This coefficient, A_2^+, in the photoproduction of π^+ mesons, as determined at Cornell[24] and the California Institute of Technology,[22,23] is tabulated in Table I. At high energies the ratio of 2 to 1 appears correct. At lower energies the agreement is less satisfactory.

VI. CONCLUSIONS

The behavior of the π^0 photoproduction cross section between gamma-ray energies of 170 Mev and 320 Mev is well fitted by a model with over 90 percent of the production going through a magnetic dipole $J=\frac{3}{2}$, isotopic spin $\frac{3}{2}$, state resonant at about 300 Mev. There is a small amount (3 percent) of S-state production consistent with direct production of π^0 mesons by interaction with the dipole moment of the proton and through charge exchange of charged mesons. The interference of S and P waves is such as to favor the α_{33} scattering phase shift going through 90°. There is a suggestion of another mode of P-wave production, perhaps by electric quadrupole radiation through a $J=\frac{3}{2}$ state or magnetic dipole through a $J=\frac{1}{2}$ state.

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Analysis of a High-Energy Cosmic-Ray Shower. I. Soft Component and Trident Process*

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An analysis is presented of the soft component arising from a high energy nuclear shower ($\sim 3\times 10^{13}$ ev) observed in stripped emulsion. The chief results obtained are: (1) The production ratio of neutral π mesons to charged shower particles is 0.50 ± 0.11; (2) the lifetime of the neutral π meson is found to be $(1_{-0.5}^{+1})\times 10^{-14}$ sec; (3) the mean free path for direct electron pair production by high-energy electrons is found to be 4.4 and 1.1 radiation units for electrons in the energy intervals 1 to 10 Bev and 10 to 100 Bev, respectively.

INTRODUCTION

THE event considered in this paper was found in a stack of twenty-four 4 in.×6 in.×400μ G-5 stripped emulsions, packed in direct contact with each other and flown for 8 hours at 102 000 ft at 55° geomagnetic latitude. The emulsions were mounted on glass before development and processed by the usual temperature methods. After development each plate was cut into four 3 in.×2 in. sections for microscopic observation. The emulsion stack was then consecutively mounted and aligned on Lucite frames using heavy nuclei as markers.[1]

The event is of the type $3+36_p$,[2] and was so situated that the shower particles traversed 3.1 cm before leaving the outside edge of the stack; the average path length per emulsion was 2 mm. Of the 36 shower particles emanating directly from the star, 25 formed a narrow cone of half opening angle$=1.08\times 10^{-2}$ radian. Due to the high degree of alignment attained, each individual track could be followed through successive emulsions without any ambiguity (target diagrams were made in each emulsion). After traversing 3.97 mm one of the shower particles underwent a nuclear interaction producing a $0+12_p$ star (I_1) in which two of the secondary particles were in the narrow cone of the original event; one of these was almost exactly in the direction of the particle producing the star I_1 and it made another nuclear interaction of the type $13+16_p(I_2)$ after traversing an additional 24 mm.

The angular distribution of the primary star is plotted in Fig. 1. On the assumption that the primary was a proton, the kinematical energy is found to be $3.7\times 10^4 Mc^2$ using the median angle method[3] and $(3.4_{-2.5}^{+3.5})\times 10^4 Mc^2$ using the statistical method of Castagnoli *et al.*[4]

ENERGY MEASUREMENTS OF VERY HIGH-ENERGY PARTICLES

Due to the very high energy in the soft component produced in this event, the conventional method of measuring the multiple Coulomb scattering fails. However, the high degree of collimation makes possible the measurement of the relative scattering between particles.[5] In this method we measure the relative separation of two tracks and derive the mean second difference ($\bar{D}'_{\mathrm{rel}}$) for a fixed cell length t. In this type of

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[1] J. Crussard *et al.*, Phys. Rev. **93**, 253 (1954).

[2] This is the notation introduced by the Bristol group; $B+S_p$ means a star with B gray+black prongs ($I/I_0\geqslant 1.5$) and S shower particles ($I/I_0<1.5$) produced by a singly charged particle (p). The subscript n denotes a neutral primary.

[3] M. F. Kaplon and D. M. Ritson, Phys. Rev. **88**, 386 (1952).

[4] C. Castagnoli *et al.*, Nuovo cimento **10**, 1539 (1953).

[5] Lord, Fainberg, and Schein, Phys. Rev. **80**, 970 (1950).

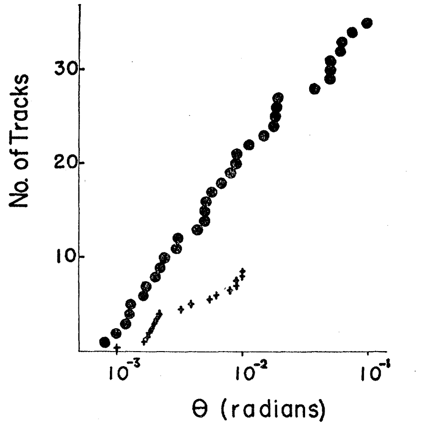

FIG. 1. Integral angular distributions of the charged particle. and electron pairs arising from the primary nuclear interactions The solid circles represent shower particles and the crosses represent electron pairs $E_p \sim (3.4_{-2.5}^{+3.5}) \times 10^4 Mc^2$.

