multiplicity distribution, range of secondaries etc., are available only up to primary energies of 10 Bev,[10] so that extrapolations had to be used. The method of calculation was that previously applied:[8,10] given the (differential) energy spectrum $s(E)$ and the multiplicity distribution $M(E)$, one computes first for each multiplicity the average numbers m, m', m'' of shower particles present at the levels of the trays D, E, and F, taking into account cascade multiplication as well as absorption and geometrical losses. From these figures, the probability $P(n,n',n'')$ is then obtained that this shower will discharge n or more, n' or more, and n'' or more counters of the three shower-detecting trays, and consequently the contribution of the energy interval (E,dE) to the events of the type $T_1P^{n,n',n''}$ is

$$dR_{n,n',n''}=s(E)\sum_M M(E)P(n,n',n'')dE.$$

Integration over E yields the "calculated rates" which may serve as a check on the reliability of the method, and similarly, the average primary energies are now directly computed. Table III summarizes the contributions R_{cal} in counts/hour for the various shower groups. In the last row, the observes rates R_{exp} of the unshielded run are added for comparison, and it is seen that the agreement is surprisingly good: in no case are the deviations larger than 20 percent.

The average energies computed from this distribution are:

$$\text{Events } T_1P^{221}: \langle E_p\rangle = 5.3 \text{ Bev},$$
$$T_1P^{222}: \langle E_p\rangle = 7.9 \text{ Bev},$$
$$T_1P^{321}: \langle E_p\rangle = 8.7 \text{ Bev},$$
$$T_1P^{332}: \langle E_p\rangle = 16.4 \text{ Bev},$$
$$T_1P^{333}: \langle E_p\rangle = 20.4 \text{ Bev}.$$

If the agreement between all the observed and calculated rates is not to be considered as wholly fortuitous, these energy values should likewise be accurate within about 20 percent.

PHYSICAL REVIEW VOLUME 97, NUMBER 1 JANUARY 1, 1955

Penetrating Showers in Light and Heavy Elements*

JOSEPH G. ASKOWITH AND KURT SITTE†
Department of Physics, Syracuse University, Syracuse, New York
(Received June 7, 1954; revised manuscript received September 17, 1954)

A large cloud chamber fitted with one thick "producer plate" of light materials and with eight ¼-in. lead plates was operated at Echo Lake, Colorado (altitude 3260 m) to record penetrating showers originating in three light elements (Li, C, Al) and in lead. The average primary energies, estimated with the "F-plot" method of Duller and Walker, were between 20 and 25 Bev. From a comparison of the shower rates in the producer plates and in the top lead plate, upper and lower limits for the transparencies of the light elements are obtained, the latter if correction is made for the differences in the primary energies. More reliable values, however, are derived from a direct comparison of the rates in the three light elements. This yields a value of $\lambda=(2.55\pm0.25)\times10^{-13}$ cm for the mean free path in nuclear matter, and accordingly transparencies of (15±7) percent, (12±4) percent, and (6±3) percent, respectively for Li, C, and Al. It is shown that second-generation effects are not negligible even in light nuclei, and that they can account for the observed differences in the average multiplicities of showers initiated in the four elements studied. At a primary energy of 25 Bev, the average multiplicity of a nucleon-nucleon collision is about 3.4. The contribution of the π mesons to the intranuclear cascade is very small. This agrees well with the observed mean free path for shower production in the lead plates by secondary particles, which varies from (380±35) g/cm² for secondaries from light elements to (475±70) g/cm² for secondaries from lead. Since nuclear scattering in the lead plates was observed for secondaries from the light elements and from lead with mean free paths of (370±35) g/cm² and (300±36) g/cm² respectively, the total interaction mean free path in lead, comprising both scattering and secondary showers, is very nearly the same for secondaries emerging from light and heavy elements—about (190±25) g/cm²—and is also close to the geometrical value.

I. INTRODUCTION

MEASUREMENTS of the cross section for shower production as a function of the atomic weight of the target nucleus are of interest for a variety of reasons, but in particular because they provide the most direct means of studying the problem of the "mean free path of nucleons in nuclear matter," and that of the significance of second-generation effects in collisions of a nucleon with a nucleus (the intranuclear cascade). From the latter effect, evidence can also be derived on the interaction mean free path of π mesons in nuclear matter.

