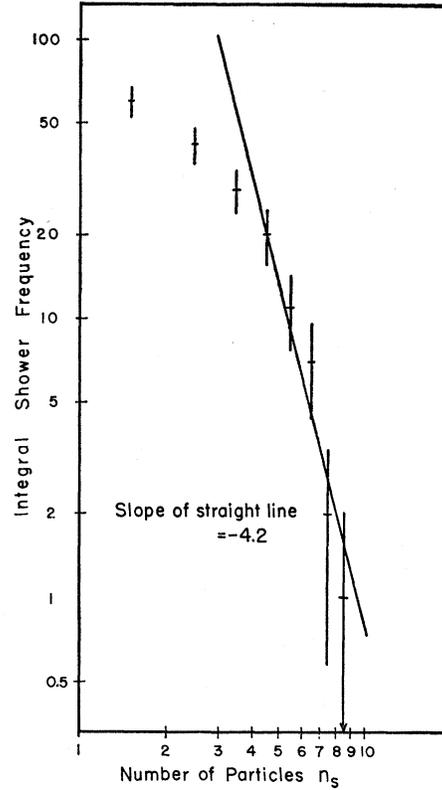


FIG. 3. Multiplicity spectrum of penetrating showers produced in graphite.

bremstrahlung had been corrected for using the results obtained in subsection (b) which will be described below. From the distribution of scintillation pulses in  $S_3$ , the multiplicity spectrum is shown in Fig. 3, showing the integral shower frequency  $\nu_s$  the number of shower particles  $n_s$ . The slope of the straight line in the log-log plot drawn through the most reliable range has the value of  $-4.2 \pm 0.5$ . This agrees satisfactorily with the value of  $-3.8 \pm 0.6$  obtained from cloud chamber experiment for carbon for  $n_s$  equal to or greater than 7, at a comparable altitude of 2960 m.<sup>18</sup> The range of validity in this experiment is  $4 < n_s < 9$ .

Figure 4 shows the integral frequency  $\nu_s$  energy of  $\pi^0$  mesons produced from cosmic-ray nuclear interactions in the graphite block  $G$ . These data were obtained with the triggering arrangement  $AB-S_2^m$  with the lead block  $PB$  in position. It can be represented by a relationship of the form  $E^{-\gamma}$ , with  $\gamma$  equal to  $-1.2 \pm 0.2$ .

In the investigation of penetrating showers produced in the polystyrene plate, the triggering arrangement  $AB-S_2^m$  was used, with and without the lead block  $PB$  in position. The results are shown in Fig. 5. The integral frequency  $\nu_s$  energy of  $\pi^0$  mesons produced from cosmic ray nuclear interactions can again be represented by an expression of the form  $E^{-\gamma}$ . The exponent  $\gamma$  has the value  $-1.7 \pm 0.2$  for the events observed without the lead filter  $PB$ , and has the value  $-1.4 \pm 0.2$  for the

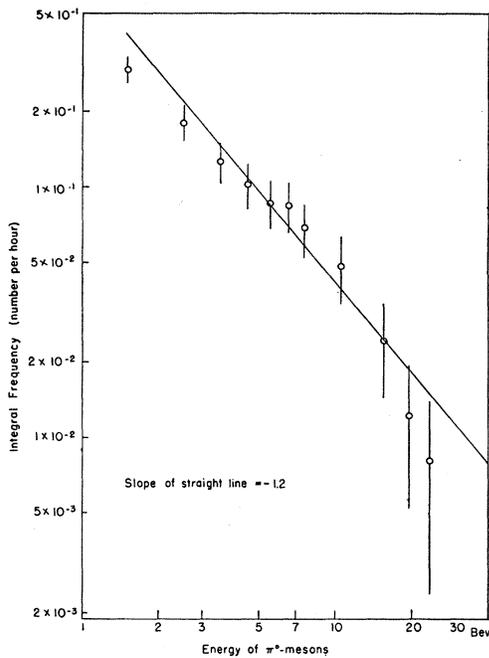


FIG. 4. Integral frequency of  $\pi^0$  mesons in penetrating showers produced in graphite.