measurement stage motion noise is entirely eliminated and the error introduced by distortion is also eliminated except for any local distortions in regions whose dimensions are of the order of the relative separation between the tracks being scattered; these local distortions are caused by small-size contaminants or crystallization and can usually be eliminated by inspection. The only sources of error left to contribute to the noise are those due to hair line setting and the random distribution of the grains along the tracks. Since we can reset the direction of the hair line in each measurement to coincide with the direction of the two tracks (which are almost parallel), this source can be reduced considerably and is evidently independent of cell size. The overall noise $\bar{N}$ resulting from the above two sources was found to be 0.1μ for the hair line setting on an individual minimum ionizing track; thus the noise for the relative scattering measurement is $\sqrt{2} \times 0.1\mu$ and the true relative scattering is $\bar{D}_{\text{rel}} = (\bar{D}'^2_{\text{rel}} - 2\bar{N}^2)^{\frac{1}{2}}$. $\bar{D}_{\text{rel}}$ may also be obtained by use of the third differences or by noise elimination between cell sizes of different length. The relative scattering in degrees/100μ between the tracks 1 and 2 is given if both particles have unit charge, by

$$\bar{\alpha}_{1,2} = 0.573 \bar{D}_{1,2} / (t/100)^{\frac{3}{2}} = K[1/(p\beta c)_1^2 + 1/(p\beta c)_2^2]^{\frac{1}{2}},$$

where t is the cell length, K is a constant slightly dependent on t,[6] p is the momentum, and βc the velocity. The lower and higher momentum of the two particles satisfy

$$(p\beta c)_{1,2} \leqslant (p\beta c)_{\text{low}} \leqslant \sqrt{2}(p\beta c)_{1,2} \leqslant (p\beta c)_{\text{high}},$$
$$(p\beta c)_{\text{low}} + (p\beta c)_{\text{high}} \geqslant 2\sqrt{2}(p\beta c)_{1,2},$$

where we define $(p\beta c)_{1,2} = K/\bar{\alpha}_{1,2}$. If relative scattering measurements are done between tracks 1, 2 and 3, then

$$\bar{D}'^2_{i,j} = \bar{D}_i^2 + \bar{D}_j^2 + 2\bar{N}^2;$$
$$\bar{D}_i^2 = (\bar{D}'^2_{i,j} + \bar{D}'^2_{i,k} - \bar{D}'^2_{j,k} - 2\bar{N}^2)/2,$$

[6] L. Voyvodic and E. Pickup, Phys. Rev. 85, 91 (1952).

and the individual momenta p_i are in principle obtainable (in practice, of course, sufficient statistics must be available to make the fluctuations smaller than the noise[7]). For the value of $\bar{N} = 0.1\mu$ and 2000μ cells one can measure values of $p\beta c$ up to $\sim$40 Bev. (it is worth noting here that for this type of measurement successive cells need not be in the same emulsion). For the measurements done in this paper 500μ cells were used over the central 1500μ of the tracks in each emulsion in order to reduce to a minimum any extreme distortions at the air or glass surfaces of the emulsion.

THE SOFT COMPONENT

The detailed survey of the shower in each emulsion showed that in addition to the original 25 shower particles in the narrow core, 29 electron pairs and 12 apparent tridents[8] were produced within 30.8 mm from

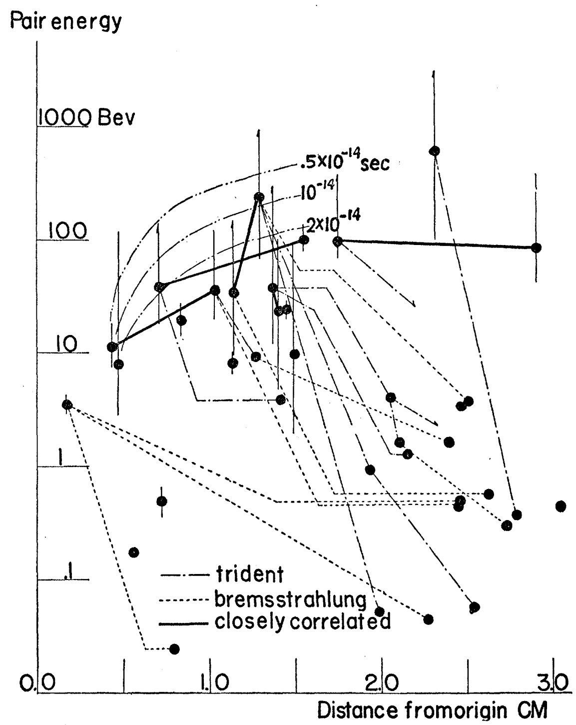

FIG. 2. Genetic relation of the electron pairs observed in the forward cone of the primary nuclear interaction.

[7] The relative fractional error in this measurement will be

$$\delta(\bar{D}_i) \approx (c^2/4q)^{\frac{1}{2}} (\Delta_{ij,i}^2 + \Delta_{jk,i}^2 + \Delta_{kj,i}^2 + 2\bar{n}_i^2)^{\frac{1}{2}},$$

where q is the number of independent measurements, $\Delta_{ij,i} = \bar{D}_{i,j}/\bar{D}_i$, $\bar{n}_i = \bar{N}/\bar{D}_i$ and $c \sim 1$. A further point to be noted here is that scattering measurements on electron tracks cannot extend over very long path lengths due to the high probability for bremsstrahlung. This approach is indeed better suited for measurements on the hard component.

