crease in elastic and misfit energy. Making the simple assumption that the mixed semicircular dislocation is half edge and half screw, one obtains approximately for U_e and U_m:[3]

$$U_e \cong [(2\pi-4-\pi\sigma)R\mu b^2/8\pi(1-\sigma)][\ln(4R/\zeta)-1], \quad (2)$$

$$U_m \cong (2\pi-4-\pi\sigma)R\mu b^2/8\pi(1-\sigma). \quad (3)$$

Here μ and σ are respectively the shear modulus and Poisson's ratio of an isotropic crystal. In writing U_e we have put ϵ, the usual lower limit of integration, approximately equal to $\zeta/2$. The general conclusions drawn later are not particularly dependent on this choice.

The critical yield stress, τ_c, will correspond to the value of τ when $\partial U/\partial R=0$. Thus we obtain

$$\tau_c \cong (k/\pi Rb)\ln[(4R/\zeta)+1], \quad (4)$$

where

$$k=(2\pi-4-\pi\sigma)\mu b^2/8\pi(1-\sigma). \quad (5)$$

Equations (4) and (5) are derived for an initial edge dislocation because it may easily be shown that a screw or mixed straight dislocation will give a higher value for τ_c.

If we take for ζ the result obtained by Foreman, Jaswon, and Wood[4] that

$$\zeta=\mu b/2\pi(1-\sigma)\tau_m, \quad (6)$$

where τ_m is the theoretical shear strength, and use for τ_m the lowest value thus far derived,[5] i.e., about $\mu/30$, we obtain

$$\zeta \cong 15b/\pi(1-\sigma). \quad (7)$$

Now, from the theory of Fürth, which assumes that melting is due to the break up of a block structure, we may write that

$$2R \cong 6b\Lambda/Q, \quad (8)$$

where Λ is the heat of sublimation and Q is the heat of melting. Using Eqs. (7) and (8) in (4), we obtain

$$\tau_c \cong [(2\pi-4-\pi\sigma)\mu Q/24\pi^2(1-\sigma)\Lambda] \times \ln\{[12\pi(1-\sigma)\Lambda/15Q]+1\}. \quad (9)$$

This formula is in principle applicable to unworked materials close to the absolute zero. The yield strengths of a few metal crystals such as Zn and Cd and of the ionic crystal NaCl have been measured at very low temperatures. One may also very roughly extrapolate the data on other crystals to the absolute zero. The yield strengths derived from formula (9) are at least an order of magnitude too high. One may lessen the discrepancy a bit by using the suggestion of Fisher[6] that a single-ended source near the surface should start to operate at one half the stress needed for a double-ended source of the same length.

There are two further ways to achieve lower results. One is to assume that the crystal is not homogeneously blocked and that there are some much longer blocks which are operative. Another, which is more interesting and more likely, is to modify Fürth's formula to read, say,

$$2R \cong 6(\zeta/2)\Lambda/Q. \quad (10)$$

This assumes that the mechanism of melting is tied intrinsically not only to the block size, but also to a width between blocks, where the atoms are misfit and may be expected to enter into the mechanism first.

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[1] R. Fürth, Phil. Mag. **40**, 1227 (1949), and other references cited there.
[2] J. Frenkel, *Kinetic Theory of Liquids* (Oxford University Press, London, England, 1946), p. 101.
[3] See for example A. Cottrell, *Dislocations and Plastics Flow in Crystals* (Oxford University Press, London, England, 1953) for basic formulas.
[4] Foreman, Jaswon, and Wood, Proc. Phys. Soc. (London) **A64**, 156 (1951).
[5] J. Mackenzie, thesis, University of Bristol, 1949 (unpublished).
[6] See reference 3, p. 86.

Theory of the Meissner Effect in Superconductors

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THE general features of the superconducting state are now well-established, although a good mathematical or detailed physical description is lacking. Pippard[1] has shown that the wave functions (range-of-order) of the electrons in the superconducting state extend over relatively large distances ($\sim 10^{-4}$ cm) and that the penetration depth does not vary much with magnetic field. The latter implies that a linear theory, in which only first-order changes of wave functions produced by the magnetic field are included, should be satisfactory. As pointed out particularly by Slater,[2] wave functions extending over large areas are favorable for a large diamagnetism. While it is thought that the Meissner effect ($\mathbf{B}=0$) follows rather generally from these considerations, it has been difficult to treat a specific model. One model, which is a modification of a degenerate free-electron gas, is discussed below.

We assume that in the superconducting state a finite energy $\epsilon \sim kT_c$ is required to excite electrons from the surface of the Fermi sea, and that electrons so excited behave much like excited electrons in the normal state. This model has been discussed in a qualitative way by Welker[3] and others. An adequate description of the "condensed" superconducting state probably requires going beyond a one-particle model.[4]

