it does indicate that the magnitude predicted theoretically is of the right order.

Finally, we observe that using (2) with $\bar{m}=0.21m_0$, which brings the values of n_i of (2) and of Morin and Maita into line at 291°K, and values of n_i given by Morin and Maita at temperatures above 291°K, we have deduced E_G at higher temperatures. The results are shown dotted in Fig. 2, from which it will be seen that the two branches fit together with very little discontinuity of slope. We would remark on the quadratic behavior of E_G at low temperature.

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Ultrasonic Attenuation in Metals by Electron Relaxation

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A DIFFERENCE in the ultrasonic attenuation between lead in the normal and superconducting states has been reported recently by Bömmel.[1] In this note it will be shown that the magnitude and temperature dependence of the attenuation in the normal state can be explained reasonably in terms of an incomplete adjustment of the Fermi distribution with respect to the elastic deformation, and consequently such an attenuation is to be expected in all metals at low temperatures when the mean free path becomes relatively long.[2]

In the free electron gas model of a metal, the Fermi surface is a sphere. With this gas we can associate an internal kinetic pressure given by $p_f=(2/5)nE_0$, where n is the number of electrons per unit volume and E_0 is the Fermi energy.[3] If a longitudinal compressive strain ϵ_x (in the x-direction) is produced slowly, the Fermi surface remains spherical and p_f increases uniformly because the volume decreases. On the other hand, if ϵ_x is brought about quickly enough, only the electron velocity components in the x-direction react immediately and the Fermi surface is elongated momentarily in that direction. Collisions of electrons with the lattice eventually lead to the equilibrium spherical distribution, and the stress necessary to maintain ϵ_x relaxes to its equilibrium value. The relaxation time τ of this process is the same as the one commonly used in the theory of electrical conductivity. The magnitude of the relaxing part of the stress Δp_x can be found simply by noting that the x-component of velocity is increased by a factor $(1+\epsilon_x)$, in the instantaneous application and by the factor $(1+\frac{1}{3}\epsilon_x)$ in the equilibrium case. In both situations n increases by $(1+\epsilon_x)$, and so Δp_x is found to be $(8/15)nE_0\epsilon_x\equiv b\epsilon_x$. A more detailed analysis also gives this result.

Such an effect may be expressed in terms of a relaxational elastic constant

$$k=k_0(1+b/k_0)[1+b/k_0(1+i\omega\tau)]^{-1}.$$

The attenuation constant α is the imaginary part of $-\omega(\rho_0/k)^{\frac{1}{2}}$, ρ_0 being the density. When $\omega\tau\ll 1$ we obtain, after expressing the relaxation time in terms of σ, the electrical conductivity, and neglecting b with respect to k_0,

$$\alpha=\frac{4}{15}\frac{\omega^2 mE_0\sigma}{\rho_0 c_0^3 e^2}. \tag{1}$$

Here, c_0 is the longitudinal wave velocity, e is the electron charge, and m its mass.

Van den Berg[4] observed that σ of lead in the normal state is given approximately by $1/\sigma=\rho'+6.6\times10^{-13}\,T^5$ ohm-cm, where ρ' is the residual resistance. Assuming that curve R of Fig. 1 (which also shows Bömmel's results) is an attenuation due to some other cause, Eq. (1) can be used to evaluate ρ' by fitting (α_n-R) at the lowest temperature. Using $E_0=4$ ev and $c_0=2.4\times10^5$ cm/sec we obtain $\rho'=1.0\times10^{-8}$ ohm-cm (a plausible magnitude). Attenuation at higher temperatures may be calculated using Van den Berg's results for σ and points are shown in Fig. 1.

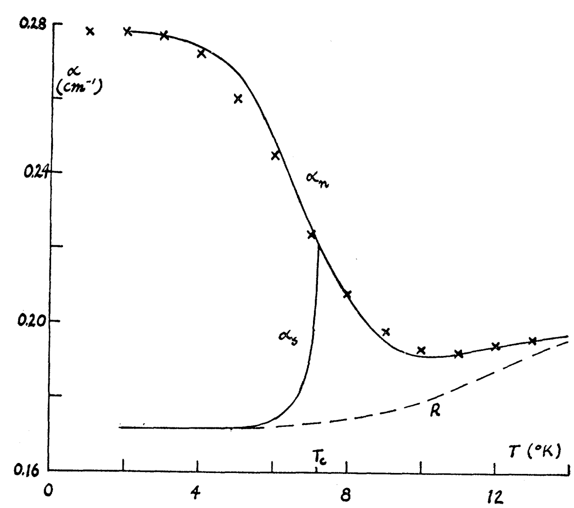

Fig. 1. Attenuation *vs* temperature in lead. The solid curves show Bömmel's measurements; curve R is assumed due to some other mechanism, and the crosses are calculated from Eq. (1).