[8] In emulsion the term trident is qualitatively applied to an event in which an electron track at minimum ionization suddenly changes into a track at $3\times$minimum ionization, this track subsequently resolving itself into three minimum ionizing electron tracks. Events of this nature were first observed by C. F. Powell, Nuovo cimento Suppl. 6, 370 (1949) and Bradt, Kaplon, and Peters, Helv. Phys. Acta 23, 24 (1950). M. Block and D. T. King

the shower origin in the cone defined by the narrow core. The energies of these events and the distance of their origin from the original star are plotted on a semi-logarithmic scale in Fig. 2. Solid broad lines connecting two high-energy pairs indicate they are spatially correlated, being less than 10μ apart in the plane normal to the shower axis. A dotted line connecting a high-energy pair with one of lower energy means that these are within 10μ of each other in the plane normal to the shower axis and a dashed line connecting two pairs means that the origin of the created pair is spatially unresolvable ($\leqslant 0.2\mu$) from the parent electron track; these are the apparent tridents. From this figure we can easily separate the high-energy electron pairs directly produced by the γ rays from π^0 decay from those of secondary origin.

LIFETIME OF THE π^0 MESON

In order to estimate the lifetime of the π^0 meson we consider the distribution $P(x)$ of converted electron pairs from π^0 decay for observations extended to an infinite length. This is given by[9]

$$P(x)dx=[2/(\lambda_c-\lambda_\tau)][\exp(-x/\lambda_c)-\exp(-x/\lambda_\tau)]dx,$$

where $P(x)$ is the average number of primary electron pairs arising from the 2γ decay of a π^0 meson of energy $\gamma\mu c^2$, the γ rays being converted in dx at a distance x from the shower origin; λ_c is the γ ray conversion length $=(97)$ (radiation length) and $\lambda_\tau=\gamma\beta c\tau$, τ being the proper lifetime of the π^0 meson. The mean conversion distance and its rms deviation are given by $\langle x\rangle=\lambda_c+\lambda_\tau$ and

$$[\langle(x-\langle x\rangle)^2\rangle]^{\frac{1}{2}}=(\lambda_c^2+\lambda_\tau^2)^{\frac{1}{2}}\approx\lambda_c;\quad \lambda_c\gg\lambda_\tau.$$

Since our observation length is $\sim\lambda_c$, λ_τ is determined by the slope of the near edge of the pair distribution as shown schematically in Fig. 3. This method has the advantage of requiring neither a knowledge of the cascade development nor an accurate knowledge of λ_c (or its energy dependence, if such exists). It suffers, however, from poor statistics, a defect which can be remedied in time. Application of this method results in a value for $\tau=(1_{-0.5}^{+1})\times 10^{-14}$ sec. (The energy-distance relations for $\tau=0.5\times 10^{-14}$, 10^{-14}, and 2×10^{-14} sec are plotted in Fig. 2.)

NEUTRAL TO CHARGE RATIO

The neutral to charge ratio R is defined as $R=N_{\pi^0}/N_s$, where N_{π^0} is the number of π^0 mesons produced in the shower and N_s is the total number of charged shower particles ($N_s=N_s'+N_{\pi^\pm}$, where N_s' thus represents

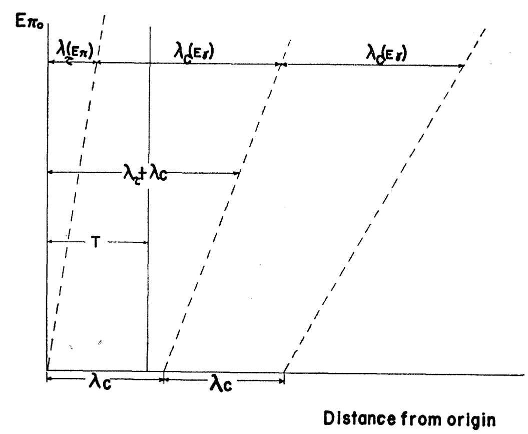

FIG. 3. Schematic representation of method of determining π^0 lifetime. (In this diagram an allowance has been made for a possible energy variation of the conversion length λ_c, of γ rays.)

those charged shower particles which are not charged π mesons). N_s is a directly measurable number but N_{π^0} must be deduced from observations on the soft component in the shower, assumed to originate from the 2γ decay of π^0 mesons. In this section we deduce the ratio R for this shower by two different methods (which however do not give statistically independent estimates).

The two methods differ only in the way of estimating N_{π^0}, the original number of neutral π mesons. From Appendix A the average number of primary pairs within a distance X from the point of emission of a single π^0 meson of energy $E_{\pi^0}=\gamma\mu c^2$ is given by

$$P(\tau,\gamma)=2\{1-[1/(1-\lambda_\tau(\gamma)/\lambda_c)] \times[\exp(-X/\lambda_c)-(\lambda_\tau(\gamma)/\lambda_c)\exp(-X/\lambda_\tau(\gamma))]\}.$$

To deduce the number of initial π^0 mesons from the above relation the number of primary pairs must be known. Inspection of Fig. 2 strongly suggests that those pairs of $E>10$ Bev are of a primary nature (arising directly from π^0 decay γ rays) while those with $E<10$ Bev are chiefly secondary. To apply the above equation those pairs with $E>10$ Bev which are not classified as bremsstrahlung or trident pairs were grouped into two classes: (1) $10\leqslant E\leqslant 100$ Bev and (2) $E>100$ Bev. The first class has ten pairs with $\langle E\rangle=42$ Bev and the second three pairs with $\langle E\rangle=320$ Bev. Using the above equation for these two energy classes, the initial number of π^0 mesons in the narrow cone containing 26 charged shower particles (one extra charged shower particle arises in this cone from the interaction I_1) is deduced and we find $R=0.53\pm0.17$ for $\tau=10^{-14}$ sec and $R=0.48\pm0.16$ for $\tau=0.5\times 10^{-14}$ sec. In this estimate no correction has been made for bremsstrahlung accompanying meson production.[10] If it is assumed that the majority of charged shower particles are π mesons this correction is quite small and reduces the above values for R by approximately 0.02.