Since Cocconi's early paper on this subject,[1] it has become customary in high-energy interactions to use the concept of nuclear transparency as a quantitative expression of the deviation of the collision cross section from the geometrical nuclear cross section. Of the numerous papers on this subject, the experiments of Rolloson[2] and Froman *et al.*[3] with water may be men-

* Supported by the Geophysics Research Division, Air Force Cambridge Research Center, Cambridge 39, Massachusetts.
† At present at the Department of Physics, Israel Institute of Technology, Haifa, Israel.

[1] G. Cocconi, Phys. Rev. **75**, 1075 (1949).
[2] G. W. Rolloson, Phys. Rev. **87**, 71 (1952).
[3] Froman, Kenney, and Regener, Phys. Rev. **91**, 707 (1953).

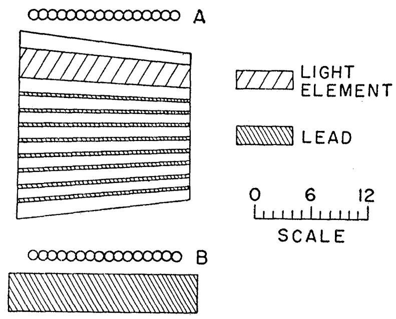

FIG. 1. Schematic diagram of the experimental arrangement.

tioned, and the attempt of Froman[4] to systematize the earlier results. The most widely studied light element is carbon, and here the results of most authors like Walker *et al.*,[5] Cool and Piccioni,[6] Boehmer and Bridge,[7] and others generally agree on a mean free path of about 80 g/cm², or a transparency of about 20 percent, even at rather high primary energy. However, Brown,[8] recording only very energetic events, found a collision cross section equal to the geometrical value.

Unfortunately, it appears that the experimental techniques applied in most of these studies are subject to serious criticism. Basically, two different methods are commonly used: in "counter-type" experiments the material under investigation is placed as an absorber on top of a set which records penetrating showers initiated by an unaccompanied primary, and the attenuation of the shower rate is measured as a function of the absorber thickness. In "chamber-type" experiments the shower production rate in light elements is directly compared with that in a heavy standard element, usually lead. The reliability of the first method, therefore, depends on the accuracy of the assumption that for all thicknesses used, every interaction that may have taken place in the top absorber is efficiently detected, and the corresponding event eliminated. This assumption is based on the premise that it is extremely unlikely for a nuclear interaction with high energy and large energy transfer to be preceded by one with a small transfer: an argument that has lost some strength in view of more recent results.[9] Indeed, it could be shown[10] that for instance the results of Boehmer and Bridge,[7] which indicate a variation with the primary energy of the collision mean free path in lead between the "geometrical" value of about 160 g/cm² and 300 g/cm², are quite compatible with the assumption of a geometrical cross section for all energies if only correction is made for the effect of small, undetected interactions in the top absorber. A similar effect should exist, and be even more pronounced, for interactions in light elements where "glancing" collisions are comparatively more frequent than in heavy elements, and this effect would falsify the conclusions on nuclear transparency derived from this type of experiment.

The cloud chamber method, on the other hand, suffers from the fact that the selection of triggering events is usually based on a certain multiplicity of shower secondaries striking the detection tray or trays. Since for a given primary energy the average multiplicity of the showers produced in heavy elements is larger than that of light-element showers,[11] this method is therefore biased in favor of detection of heavy-element showers for which it covers a larger part of the primary spectrum, and a direct comparison of the two shower rates again yields too high values for the transparency.

It was therefore decided to re-investigate the transparency problem with methods designed to avoid these pitfalls.

II. ARRANGEMENT AND PROCEDURE

The experimental arrangement, shown schematically in Fig. 1, consisted of a large cloud chamber in which the showers were produced, and a counter control selecting high-energy events. The aluminum-walled cloud chamber, of dimensions 18 in.×18 in. in its central plane and 18 in. deep, was fitted with a light-material "producer plate" near the chamber top and with eight $\frac{1}{4}$-in. lead plates below it. Three elements were used as shower producers: lithium, carbon, and aluminum. The lithium plate was encased in a thin-walled steel box and had an average thickness of 4.9 g/cm², the carbon producer was a 10.3-g/cm² sheet of pure graphite, and the aluminum plate had a thickness of 10.5 g/cm². The chamber expanded to both sides through expansion valves of a pneumatic-relay type like that used in an earlier chamber,[9] and was usually filled with an argon-oxygen mixture containing about 80 percent of the inert gas, and with vapor of an isopropyl alcohol-water mixture. Rear illumination was used, and stereoscopic pictures were taken with a two-lens camera under angles of about 12° from the axis.

