between the Fourier component $\mathbf{i}(\mathbf{k})$ of the current density of the electrons and the corresponding component $\mathbf{A}(\mathbf{k})$ of the vector potential. If the gauge in $\mathbf{A}$ is chosen so that $\text{div}\mathbf{A}=0$, then $\mathbf{A}$ and $\mathbf{k}$ are perpendicular, and $\mathbf{i}(\mathbf{k})$ is parallel to $\mathbf{A}(\mathbf{k})$. Thus we may write:

$$(4\pi/c)\mathbf{i}(\mathbf{k})=-F(k)\mathbf{A}(\mathbf{k}). \tag{1}$$

As pointed out by Klein and by Schafroth, the Meissner effect is obtained if $F(0)>0$. If $\mathbf{A}_0(\mathbf{k})$ represents the source field, the self-consistent solution is obtained from:

$$[k^2+F(k)]\mathbf{A}(\mathbf{k})=k^2\mathbf{A}_0(\mathbf{k}). \tag{2}$$

If one assumes with London that the wave functions are not modified at all by the magnetic field,

$$F(k)=\lambda_0^{-2}=4\pi e^2n/mc^2, \tag{3}$$

where $\lambda_0$ is the penetration depth and $n$ is the electron concentration. Actually, one should include first-order perturbation changes resulting from the field. We assume that the energies of excited states and matrix elements are similar to those of a normal degenerate electron gas, except for the additional energy $\epsilon$ required for each excited electron. A modification of Klein's treatment then gives

$$\lambda_0^2F(k)=1-\frac{3}{2}\frac{k_0}{k}\times\int_0^1 u(1-u^2)\log\left(\frac{k_0u+\frac{1}{2}k+m\epsilon/\hbar^2k}{|k_0u-\frac{1}{2}k|+m\epsilon/\hbar^2k}\right)du, \tag{4}$$

where $k_0$ is the magnitude of the wave vector of the Fermi surface. When $\epsilon=0$, this gives the usual Landau diamagnetism. When $\epsilon>0$, $F(k)\rightarrow\lambda_0^{-2}$ as $k\rightarrow 0$. For $k$ such that $\lambda_0k\sim 1$ and $m\epsilon/\hbar^2k^2\sim 1$, a good approximation to $F(k)$ is

$$\lambda_0^2F(k)=\frac{3}{2}\frac{m\epsilon}{\hbar^2kk_0}\log\left(1+\frac{\hbar^2kk_0}{m\epsilon}\right). \tag{5}$$

This expression (5) is valid over the range of $k$ which makes an appreciable contribution to the field for normal penetration phenomena. The penetration depth is obtained from the integral

$$\lambda=\frac{2}{\pi}\int_0^\infty\frac{dk}{k^2+F(k)}, \tag{6}$$

which may be evaluated approximately by replacing $k$ in the logarithm by an average value $\sim\lambda_0^{-1}$. We then find:

$$\lambda=0.77\lambda_0\left[\frac{3}{2}\frac{m\epsilon\lambda_0}{\hbar^2k_0}\log\left(1+\frac{\hbar^2k_0}{m\epsilon\lambda_0}\right)\right]^{-\frac{1}{3}}. \tag{7}$$

With $\epsilon\sim 5\times 10^{-16}$ ergs, $k_0\sim 10^8$ cm$^{-1}$, $\lambda_0\sim 10^{-6}$ cm, we find $\lambda\sim 2\lambda_0$. Since the logarithm is slowly varying, $\lambda$ varies approximately as $(E_FkT_c)^{-\frac{1}{3}}$ or as $n^{-2/9}\epsilon^{-\frac{1}{3}}$.