[Phys. Rev. **95**, 171 (1954)] review the observations on tridents and make a quantitative estimate of the background of pseudo-tridents. These results from the conversion of the highly collimated bremsstrahlung accompanying fast electrons and are qualitatively indistinguishable from the true tridents which represent electron pair production by an electron.

[9] This equation is obtained by differentiating Eq. (2A) of Appendix A.

[10] L. Schiff, Phys. Rev. **76**, 89 (1949).

An alternative way of estimating N_{π^0} which is free of this possible ambiguity is to use those 5 groups of pairs in Fig. 2 (those connected by a solid line) which are strongly spatially correlated and for which each pair of the group can be considered as a directly converted pair from the same π^0 decay. For this estimation the resulting expression for P_2 of Appendix A must be used. With $\langle E_{\pi^0}\rangle = 150$ Bev and $\lambda_c = 3.75$ cm we find using P_2 that $R = 0.71 \pm 0.28$ for $\tau = 10^{-14}$ sec and $R = 0.66 \pm 0.26$ for $\tau = 0.5 \times 10^{-14}$ sec. The values of R obtained by the above two methods are rather sensitive to the value of λ_c (in both instances we use $\lambda_c = 3.75$ cm in emulsion); if λ_c is reduced by a factor of two the values of R obtained are also reduced by approximately the same factor.

A check on the determinations of R given above can be made by making use of only those observations within distances small compared to the conversion length.[11] This method is essentially identical to the first except that it restricts itself to consideration only of those pairs lying within the first 13 mm; this restriction further precludes the possibility of including secondary γ ray pairs. We have applied this method for the distances 5, 7, 9, 11, and 13 mm from the shower origin with the inclusion of all γ-ray pairs of $E > 1$ Bev which do not appear to be of a secondary nature and obtain for the corresponding values of R: 0.48, 0.47, 0.46, 0.46, and 0.53. In applying this method a correction has been made for bremsstrahlung accompanying meson production (assuming mainly $\pi^\pm$ mesons) and a finite π^0 lifetime of 5×10^{-15} sec has been taken into account by decreasing the available γ ray path length assuming $\langle E_{\pi^0}\rangle \approx 50$ Bev. The average value of R determined this way is 0.48 ± 0.20.

From the above observations we deduce the best value of R as 0.50 ± 0.11 (for $\tau = 5 \times 10^{-15}$ sec) in excellent agreement with previous results for very high energy interactions reported from this laboratory,[3,11] but rather different from the result of $R = 0.25$ reported by the Bristol group.[12]

TRIDENT PROCESS

By an apparent trident we mean an event where an electron pair originates in the emulsion so close to another electron that its origin is spatially unresolvable ($\leqslant 0.2\mu$) from the accompanying electron. As has been pointed out previously,[8] the true trident process, electron pair production by an electron, is qualitatively indistinguishable from the pseudo-trident process which is the conversion of highly collimated accompanying bremsstrahlung. An attempt to account for this background on a statistical basis has been made by Block and King;[8] an extension of their work will be presented in Appendix B.

We attempt here at first to eliminate this background (referred to as B.S. pairs) by the imposition of certain criteria which must be satisfied by an apparent trident for it to be classified as direct pair production by an electron. The criteria are:

$$\theta_i > [\bar{\alpha}_i^2(t) + \bar{\alpha}_N^2]^{\frac{1}{2}} \quad \text{for all } i = 1, 2, 3, \tag{1}$$

$$\theta_1 > [\bar{\alpha}_1^2(t) + \bar{\alpha}_N^2]^{\frac{1}{2}} \quad \text{for } E_1 > E_2, E_3. \tag{2}$$

In the above, θ_i is the deflection angle of the ith electron, $\bar{\alpha}_i(t)$ the deflection due to its multiple Coulomb scattering in the distance t used for measurement of deflection, $\bar{\alpha}_N$ the contribution as noise of the scattering of the the reference track used to measure the deflection, and E_i the energy of the ith electron; here $i = 1$, 2, and 3 refer to the three electrons forming the trident. The first criterion is to be applied when no energy distinction can be made between the three electrons and ensures that at least the incident electron has been deflected at the point of origin of the trident; the second criterion applies to the case in which the highest energy electron of the trident has been deflected but does not necessarily insure that the trident primary was deflected. In Table I all apparent tridents are listed. Column 1 identifies the event listing the three members of the trident T_x^i ($i = 1$, 2, 3), column 2 gives the primary energy in Bev and the origin of the incident electron (o refers to the energy estimate by opening angle and s by scattering), column 3 the distance of the parent electron from its source, column 4 the projected deflection angle in milliradians, column 5 the energy in Bev of each member of the trident, and column 6 the identification as a true trident using criteria (1) or (2) or a ? indicating no verdict.