The three counter trays A, B, and C shown in Fig. 1 selected the triggering events. Showers were recorded when a single particle incident at A struck in coincidence with at least three particles at B and two or more at C. Tray C was separated from B by a layer of 4-in. Pb. In this way, only high-energy local interactions (and the unavoidable background of air showers) triggered the chamber. The experiment was carried out at Echo Lake, Colorado, altitude 3260 m.

While thus the experimental arrangement was of the

[4] D. Froman, Phys. Rev. **88**, 172(A) (1952).
[5] Walker, Walker, and Greisen, Phys. Rev. **80**, 546 (1950).
[6] R. L. Cool and O. Piccioni, Brookhaven National Laboratory Report No. 119, 1952 (unpublished); O. Piccioni and R. L. Cool, Phys. Rev. **87**, 216(A) (1952).
[7] H. W. Boehmer and H. S. Bridge, Phys. Rev. **85**, 863 (1950).
[8] R. R. Brown, Phys. Rev. **87**, 999 (1952).
[9] Froehlich, Harth, and Sitte, Phys. Rev. **81**, 504 (1952).
[10] K. Sitte, Acta Phys. Austriaca **6**, 167 (1952).
[11] See, for example, Lovati, Mura, Salvini, and Tagliaferri, Nuovo cimento **7**, 943 (1950).

conventional type, the method of evaluation differed from that previously used. A comparison of the shower rates in the light elements with the rate of showers starting in the first lead plate—the only one for which the geometry is still sufficiently close to that of events from the light-element producers—served only to derive maximum and minimum values for the interaction mean free paths. A value that can be considered much more reliable was obtained from a comparison of the rates of events in the three light elements, using as standards in each case the total number of interactions initiated, during the respective operating times, in the eight lead plates. In this way, the error due to the energy bias mentioned above is minimized, and the statistical accuracy improved. Besides, the average primary energies could be estimated, and thus the appropriate small corrections could be made.

For essential parts of the discussion, the angular distribution of the shower particles and the multiplicity distribution of the showers are likewise needed. Again it was assumed that showers initiated in the producer plates, and those starting from the first lead plate, are detected with equal probability, so that a comparison of these distributions can be made without further corrections.

III. EXPERIMENTAL RESULTS

1. Angular and Multiplicity Distribution. Average Primary Energies

In the lithium run, a total of 97 showers originating in the producer plate were obtained, together with 428 showers starting in the lead plates, 60 of which came from the first plate. The corresponding figures for the carbon run were 94, 227, and 34, and for the aluminum run 86, 255, and 34. In this survey, events with less than three shower particles ejected in the primary collision were omitted even if, owing to secondary interactions in the material between producer plate and counters, they were able to trigger the arrangement.

A qualitative argument may be introduced here in order to stress the importance of recording the distributions in multiplicity and angular spread. In discussing transparency of the nuclei and mean free path of the secondaries, it is essential to assess correctly the influence of second-generation effects. Now evidently both the multiplicity distribution and the angular distribution of the showers originating in different elements will remain identical as long as these effects are negligible, while both will be affected if the shower is the result of an intranuclear cascade rather than of a nucleon-nucleon collision.

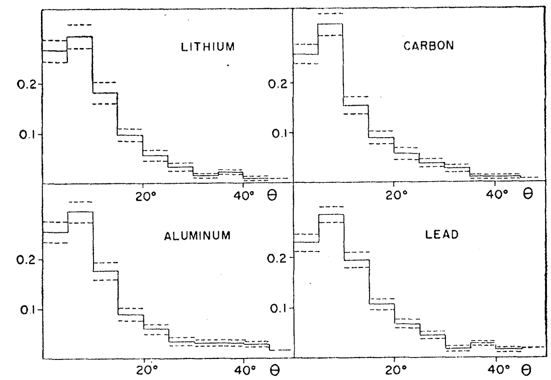

FIG. 3. Angular distributions $\Delta N/\Delta\theta$ of shower particles as a function of the laboratory zenith angle θ.