The fair agreement between Eq. (1) and Bömmel's results in spite of the simple model used, lends support to the original assumption of a relaxation of the Fermi distribution. Other consequences of this mechanism are: (a) The effect should be observed in all metals at low enough T, provided ρ' is small. (b) An absorption of comparable magnitude will occur for shear waves (as found by Bömmel) since a shear strain can be considered equivalent to simultaneous compressive and extensive longitudinal strains of the type discussed above. (c) Equation (1) provides no reasonable explanation of the rapid drop of the attenuation in the superconducting region, as long as only normal electrons are considered. This suggests that even a small number of superconducting electrons has a large effect in speeding the equilibrium between normal electrons and the lattice.

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[1] H. Bömmel, Phys. Rev. **96**, 220 (1954).
[2] Since preparing this note it has been brought to the author's attention that W. P. Mason has discussed Bömmel's results in terms of the equivalent shear viscosity of the electron gas [Phys. Rev. **97**, 557 (1955)]; this discussion leads to results similar to those derived here. The approach used in the present paper clearly involves the same physical mechanism as does shear viscosity, and, at low frequencies, is an alternative way of considering the same effect. The two differ, however, at high frequencies in the same way that a relaxation type absorption differs from a viscous absorption. It is felt that the present approach gives a somewhat clearer picture of the mechanism involved.
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Superfluidity in Unsaturated Helium Films above the λ Temperature

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DIRECT flow observations[1] as well as heat conductivity measurements[2,3] have shown that superfluidity appears in unsaturated helium films at a given temperature only above a critical value of the saturation P/P_0, P being the gas pressure in equilibrium with the film, P_0 the vapor pressure of the bulk liquid. By means of the adsorption isotherm the values of P/P_0 can be transformed into film thicknesses, eventually expressed in numbers of statistical layers.

An analysis of detailed heat conduction experiments on unsaturated films to be published soon showed that at 1.3°K about 2 layers are not showing superfluidity, this number increasing to 10 layers at 2.0°K. An extrapolation to the λ-temperature led to the result that about 20 layers should be immobile at this temperature. Since the film thickness can exceed 20 layers appreciably[4] and thermodynamic considerations suggested the possibility that superfluidity can occur in unsaturated films above T_λ,[4,5] we investigated therefore whether or not films of more than 20 layers would exhibit superfluidity at or above T_λ.

The heat conduction apparatus of reference 2 was used. The bath temperature was kept at $1\frac{1}{2}\pm0.2$ millidegrees below the λ-point using $P_\lambda=38.1$ mm Hg,[6,7] corresponding to 2.183°K of the conventional scale, and $dP/dT=0.094$ mm Hg/10^{-2} deg.

Without contribution of superfluidity the conductance of the apparatus is $\sim10\mu$ watts/deg, and its capacity $\sim1\times10^{-2}$ joule/deg so that 0.3-μwatt heating produces a warming rate at the warm end of ~2 millidegrees/minute as established at $P/P_0\sim0.8$ corresponding to about 7 layers. Using then (a) $P/P_0=0.9996$ ($P_{\text{bath}}-P_{\text{system}}=0.3$ mm oil, P_{bath} 560 mm oil), estimated film thickness ~30 layers, (b) about 15 percent more gas in the apparatus than necessary to make $P_{\text{system}}=P_{\text{bath}}$, (estimated film thickness ~50 layers), we obtained the following result:

Up to 2 μwatts heat input did not produce any measureable temperature difference up to temperatures of 2.185° of the conventional scale, i.e., 2 millidegrees above the real λ-point. Higher heating rates up to 7.3 μwatts did produce increasing temperature differences. However the warming rate with these heat inputs was up to 2.195°K of the conventional scale only a fraction of that derived from the experiments at $P/P_0\sim0.8$ (without contribution of superfluidity) suggesting that superfluid heat transport is active up to these temperatures.

The results seem therefore to indicate that films of more than 20 layers are showing superfluid behavior at and above the λ-point of the bulk liquid. A detailed account will be given in the forthcoming paper.

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Electronic Density of States of Graphite*

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IT has been shown by Carter and Krumhansl[1] that the electronic density of states of graphite near the top of the filled band is asymmetric about the energy where this band touches the next, unfilled, band. This conclusion is based on a modification of the Wallace[2] band structure. This modification consists of recognizing the difference in the number of neighbors between the