In Table II the tridents are divided into two groups according to the primary electron energy (column 1), the second and third columns give the average and the total track length of all the electrons in each energy region (in radiation units), the fourth the number of observed B.S. pairs, the fifth the number of observed apparent tridents, the sixth the total number of B.S. pairs corrected by the method of Appendix B, the seventh the number of true tridents, the eighth and ninth the mean free path in radiation units for B.S. pairs and tridents, and the tenth an upper limit for the trident mean free path using criteria (1) and (2). (In Table II we have not included one of the observed pairs in the soft development as a B.S. pair since there is a strong indication from its direction that it results from the secondary interaction I_1.)

In applying the criteria (1) and (2) given previously

[11] Kaplon, Walker, and Koshiba, Phys. Rev. **93**, 1424 (1954).

[12] R. R. Daniel *et al.*, Phil. Mag. **43**, 753 (1952); J. H. Mulvey, Proc. Roy. Soc. (London) **221**, 367 (1954). Both find $R = 0.25$ in marked contrast with our result. It should be pointed out that both these experiments were done on glass backed emulsions. In particular, the latter experiment, which derives its result from the analysis of a very large star, suffers particularly from this disadvantage in that all electron pairs could not be positively identified due to the interposition of the glass backing. In addition, in this analysis, the use of cascade theory, especially at the beginning of the development is subject to error since the slope of the rise in the development is quite sensitive to a small admixture of high-energy photons.

TABLE I. Observational data on the parent electrons forming tridents and the electron members of the tridents.

Trident	Primary origin and energy (Bev)		Distance (mm) of primary from its origin	Projected angles of deflection (milliradians)	Energies of trident members (Bev)	Identi-fication
T_1^1				10 ± 8.5	0.082	
T_1^2	P_5^2	$18(o)_{-8}^{+42}$	2.4	-1.02 ± 1.8	0.83	(2)
T_1^3		$>17(s)$		1.67 ± 0.22	>9.0	
T_2^1				-0.38 ± 0.96	1.2 ± 0.4	
T_2^2	P_{10}^1	$18(o)_{-13.3}^{+\infty}$	7.15	-1.08 ± 0.5	$2.6_{-2.05}^{+3.6}$	(a)
T_2^3		$5(s)_{-1.7}^{+5.7}$		-0.2 ± 0.36	$4.6_{-1.0}^{+2.0}$	
T_3^1				...	$1.7_{-0.4}^{+0.9}$	
T_3^2	P_{11}^2	$19(o)_{-13}^{+\infty}$	6.85	...	>3.7	(?)
T_3^3		$3.0(s)_{-1.2}^{+6.1}$		...	$3.35_{-0.8}^{+1.8}$	
T_4^1				...	>2.3	
T_4^2	P_{11}^1	$19(o)_{-13}^{+\infty}$	7.8	...	$0.6_{-0.15}^{+0.3}$	(?)
T_4^3		$2.8_{-1.0}^{+4.8}$		...	$0.67_{-0.22}^{+0.24}$	
T_5^1				0.34 ± 0.3	>17	
T_5^2	P_{19}^1	$>50(o)$	4.7	0.14 ± 3.6	0.27	(2)
T_5^3		$\sim30(s)$		14.3	<0.1	
T_6^1				1.7 ± 1.5	...	(2)
T_6^2	P_{18}^2	$\sim100(o)$	11.3	-5.0 ± 1.5	...	
T_6^3		$>13.5(s)$		-24.3 ± 1.5	...	
T_7^1				...	$1.75_{-0.4}^{+0.9}$	
T_7^2	T_3^3	$3.35_{-0.8}^{+1.8}$	0.41	...	$1.05_{-0.27}^{+0.55}$	(?)
T_7^3				...	$0.55_{-0.14}^{+0.28}$	
T_8^1				24.6 ± 0.75	...	
T_8^2	T_3^2	>3.7	6.93	37.4 ± 0.75	...	(1)
T_8^3				57.2 ± 0.75	...	
T_9^1				-3.7 ± 0.75	...	
T_9^2	P_{12}^2	$<92(o)$	14.1	0.65 ± 0.75	...	(?)
T_9^3		$>68(s)$		11.3 ± 0.75	...	
T_{10}^1				16.1 ± 2.0	0.35	
T_{10}^2	P_7^2	$\sim200(o)$	6.39	6.9 ± 1.7	0.59	(?)
T_{10}^3		$>35(s)$		0.08 ± 0.15	...	
T_{11}^1				0.013 ± 0.2	...	
T_{11}^2	P_7^1	$26–35(s)$	6.88	53.7 ± 50	0.0091	(?)
T_{11}^3				-27.5 ± 3.7	0.042	
T_{12}^1				...	0.022	
T_{12}^2	$T_{10}^3(P_7^2)$	$\sim200(o)$	$12.49(P_7^2)$	...	0.035	(?)
T_{12}^3		$>35(s)$	$6.1(T_{10}^3)$	...	...	

[a] See text.

we have given criterion (1) a weight factor of unity since it certainly ensures that the incident electron was deflected at the point of origin of the trident; a weight factor of $\frac{1}{2}$ is given criterion (2) due to its weaker nature (this is due to the impossibility of determining in the trident the continuation of the incident electron). In addition event T_2 is given a weight of $\frac{1}{2}$ because of the low probability of finding a B.S. pair as an apparent trident for the energy and distance from the parent of this event. We then find $N_T \geqslant 1.5$ for both energy ranges resulting in $\lambda_T \leqslant 10.45$ radiation units for $1 \leqslant E \leqslant 10$ Bev and $\lambda_T \leqslant 4.72$ radiation units for $10 < E \leqslant 100$ Bev.