The nature of the excited states in actual superconductors is indicated by the temperature variation of the specific heat, thermal conduction, and electrical conduction (observed in the skin depth at microwave frequencies). These all indicate a density of "normal" electrons in excited states, such as would follow from our model. The energy $\epsilon$ undoubtedly depends on temperature and goes to zero at the transition point. Semi-empirical expressions for free energy and critical field derived from a model of this sort are in good agreement with experiment.[7] There is less justification for assuming that the matrix elements of the magnetic interaction are unchanged by the transition, but one would not expect a change in matrix elements to alter the results in a drastic way. Thus any model which gives correctly the thermodynamical properties of the superconducting state will most likely give the Meissner effect.

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## Energy Distribution of Protons Due to Collision Energy Loss

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EXPERIMENTS have shown that the Landau distribution is too narrow for electrons (e.g., Rothwell,[1] West,[2] Birkhoff[3]). Blunck and Leisegang,[4] Blunck[5] and others, using essentially the same method as Landau,[6] have attempted to improve the calculated distribution by inclusion of the so-called second moment term which they obtained in an approximate form by using the semiclassical Bohr treatment of collision loss. Apart from this improvement, however, their treatment suffers from the same defects as the Landau theory.

Recently Fano[7] has given a complete formulation of the general problem of the passage of charged particles through layers of material thick enough to contain the whole range of the particles, although the only solution quoted which is specific to the Landau problem (thin layers) is that of Landau himself. Fano states two of the errors in the Landau treatment, *viz.*, use of approximate probability distributions for energy loss and extension of the upper limit of integration over $\epsilon$ (energy loss) to infinity. It is the purpose of the present note to indicate an accurate method of solving the problem and to apply this method to the energy distribution of protons.

The transport equation is

$$\frac{\partial g(x,E)}{\partial x}=\int_{\epsilon_{\min}}^{E_0-E} g(x, E+\epsilon)w(E+\epsilon, \epsilon)d\epsilon - \int_{\epsilon_{\min}}^{\epsilon_{\max}} g(x,E)w(E,\epsilon)d\epsilon, \quad (1)$$

where $g(x,E)dE$ is the number of particles in $(E,dE)$, $E_0$ is the initial energy, and $\epsilon_{\min}$, $\epsilon_{\max}$ are, respectively, the minimum and maximum amounts of energy which can be transferred in a single collision. If the layer of absorbing material is thin enough, it is justifiable to regard the function $w(E,\epsilon)$—the probability per unit length of an energy loss $\epsilon$—as independent of the particle energy $E$.

Instead of the Laplace transform used by Landau we apply the Mellin transform to define a transform function $G(x,s)$ satisfying

$$\frac{\partial G(x,s)}{\partial x}=\int_0^\infty dL\int_{\epsilon_{\min}}^{E_0-L+\epsilon} L^{s-1}\left(1-\frac{\epsilon}{L}\right)^{s-1} \times g(x,L)w(\epsilon)d\epsilon-G(x,s)\int_{\epsilon_{\min}}^{\epsilon_{\max}} w(\epsilon)d\epsilon, \quad (2)$$

with $L=E+\epsilon$.

The range of integration over $\epsilon$ is split up by choosing $\epsilon_1$ such that the first three terms of the binomial expansion are adequate for all values of $\epsilon$ between $\epsilon_{\min}$ and $\epsilon_1$:

$$\frac{\partial G(x,s)}{\partial x}=-G(x,s)\int_{\epsilon_1}^{\epsilon_{\max}} w(\epsilon)d\epsilon -(s-1)G(x, s-1)\int_{\epsilon_{\min}}^{\epsilon_1} \epsilon w(\epsilon)d\epsilon +\frac{(s-1)(s-2)}{2}G(x, s-2)\int_{\epsilon_{\min}}^{\epsilon_1} \epsilon^2 w(\epsilon)d\epsilon +\int_0^\infty dL\int_{\epsilon_1}^{E_0-L+\epsilon} L^{s-1}\left(1-\frac{\epsilon}{L}\right)^{s-1} g(x,L)w(\epsilon)d\epsilon. \quad (3)$$

The first three terms on the right hand side of (3) provide a Gaussian approximation to $g(x,E)$ with roughly the correct half-width.[8] The third (second moment) term must be worked out by an accurate quantum mechanical method, analogous to the Bethe stopping-power calculation for the first moment.