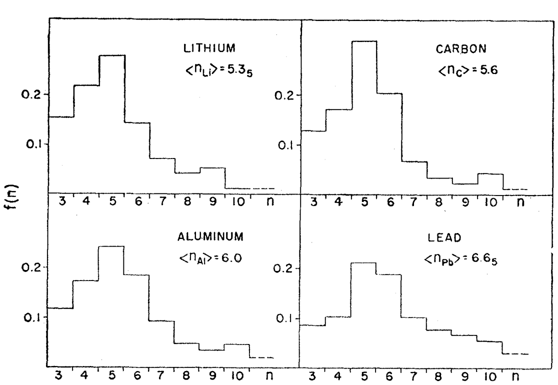

FIG. 2. Multiplicity distributions and average multiplicities $\langle n\rangle$.

The multiplicity distributions and angular distributions observed in this experiment are reproduced in Fig. 2 and Fig. 3. An inspection of these histograms shows significant differences between the light-element showers and those originating in lead (the lead date are taken from all three runs). This is further borne out by the trend in the average particle numbers obtained from the multiplicity distributions of Fig. 2, which range from 5.35 for lithium to 6.65 for lead. An even better proof for the reality of the difference can be given if the "F plots" of Duller and Walker[12] are constructed. These authors have shown that for a nucleon-nucleon collision, under the assumption of isotropic distribution of the secondaries in the center-of-mass system, the fraction $F(\theta)$ of shower particles emitted, in the laboratory system, under an angle less than θ satisfies the relation,

$$\tan^2\theta = (1/\gamma_c^2)[F(\theta)/1-F(\theta)], \qquad (1)$$

where γ_c is the energy of the primary in the center-of-mass system, expressed in multiples of its rest mass. Plotting the observable quantity $F/(1-F)$ against $\tan^2\theta$ one expects, therefore, to obtain a straight line, the slope of which defines the average primary energy. This was verified by Duller and Walker on their own data and on those taken from Bristol stars, and it was again found true with the data obtained in this experiment as shown in Fig. 4 where the quantities $F(\theta)/[1-F(\theta)]$ corresponding to the distributions of Fig. 3 are reproduced in a logarithmical plot. Straight lines that fit the experimental points can be found for all three

[12] N. M. Duller and W. D. Walker, Phys. Rev. 93, 215 (1954).

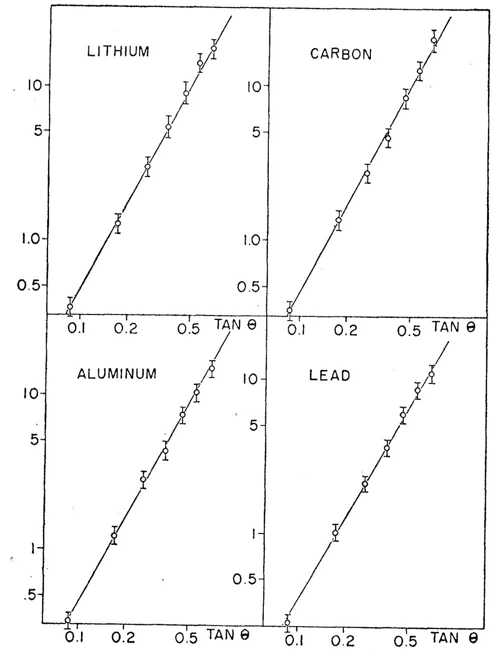

FIG. 4. "F plots": $\log(F/1-F)$ *vs* $\log(\tan\theta)$, for all four elements.

elements; however, the average primary energies derived from these graphs for the showers from the various elements differ noticeably. The average kinetic energies obtained from Eq. (1) are 25.3 Bev, 25.0 Bev, 23.6 Bev, and 21.3 Bev, respectively for Li, C, Al, and Pb, with probable errors lying between 0.5 and 0.8 Bev. Thus, no significant difference appears to exist between the primary energies of showers originating in Li and C, while the Al showers and even more so the Pb showers, are on the average initiated by somewhat less energetic primaries.

These kinetic energies will later on be used to make the appropriate corrections for the differences in the primary flux of the showers starting in the four elements.

2. Secondary Interactions

Nuclear interactions of the shower secondaries in the lower lead plates were divided into two groups: a process was defined as a "secondary shower" if at least two relativistic particles emerged from the plate, or at least one additional meson—relativistic or not—was created in the interaction. A process was called "nuclear scattering" if the incident shower particle, in traversing the plate, was deflected by 10°, or if one or more non-relativistic heavy "star particles" were ejected from the plate but no mesons were created.