To improve the direct estimation of an upper limit for λ_T we have calculated in Appendix B the contribution of the converted accompanying bremsradiation to the apparent trident cases. Applying the results of

TABLE II. Classification of electrons into energy groups and the numbers of tridents and converted bremsstrahlung γ rays (B.S. pairs) associated with these electrons.

Energy range of electrons in Bev	Average electron track length (rad. lengths)	Total track length (rad. lengths)	No. of observed B.S. pairs	No. of apparent tridents	Corrected No. of B.S. pairs	Corrected No. of tridents	$\lambda_{B.S.}$ (rad. lengths)	λ_T (rad. lengths)	Upper limit for λ_T by direct identification
10–100	0.6	7.08	4	8	5.6	6.4	1.26	1.1	4.72
1–10	0.56	15.69	8	4	8.5	3.5	1.85	4.5	10.45

Appendix B we find that 1.6 of the apparent tridents in the high energy range and 0.5 in the low energy range could be due to B.S. pairs. The resulting mean free paths turn out to be $\lambda_T = 4.5$ and 1.1 radiation units for $1 \leqslant E \leqslant 10$ Bev and $10 < E \leqslant 100$ Bev respectively and the mean free path for B.S. pairs in radiation units is $\lambda_{\text{B.S.}} = 1.85$ and 1.26 respectively for the above two energy ranges.

A check on the above figures is furnished as follows. A high energy electron in one radiation unit will emit on the average ≈ 3.2 photons with energy $> (1/10)$ the primary electron energy, so that $\lambda_{\text{brems}} \approx 1/3.2$. Since $\lambda_{\text{B.S.}}$ should approximately equal $\lambda_{\text{brems}} + \lambda_{\text{conv}}$, we find $\lambda_{\text{B.S.}} \approx 0.3 + 9/7 = 1.6$ in good agreement with the values for $\lambda_{\text{B.S.}}$ obtained above.

CONCLUSIONS

The analysis of the soft component arising from a very high-energy nuclear interaction has given the following results.

(a) The production ratio of neutral π mesons to charged shower particles for this event is determined as $R = 0.50 \pm 0.11$ in agreement with previous results from this laboratory[3,11] and others,[13] but in disagreement with a lower value reported by the Bristol group.[12] The determination of this ratio is of importance for its implications with respect to heavy-meson production at high energies. Since

$$R = N_{\pi^0}/(N_{\pi^\pm} + N_s') = (N_{\pi^0}/N_{\pi^\pm})/(1 + N_s'/N_{\pi^\pm}),$$
$$N_s'/N_{\pi^\pm} = (N_{\pi^0}/N_{\pi^\pm})/R - 1,$$

a knowledge of R and the ratio $N_{\pi^0}/N_{\pi^\pm}$ will set an upper limit to the relative production efficiency of heavy mesons as compared to $\pi^\pm$ mesons (it is an upper limit since N_s' may contain some protons as well as heavy mesons). We assume that charge independence holds at these high energies so that $N_{\pi^0}/N_{\pi^\pm} = 0.5$; we then find that $N_s'/N_{\pi^\pm} < 0.28$ implying that particles other than π mesons are quite inefficiently produced at these high energies. (In this connection it is worth noting that Sitte, Froehlich, and Nadelhaft,[14] in a study of electron production in high-energy nuclear interactions, $\sim$10 to 100 Bev, have found that the fractional energy transfer to the electron component is constant or slightly increasing for energies above 20 Bev. This is inconsistent with the result expected if the Bristol value of $R = 0.25$ is correct, since then heavy meson production should compete with π meson production and if charge independence is assumed to hold, the fractional energy transfer to the electron component should decrease by about $\frac{1}{2}$ in the absence of any other source of the soft component. It seems to us that their results favor the observations reported in this paper.)

The conclusions stated in this paragraph rest upon the validity of charge independence at high energies and the assumption that directly produced π^0 mesons are the sole source of high-energy γ rays in nuclear interactions. If other sources of γ rays or π^0 mesons should exist, these conclusions are invalid; however in this case the γ rays must be either directly produced or be the decay products of very short-lived particles other than π^0 mesons having a lifetime $< 10^{-14}$ sec and the π^0 mesons if secondary in nature must be decay products of unstable particles of even shorter lifetime.

(b) The lifetime of the π^0 meson is estimated to be $(1_{-0.5}^{+1}) \times 10^{-14}$ sec in good agreement with previous values reported in the literature.[15]

(c) The mean free path for an electron to produce an electron pair (trident process) was found to be 4.5 and 1.1 radiation lengths for electron energies in the range 1 to 10 and 10 to 100 Bev, respectively.[16] Though the statistics are low it is believed that these values are not in error by more than a factor of two. This result, which for the high energy range (>10 Bev) is in rather marked disagreement with theory,[17] implies that for these energies the trident process becomes of comparable importance with bremsstrahlung and pair conversion in the soft cascade development. If further work should substantiate the validity of the results reported in this paper, the high-energy soft cascade (as well as the theory of the trident process) should certainly be reassessed.

We should like to express our appreciation to Miss B. Hull for her assistance in scanning and to the Office of Naval Research for its aid in obtaining the balloon exposure.