For protons, in general, $E_0-L>\epsilon_{\max}$ and the upper limit of integration in the last term of (3) must be taken as $\epsilon_{\max}$ and not $E_0-L$. It has been found essential to use the correct upper limit here and this fact indicates a serious defect in the Landau treatment since the latter cannot be adapted for use with the correct upper limit.

The last term in (3) may be evaluated by expanding out the binomial series (including 4 terms of the expansion), inserting the expression for $w(\epsilon)$ (see Bethe),[9] and carrying out the integrations. After combining the result with the other terms of (3), one obtains

$$\frac{\partial G(x,s)}{\partial x}=-\beta(s-1)G(x, s-1) +\gamma\frac{(s-1)(s-2)}{2}G(x, s-2) -\eta\frac{(s-1)(s-2)(s-3)}{3\times2\times2}G(x, s-3), \quad (4)$$

in which

$$\beta=\int_{\epsilon_{\min}}^{\epsilon_{\max}} \epsilon w(\epsilon)d\epsilon;\quad \gamma=\int_{\epsilon_{\min}}^{\epsilon_{\max}} \epsilon^2 w(\epsilon)d\epsilon;$$

$$\eta=\frac{2\pi Ne^4Z}{mv^2}\left\{\frac{2mv^2}{1-v^2/c^2}\right\}^2\left\{1-\frac{2}{3}\frac{v^2}{c^2}\right\}. \quad (5)$$

A good approximate solution (correct to terms in $x^2$) for Eq. (4) is

$$G(x,s)=(E_0-x\beta)^{s-1}\exp[a_1(s)x+a_2(s)x^2], \quad (6)$$

in which

$$a_1(s)=\gamma\frac{(s-1)(s-2)}{2E_0^2}-\eta\frac{(s-1)(s-2)(s-3)}{3\times2\times2E_0^3}, \quad (7)$$

and $a_2(s)$ gives small correction terms:

$$a_2(s)=-\tfrac{1}{2}\gamma^2\frac{(s-1)^2(s-2)^2}{4E_0^4}+\frac{\beta(s-1)}{2E_0}\times\left\{\frac{\gamma(s-1)(s-2)}{2E_0^2}-\frac{(s-1)(s-2)(s-3)}{3\times2\times2E_0^3}\eta -\frac{\gamma(s-2)(s-3)}{2E_0^2}+\frac{(s-2)(s-3)(s-4)}{3\times2\times2E_0^3}\eta\right\} +\gamma^2\frac{(s-1)(s-2)(s-3)(s-4)}{2\times4E_0^4}+\frac{3\beta\gamma(s-1)(s-2)}{4E_0^3} -\frac{3\beta\eta(s-1)(s-2)(s-3)}{3\times2\times4E_0^4}-\frac{\beta\gamma(s-1)(s-2)}{4E_0^3}. \quad (7a)$$

The solution (6) is inserted into the inversion formula for the Mellin transform to yield a relation for the required distribution function:

$$g(x,E)=\frac{1}{2\pi i}\int_{-i\infty+\delta}^{i\infty+\delta}\exp[-s\log E+(s-1)\log(E_0-x\beta)+a_1(s)x+a_2(s)x^2]ds. \quad (8)$$

An experimental number *versus* energy plot for protons in Al, taken from the work of Reynolds *et al.*,[10] is shown in Fig. 1. In order to compare this with results obtained from the numerical evaluation of (8), one plots the quantities $N(x,E)$ defined by

$$N(x,E)=\int_E^{E_0} g(x,E)dE. \quad (9)$$

The integral distribution obtained in this way from the present calculations is seen to fit the experimental result very closely. The agreement is even better when one notes the initial energy spread of the incident particles shown on the right of Fig. 1. The slight asymmetry of the experimental curve is present also in the theoretical result.