events observed with the lead filter  $PB$  placed in position. This latter value can be compared with the value of  $-1.2 \pm 0.2$  for a similar expression for the integral frequency  $vs$  energy of  $\pi^0$  mesons produced in the graphite block  $G$  as discussed previously, since both sets of data were obtained under comparable condition of being both with the lead block  $PB$  in position. The curves seem to indicate that the exponent  $\gamma$  is slightly smaller (in absolute value) in the measurements obtained with the lead block  $PB$  in position than in those obtained without the lead block. The reason is probably that with the lead block in position the nuclear events observed consisted of both events produced by cosmic-ray primaries in the air and by secondaries produced in the lead block  $PB$ , which were of somewhat lower energies than the primaries. It is a known fact that the exponent  $\gamma$  decreases for  $\pi^0$  mesons of lower energies, which are mostly produced by cosmic rays of lower energies. In the present experiment, the exponent  $\gamma$  in the expression  $E^{-\gamma}$  for the integral frequency  $vs$  energy of  $\pi^0$  mesons relationship for polystyrene has been found to be  $-1.7 \pm 0.2$ , as mentioned above. The same exponent has been reported to be  $-1.5 \pm 0.2$ , for carbon obtained in cloud chamber experiment at approximately the same altitude.<sup>18</sup> Entirely different experimental techniques were used. At least in the high energy range, the method used in the present experiment should give more reliable information because of the intrinsic limitation of cloud chamber technique with respect to recording large number of particles.

It was attempted to make an estimate of the frequency of nuclear interactions by high energy cosmic-ray particles with the target materials, for events in which photons (presumably originated from the decay of the  $\pi^0$  mesons) of a total energy of at least 3 Bev were observed. When the lead block  $PB$  was in position, the contribution of  $PB$  to these observed events is estimated to be equal to 0.03 per hour, or  $0.013 \text{ cm}^{-2} \text{ sterad}^{-1} \text{ hour}^{-1} L^{-1}$ , where  $L$  is the geometric collision length of the nuclei. A summary of the results is given in Table I. It is seen that they are not inconsistent with the values of the intensity of protons of comparable energy at the same altitude,<sup>27</sup> account must be taken of the fact that we detected and observed only those nuclear interactions in which at least one  $\pi^0$  meson was produced which in turn decayed into photons which developed in the lead plate between the scintillators  $S_3$  and  $S_2$  into a cascade shower of at least about 16 electrons in order to trigger the recording system.

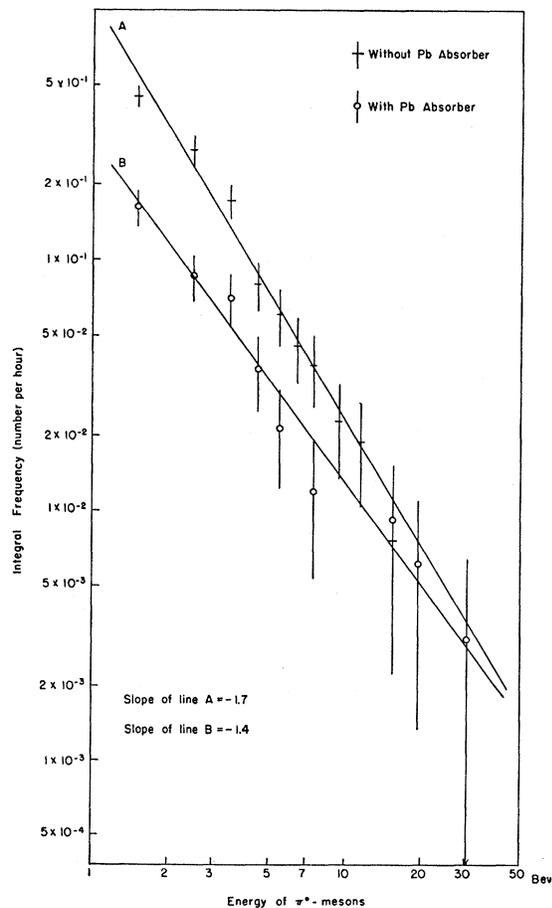


FIG. 5. Integral frequency of  $\pi^0$  mesons in penetrating showers produced in polystyrene.

<sup>27</sup> B. Rossi, *Revs. Modern Phys.* **20**, 537 (1948).