APPENDIX A

We consider a π^0 meson of energy $E_{\pi^0} = \gamma\mu c^2$ and proper lifetime τ. Let γ_c be the γ-ray conversion length and $\lambda_\tau = \gamma\beta c\tau$ the decay length for the π^0 meson decay into two γ rays. The probability that the π^0 meson decays in dy at a distance y from its point of origin, the first of the two γ rays converts at t_1 in dt_1 and the

[13] Lal, Pal, and Rama (private communication); they find $R = 0.4 \pm 0.08$ for the energy region 50 to 250 Bev/nucleon. P. Freier and J. Naugle, Duke Conference on Cosmic Radiation (unpublished), find $R = 0.46 \pm 0.09$ from the analysis of a very high-energy interaction produced by a heavy nucleus.

[14] Sitte, Froehlich, and Nadelhaft, Phys. Rev. (to be published). We are indebted to the authors for a prepublication copy.

[15] Kaplon, Peters, and Ritson, Phys. Rev. **85**, 932 (1952); R. R. Daniel *et al.*, Phil. Mag. **43**, 753 (1952); J. J. Lord *et al.*, Phys. Rev. **87**, 538 (1952), B. M. Amand, Proc. Roy. Soc. (London) **220**, 183 (1953): the last paper reviews the results, to that date, on the lifetime of the π^0 meson and has a complete bibliography.

[16] P. Freier and J. Naugle, Duke Conference on Cosmic Radiation and private communication; for electron energies $E > 2$ Bev, and extending up to 50 Bev, they find the trident mean free path $\lambda_T \leqslant 11$ cm of emulsion or 3.7 radiation units in excellent agreement with the results reported here.

[17] Block, King, and Wada, Phys. Rev. **96**, 1628 (1954), have recently evaluated and compared the theoretical predictions for the true trident process. From their results we obtain $\lambda_T \approx 14$ and 8 radiation units for electron energies of 10 and 100 Bev respectively. These values correspond to the cross sections with inclusion of screening. The corresponding values for the unscreened cross sections are approximately 12 and 5.5 radiation units for electron energies of 10 and 100 Bev. We are indebted to Drs. Block, King, and Wada for a prepublication copy of their paper.

second at t_2 in dt_2 (measured from the origin) is given by:

$$P(y,t_1,t_2)dydt_1dt_2=\exp(-y/\lambda_\tau)(dy/\lambda_\tau)\times\exp[-(t_1-y)/\lambda_c](dt_1/\lambda_c)\times\exp[-(t_2-y)/\lambda_c](dt_2/\lambda_c). \quad (1A)$$

The probability of observing both converted γ rays from the single π^0 meson within a distance X from the origin is given by:

$$P_2=\int_0^X dy\int_y^X dt_1\int_y^X dt_2P(y,t_1,t_2).$$

The probability that only one γ ray is converted within the distance X from the origin is given by

$$P_1=2\int_0^X dy\int_y^X dt_1\int_X^\infty dt_2P(y,t_1,t_2).$$

The above integrations are easily performed and one then finds that the probable number of pairs observed at a distance X from the origin arising from a single π^0 meson is:

$$2P_2+P_1=2\{1-[1/(\lambda_c-\lambda_\tau)]\times[\lambda_c\exp(-X/\lambda_c)-\lambda_\tau\exp(-X/\lambda_\tau)]\}. \quad (2A)$$

APPENDIX B

In this appendix we calculate the background contribution from the conversion of the bremsstrahlung accompanying high energy electrons to the trident process (pseudo-tridents).[18] To calculate the probability of finding a converted bremsstrahlung pair within a certain distance of the parent electron, we need to consider the angles of emission of the γ rays as well as the scattering of the parent electron. The former effect is important for those γ rays which have not travelled far before conversion into electron pairs, whereas the latter is important for those γ rays which have travelled a considerable distance before conversion into electron pairs.

Consider a fast electron moving in the x direction incident on an absorber in the xy plane. The distribution in y of the displacement due to multiple Coulomb scattering of the electron projected onto the xy plane after traversing a distance t has been derived by Rossi and Greisen[19] and is given by

$$H(t,y)=(1/\pi)^{\frac{1}{2}}(3w^2/4t^3)^{\frac{1}{2}}\exp(-3w^2y^2/4t^3), \quad (B1)$$

where the notation above is that of Rossi and Greisen.

[18] The analysis given here is an extension of that done by Block and King (see reference 8) in that an attempt has been made to take into account the characteristic angle of emission and the origin of the bremsstrahlung γ rays whose conversion contributes to the background of tridents. We are indebted to them for a prepublication copy of their manuscript.

[19] B. Rossi and K. Greisen, Revs. Modern Phys. **13**, 240 (1941).

If the reference axis is inclined to the x direction by a small angle θ_0, the distribution is given by

$$H(t,y,\theta_0)=(1/\pi)^{\frac{1}{2}}(3w^2/4t^3)^{\frac{1}{2}}\exp[-3w^2(y-\theta_0t)^2/4t^3]. \quad (B1')$$

We assume a Gaussian distribution for the angle of emission of the γ rays by the electrons with the root-mean-square angle α:

$$P(\theta_0)d\theta_0=(1/2\pi\alpha^2)^{\frac{1}{2}}\exp(-\theta_0^2/2\alpha^2). \quad (B2)$$