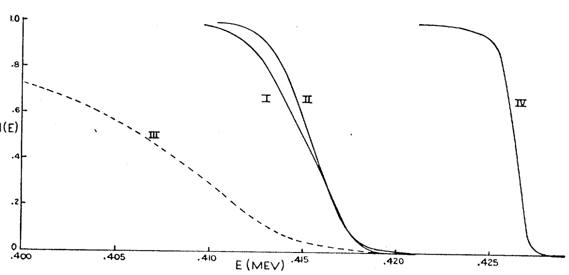


FIG. 1. Curves for the energy distribution of 0.4263-Mev protons after passing through $3.795\times10^{-5}$ g/cm$^2$ of Al. $N(E)$ is the number of particles with energy greater than $E$, normalized to 1 incident particle. Curve I gives experimental results from the work of Reynolds *et al.* Curve II has been calculated on the basis of the present theory. Curve III has been calculated on the basis of the Landau theory. Curve IV shows the initial energy spread of the protons of curve I.

The distribution obtained from Landau's theory is also shown in Fig. 1. In view of the important omissions in this treatment it is not surprising that the Landau curve is unsatisfactory. It is worth noting that for electrons the Landau theory gives a distribution which is too narrow, whereas for protons the distribution is much too wide.

The author is indebted to Dr. D. N. F. Dunbar for making the experimental results available prior to publication.

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[8] For electrons the last term in (3) introduces large corrections and must be worked out to a sufficient degree of approximation. This term gives rise to the characteristic asymmetry of the distribution. In all cases for electrons $\epsilon_{\max}=\frac{1}{2}(E-mc^2)\gg E_0-L$, the upper limit of integration must be taken as $E_0-L$ and so contains the transform variable itself. The result is a considerably more complicated transform equation than (3). Furthermore for electrons the zero, first and second order moment terms will not be the same as they were for protons and a different $w(\epsilon)$ must be used in calculating the last term of (3). In the accurate calculations, therefore, it is not possible to use the same method of solution for heavy particles and electrons as is done by Landau. The solution for electrons will be presented elsewhere.
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# Errata

**Search for 15-Mev Gamma Radiation from $N^{14}+d$ and $Be^9+\alpha$,** V. K. RASMUSSEN, JOHN R. REES, M. B. SAMPSON, AND N. S. WALL [Phys. Rev. **96**, 812 (1954)]. The subscripts $\alpha$ and $\gamma$ were interchanged at the bottom of the first column of page 813. The statement as to the relative probability of $\alpha$ and $\gamma$ decay should read "$\Gamma_\alpha$ is certainly less than 100 $\Gamma_\gamma$ and is probably less than 10 $\Gamma_\gamma$."

**Recombination Processes in Insulators and Semiconductors,** ALBERT ROSE [Phys. Rev. **97**, 322 (1955)]. In item 5 of the section labeled "Summary" on page 333, read "usually" instead of "always."

**Average Number of Neutrons Emitted During the Spontaneous Fission of $Cf^{252}$,** W. W. T. CRANE, G. H. HIGGINS, AND S. G. THOMPSON [Phys. Rev. **97**, 242 (1955)]. The first sentence should read "The average number of neutrons per spontaneous fission of $Cf^{252}$ has been found to be $3.53\pm0.15$ $\cdots$" instead of "The average $\cdots$ has been found to be $3.10\pm0.15$ $\cdots$."

**Origin of Nitrous Oxide in the Atmosphere,** P. HARTECK AND S. DONDES [Phys. Rev. **95**, 320 (1954)]. It has come to the attention of the authors that Adel[1] (the discoverer of nitrous oxide in the atmosphere) has also discussed the origin of nitrous oxide in the atmosphere. Bates and Witherspoon[2] have theoretically examined the photochemistry of the constituents of the atmosphere. The results of Bates and Witherspoon quantitatively reflect the idea of photochemical processes causing the presence of nitrous oxide in the atmosphere, similar to our own views, and question the adequate