**(b) Bremsstrahlung Produced in the Lead Block  $PB$  by Cosmic-Ray  $\mu$  Mesons**

In analyzing the data from the set of experiment using the triggering arrangement  $AB-S_2^m$  with the lead block  $PB$  in position, events were found which could be identified as due to bremsstrahlung produced in the lead block by the  $\mu$  mesons. The procedure adopted for the analysis was described in subsection III (2) above. The results are shown in Fig. 6. These results will be compared with the theoretically expected values. Since it is generally assumed that  $\mu$  mesons have spin  $\frac{1}{2}$ , the interaction cross section for complete screening<sup>28</sup> of bremsstrahlung production for  $\mu$  mesons of spin  $\frac{1}{2}$  is assumed. Also it is assumed that the primary differential energy spectrum of the high-energy  $\mu$  mesons is given by an expression of the form<sup>29</sup>

$$N(E)dE \propto (E+E_0)^{-3}dE,$$

where  $E_0$  is a constant denoting the ionization loss of the  $\mu$  mesons from the point of production up to the altitude considered. Integrating numerically, we obtain the cross section for the production of photons of energies between  $E'$  and  $E'+dE'$  averaged over the whole  $\mu$ -meson spectrum given by the expression

$$\bar{\sigma}(E')dE' = \text{const}(E')^{-2.85}.$$

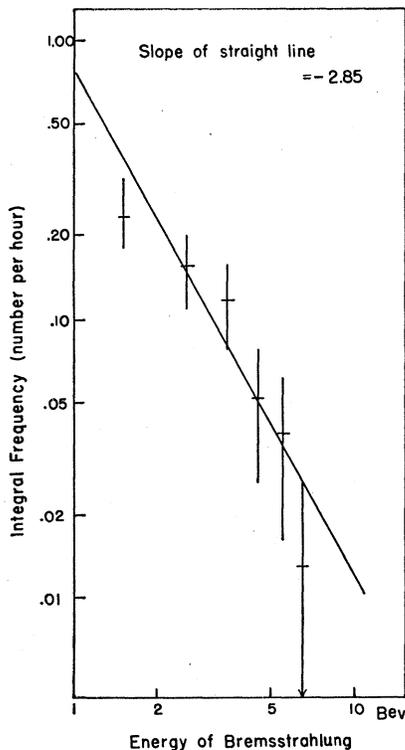


FIG. 6. Energy spectrum of bremsstrahlung produced by  $\mu$  mesons in lead.

<sup>28</sup> R. F. Christy and S. Kusaka, *Phys. Rev.* **59**, 414 (1941).

<sup>29</sup> Barrett, Bollinger, Cocconi, Eisenberg, and Greisen, *Revs. Modern Phys.* **24**, 133 (1952).

TABLE I. Frequency of nuclear interactions by high-energy cosmic rays in the target material in which photons were originated (presumably from the decay of  $\pi^0$  mesons) with a total energy of at least 3 Bev.

Target material	With or without lead block $PB$	Rate per hour	Rate (in $\text{cm}^{-2} \text{sterad}^{-1} \text{hour}^{-1} L^{-1}$ ) <sup>a</sup>
Graphite block $G$	with	0.15	0.027
Plastic scintillator $S_3$	without	0.17	0.23
Plastic scintillator $S_3$	with	0.037	0.051

<sup>a</sup>  $L$  denotes the geometrical collision length of the nuclei of the target material.

This expression should give the energy distribution of the bremsstrahlung produced by the assumed spectrum of  $\mu$  mesons. The straight line drawn in Fig. 6 is of a slope equal to  $-2.85$ . It is seen that the agreement is satisfactory within the statistical errors. We integrate the preceding cross section for photons of energies greater than 2 Bev and obtain the total cross section given by

$$\sigma_{\text{theoret}} = 3.2 \times 10^{-28} \text{ cm}^2.$$

This calculated total cross section is made use of, and the observed intensity of bremsstrahlung with energies higher than 2 Bev is compared with the observed flux intensity of  $\mu$  mesons underground.<sup>30</sup> It is found that the flux intensity corresponds to that of  $\mu$  mesons of energies higher than about 10 Bev. This latter value is of expected order of magnitude and serves as a check of the general correctness of the theory and the assumptions adopted in the calculations.