To avoid introduction of a boundary, we follow the method of Klein[5] and Schafroth[6] in which an infinite medium is considered and the sources of the magnetic field are introduced in the interior. A relation is derived between the Fourier component $\mathbf{i}(\mathbf{k})$ of the current density of the electrons and the corresponding component $\mathbf{A}(\mathbf{k})$ of the vector potential. If the gauge in $\mathbf{A}$ is chosen so that $\mathrm{div}\mathbf{A}=0$, then $\mathbf{A}$ and $\mathbf{k}$ are perpendicular, and $\mathbf{i}(\mathbf{k})$ is parallel to $\mathbf{A}(\mathbf{k})$. Thus we may write:

$$(4\pi/c)\mathbf{i}(\mathbf{k})=-F(k)\mathbf{A}(\mathbf{k}). \quad (1)$$

As pointed out by Klein and by Schafroth, the Meissner effect is obtained if $F(0)>0$. If $\mathbf{A}_0(\mathbf{k})$ represents the source field, the self-consistent solution is obtained from:

$$[k^2+F(k)]\mathbf{A}(\mathbf{k})=k^2\mathbf{A}_0(\mathbf{k}). \quad (2)$$

If one assumes with London that the wave functions are not modified at all by the magnetic field,

$$F(k)=\lambda_0^{-2}=4\pi e^2n/mc^2, \quad (3)$$

where λ_0 is the penetration depth and n is the electron concentration. Actually, one should include first-order perturbation changes resulting from the field. We assume that the energies of excited states and matrix elements are similar to those of a normal degenerate electron gas, except for the additional energy ϵ required for each excited electron. A modification of Klein's treatment then gives

$$\lambda_0^2F(k)=1-\frac{3}{2}\frac{k_0}{k}\times\int_0^1 u(1-u^2)\log\left(\frac{k_0u+\frac{1}{2}k+m\epsilon/\hbar^2k}{|k_0u-\frac{1}{2}k|+m\epsilon/\hbar^2k}\right)du, \quad (4)$$

where k_0 is the magnitude of the wave vector of the Fermi surface. When $\epsilon=0$, this gives the usual Landau diamagnetism. When $\epsilon>0$, $F(k)\rightarrow\lambda_0^{-2}$ as $k\rightarrow 0$. For k such that $\lambda_0k\sim 1$ and $m\epsilon/\hbar^2k^2\sim 1$, a good approximation to $F(k)$ is

$$\lambda_0^2F(k)=\frac{3}{2}\frac{m\epsilon}{\hbar^2kk_0}\log\left(1+\frac{\hbar^2kk_0}{m\epsilon}\right). \quad (5)$$

This expression (5) is valid over the range of k which makes an appreciable contribution to the field for normal penetration phenomena. The penetration depth is obtained from the integral

$$\lambda=\frac{2}{\pi}\int_0^\infty\frac{dk}{k^2+F(k)}, \quad (6)$$

which may be evaluated approximately by replacing k in the logarithm by an average value $\sim\lambda_0^{-1}$. We then find:

$$\lambda=0.77\lambda_0\left[\frac{3}{2}\frac{m\epsilon\lambda_0}{\hbar^2k_0}\log\left(1+\frac{\hbar^2k_0}{m\epsilon\lambda_0}\right)\right]^{-\frac{1}{3}}. \quad (7)$$

With $\epsilon\sim 5\times 10^{-16}$ ergs, $k_0\sim 10^8$ cm^{-1}, $\lambda_0\sim 10^{-6}$ cm, we find $\lambda\sim 2\lambda_0$. Since the logarithm is slowly varying, λ varies approximately as $(E_FkT_c)^{-\frac{1}{3}}$ or as $n^{-2/9}\epsilon^{-\frac{1}{3}}$.

The nature of the excited states in actual superconductors is indicated by the temperature variation of the specific heat, thermal conduction, and electrical conduction (observed in the skin depth at microwave frequencies). These all indicate a density of "normal" electrons in excited states, such as would follow from our model. The energy ϵ undoubtedly depends on temperature and goes to zero at the transition point. Semi-empirical expressions for free energy and critical field derived from a model of this sort are in good agreement with experiment.[7] There is less justification for assuming that the matrix elements of the magnetic interaction are unchanged by the transition, but one would not expect a change in matrix elements to alter the results in a drastic way. Thus any model which gives correctly the thermodynamical properties of the superconducting state will most likely give the Meissner effect.

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[2] J. C. Slater, Phys. Rev. **51**, 195 (1937); **52**, 214 (1937).
[3] H. Welker, Z. Physik **114**, 525 (1939).
[4] A completely filled band yields only a small diamagnetism even when the energy gap is small. The author [Phys. Rev. **81**, 829 (1951)] has proposed a one-particle model in which the electrical properties can be described by a small number of particles with very small effective mass. Klein's method as applied to this model gives a large but not perfect diamagnetism. As pointed out by H. Fröhlich [Nature **168**, 280 (1951)], there is a small but finite residual field in the interior of a massive specimen. Although such a field penetration is not ruled out by experiments, it seems unlikely to occur. This is probably as close as one can come to the Meissner effect using a purely individual particle description. The assumption of a "condensed state" goes beyond such a description.
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[7] W. L. Ginsburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **14**, 134 (1946), Fortschr. Physik **1**, 101 (1953).

Energy Distribution of Protons Due to Collision Energy Loss

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EXPERIMENTS have shown that the Landau distribution is too narrow for electrons (e.g., Rothwell,[1] West,[2] Birkhoff[3]). Blunck and Leisegang,[4] Blunck[5] and others, using essentially the same method as Landau,[6] have attempted to improve the calculated distribution by inclusion of the so-called second moment term which they obtained in an approximate form by using the semiclassical Bohr treatment of collision loss. Apart from this improvement, however, their treatment suffers from the same defects as the Landau theory.

Recently Fano[7] has given a complete formulation of the general problem of the passage of charged particles through layers of material thick enough to contain the whole range of the particles, although the only solution quoted which is specific to the Landau problem (thin layers) is that of Landau himself. Fano states two of the