The probability distribution of the separation between the γ ray and its parent electron at a distance t from the point of emission of the γ ray will be given by

$$Q(t,y)=\int_{-\infty}^\infty P(\theta_0)H(t,y,\theta_0)d\theta_0 =(1/\pi t^2)^{\frac{1}{2}}(1/(4t/3w^2+2\alpha^2)^{\frac{1}{2}})\times\exp[-(y/t)^2(1/(4t/3w^2+2\alpha^2)]. \quad (B3)$$

A fast electron traversing an absorber will emit a spectrum of bremsstrahlung γ rays; we denote the mean free path for emitting photons of energy exceeding some critical value by λ' and we may consider that for each portion of the electron track there are dz/λ' photons emitted. (z as well as all other quantities having the dimensions of a length are measured in radiation units.) The total number of pairs converted in dt at a distance t from the origin of the parent electron will then be

$$p(t)dt=\int_0^t(dz/\lambda')\exp[-(7/9)(t-z)](7/9)dt =(dt/\lambda')[1-\exp(-7t/9)]. \quad (B4)$$

The probability of these converted pairs lying within a distance y at t from the parent electron will be

$$M(y,t)=(1/p(t)dt)\int_0^t(dz/\lambda')\times\exp[-(7/9)(t-z)](7/9)dt\int_{-y}^{y}Q(t-z,\,y)dy, \quad (B5)$$

and the probability of finding an electron pair within a distance y of all those pairs which were converted at distances extending from the electron origin up to T will be

$$N(y,T)=\int_0^T M(y,t)p(t)dt\Big/\int_0^T p(t)dt. \quad (B6)$$

We note that in these expressions for $M(y,t)$ and $N(y,T)$ the mean free path λ' does not appear; this is a consequence of the assumption that the distribution in θ_0 is not dependent on the energy of the emitted γ ray.

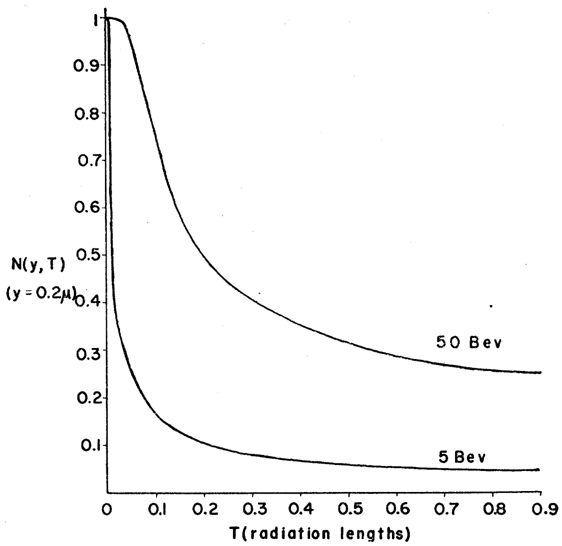

FIG. 4. Fraction of converted bremsstrahlung (B.S. pairs) lying within 0.2μ projected distance of the parent electron as a function of distance of the electron in absorber.

The above integrations are performed to obtain

$$M(y,t)=(7/9)\{1/[1-\exp(-7t/9)]\}\times\int_0^t d\epsilon\,\exp(-7\epsilon/9)E(y,\epsilon),\quad\text{(B5}')$$

$$N(y,T)=\frac{7}{9}\frac{1}{T-(9/7)[1-\exp(-7T/9)]}\times\int_0^T d\epsilon(T-\epsilon)\exp(-7\epsilon/9)E(y,\epsilon),\quad\text{(B6}')$$

where $E(y,\epsilon)$ is defined as

$$E(y,\epsilon)=(4/\pi)^{\frac{1}{2}}\int_0^{y/f(\epsilon)}\exp(-\phi^2)d\phi,\quad\text{(B7)}$$

$$f(\epsilon)=\epsilon(4\epsilon/3w^2+2\alpha^2)^{\frac{1}{2}}$$
$$=\epsilon(\tfrac{2}{3}\langle\theta^2{}_{\text{scatt}}\rangle+2\langle\theta^2{}_{\text{open}}\rangle)^{\frac{1}{2}}.$$

For emulsion we use the value of the scattering constant $K=26$ corresponding to 7.75 Mev radians/(radiation length)$^{\frac{1}{2}}$ and we have

$$(\langle\theta^2{}_{\text{scatt}}\rangle)^{\frac{1}{2}}=(7.75/p\beta)(t)^{\frac{1}{2}}\approx(7.75/E_0)(t)^{\frac{1}{2}}.\quad\text{(B8)}$$

For $(\langle\theta^2{}_{\text{open}}\rangle)^{\frac{1}{2}}$ we use the results of Stearns,[20] for $k/E_0=0.5$ and obtain

$$(\langle\theta^2{}_{\text{open}}\rangle)^{\frac{1}{2}}=0.67(mc^2/E_0)\log(E_0/mc^2).\quad\text{(B9)}$$

The average opening angle given by expression (B9) refers to the angle of emission of γ rays with respect to the incident electron; the angle between the γ ray and the secondary electron, which is the one of interest here, will be given approximately by increasing (B9) by a factor of $\sqrt{2}$. This factor is cancelled by another factor of $\sqrt{2}$ which is required to convert the spatial angle to a projected angle. It is also possible that the scattering constant should be increased since the physical case in question here corresponds to larger cell lengths and no cutoff in scattering.[6]

The numerical integration for (B6′) was carried out for $y=0.2\mu$ and for $E_0=50$ and 5 Bev. The results are given in Fig. 4.

[20] M. Stearns, Phys. Rev. **76**, 836 (1949).