**V. CONCLUDING REMARKS**

Part of the present results obtained with large plastic scintillation counter measurements concerning the production of penetrating showers in light materials by high energy cosmic rays are in agreement with some earlier results of others using different methods of measurement. The integral frequency *vs* energy of the  $\pi^0$  mesons produced in polystyrene relationship can be represented by a power law with an exponent equal to  $-1.7 \pm 0.2$  when no heavy filter was used. The validity of this relationship probably extends over an energy range of the  $\pi^0$  mesons from 2 to 20 Bev. At the high energy end, our conclusions seem still reliable in contrast to other methods. The results obtained with the lead block  $PB$  in position and also the results obtained with the graphite block  $G$  as target material check the internal consistency of the method of analysis of the data obtained with the present apparatus. The relationship of the integral frequency *vs* the multiplicity of the charged mesons in the penetrating showers produced in the graphite block  $G$  can be described by a power law with an exponent equal to  $-4.2 \pm 0.5$  in the range of 4–9 particles. In the analysis of the pene-

<sup>30</sup> J. G. Wilson, *Progress in Cosmic Ray Physics* (Interscience Publishers, New York, 1952), E. P. George, pp. 395–450.

trating showers produced in the light materials, the effect of heavy meson production is considered small. This is justified from the cloud chamber investigations carried out at approximately the same altitude and in approximately the same energy range.<sup>18</sup>

The results concerning the bremsstrahlung produced

by high energy cosmic-ray  $\mu$  mesons confirm the essential correctness of the assumptions based upon which the calculations were made.

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## New Formulation of the General Theory of Relativity

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The principle of equivalence is partly abandoned as a basis for general relativity, and cosmic time is introduced as a new field variable. Field equations are obtained which take account of the self-energy of the gravitational field. The central symmetrical solution of the new field equations shows a significant deviation from the well-known Schwarzschild solution. It is free from singularities and gives a slightly smaller value for the perihelion motion of planetary orbits. Other consequences of the new formalism are:

- (a) A rigorous definition can be given to the concept of ether.
- (b) The energy-stress tensor of gravitational fields can be defined in a satisfactory manner.
- (c) The gravitational field energy of a particle is distributed continuously over the space and its integral is equal to the gravitational mass of the particle.
- (d) There are proper gravitational waves, generated by oscillating matter and propagating with the velocity of light.
- (e) There is a noticeable ether drift which tends to increase the gravitational mass of a body of given inertial mass.
- (f) The ratio of gravitational and inertial mass of radiating energy is twice the corresponding ratio for neutral static matter.
- (g) Hubble's recession constant is equal to the reciprocal of the age of the universe.

An outstanding problem is to determine the coupling constant between the gravitational field and the cosmic time field. The value  $\beta=1$  is strongly suggested by cosmological considerations. An experimental determination is possible if the rate of advance of the perihelion of the Mercury orbit is known more accurately.

### 1. INTRODUCTION

THE general theory of relativity rests upon the so-called *principle of equivalence* which states that: (a) It is possible to choose at every point of the space-time continuum a frame of reference which is Galilean at that particular point, i.e., in which special relativity holds in the immediate neighborhood of the point. (b) All frames of reference are equivalent in the sense that there is no general physical property which would distinguish one particular frame (or even a whole class of frames) from among the others.<sup>1</sup>

The two statements have an entirely different standing. Whereas postulate (a) is firmly established and supported by a considerable mass of experimental evidence, postulate (b) rests upon an essentially negative statement which obviously cannot be verified directly. In fact its validity has often been challenged

<sup>1</sup> A third postulate, often quoted in connection with the principle of equivalence, requires that physical laws should have a form which does not depend on the particular frame which one happens to use. This is not really a physical but an epistemological postulate; it expresses the belief that physical laws can be put in a particular mathematical form, the desirability of which can hardly be disputed.

in view of certain conceptual difficulties which arise from it both on the cosmical and local scales.

On the cosmical scale, postulate (b) is clearly in conflict with one of the most important cosmological principles known as *Weyl's postulate*, which necessarily leads to the notion of absolute cosmic time.<sup>2</sup> Although no formulation of Weyl's postulate has ever been given which would reveal that cosmic time has any noticeable physical effects, the conceptual conflict between the two principles can hardly be denied.

On the local scale, it is a well-known weakness of general relativity that it is incapable of defining the energy-stress tensor of the gravitational field in a satisfactory manner. The only known quantity which can be regarded as a substitute for the energy tensor is a pseudo-tensor which, if the principle of equivalence is accepted, can be transformed away in a suitable frame of reference. Closely connected with this is the following observation which was actually the starting point of the present investigations.

<sup>2</sup> See H. Bondi, *Cosmology* (Cambridge University Press, Cambridge, 1952), p. 70.