The survey of the lithium run showed 36 scatterings and 29 secondary showers in 1637 traversals. In the carbon run, 1649 observed traversals yielded 31 scatterings and 33 showers, while the corresponding figures for the aluminum run were 1609, 30, and 32. Since no significant difference is found between the data for the three light elements, they will be lumped together and compared, as the secondary interactions of shower particles from light elements, with the total of the interactions found in all three runs for the secondaries of showers initiated in the first lead plate. For these events, 2398 traversals gave 58 scatterings and 37 showers. In all cases, only events were included in the survey for which a charged relativistic secondary particle could be definitely established as the interacting particle.

If the average inclination of the tracks is taken into account, the 97 scattering events in 4895 traversals for the light-element showers correspond to a mean free path for nuclear scattering $\lambda_{sc}=(370\pm35)$ g/cm^2, and similarly, the 95 secondary showers give a mean free path $\lambda_{sh}=(380\pm35)$ g/cm^2. From the lead data, one computes $\lambda_{sc}=(300\pm36)$ g/cm^2, and $\lambda_{sh}=(475\pm70)$ g/cm^2. It is of interest to note that, although the distribution between scattering and shower production in lead may differ for secondaries from light elements and those from lead, the total interaction mean free path comprising both kinds of events is the same, well within the experimental errors: the figures quoted above give $\lambda_{int}=(190\pm15)$ g/cm^2 for secondaries from light elements, and $\lambda_{int}=(185\pm20)$ g/cm^2 for secondaries from lead, both close to the geometrical mean free path of 164 g/cm^2.

During the same run, a total of 54 interactions definitely induced by neutral particles was observed for light-element showers (19+21+14), against 29 interactions for lead-shower secondaries. Assuming that the interaction mean free path of neutrons is the same as that for charged particles, one computes with these figures a ratio of shower neutrons to charged secondaries of (0.28±0.05) for the light elements, and (0.31±0.07) for lead. If the number of protons emitted in a shower is equal to that of the neutrons, the light-element showers—taking an average multiplicity of 5.6—would thus consist on the average of 4 mesons and 1.6 protons, while for lead showers a meson multiplicity of about 4.5 is obtained. However, the assumption of an equal mean free path may not be correct; if the interaction cross section of the neutrons were geometrical, the π-meson multiplicities would be slightly higher.

As far as a comparison can be made, these results are in satisfactory agreement with those of Duller and Walker[12] who report mean free paths for secondary shower production decreasing with increasing energy, and approaching the geometrical value for particle energies $\gtrsim 4$ Bev. In order to substantiate this point, showers produced by secondaries from all four elements were grouped according to the angle of emission of the shower-initiating particle. For angles below 10°, 91 secondary showers were found in 4238 traversals. This corresponds to a mean free path of (340±36) g/cm^2. The on-the-average less energetic secondaries ejected at angles of more than 10° gave 40 showers in 3055 traversals, and hence a mean free path of (560±90) g/cm². A more direct attempt to determine the dependence on the secondary energy of the mean free path was not deemed worth while with the material on hand.

IV. DISCUSSION

1. Primary Collision Mean Free Path

Two ways are now open to determine the mean free path for shower production by the primaries. Although its results have only a limited value, it is instructive to apply first the usual method of comparing, for each run, the rates of showers produced in the light element and in the first lead plate. Writing n_{el} and n_{Pb} for the numbers of events recorded, x_{el} and x_{Pb} for the respective producer thicknesses, and λ_{el}, λ_{Pb} for the mean free paths, one has obviously

$$n_{el}/n_{Pb}=[1-\exp(-x_{el}/\lambda_{el})]/\exp(-x_{el}/\lambda_{el})\times[1-\exp(-x_{Pb}/\lambda_{Pb})]$$
$$=[\exp(x_{el}/\lambda_{el})-1]/[1-\exp(-x_{Pb}/\lambda_{Pb})], \quad (2)$$

if no correction is made for the difference in the average primary energy. This error may now be eliminated. Under the assumption of a power law for the energy spectrum, a more correct expression for the ratio n_{el}/n_{Pb} is

$$n_{el}/n_{Pb}=(E_{Pb}/E_{el})^{\gamma}[\exp(x_{el}/\lambda_{el})-1]/[1-\exp(-x_{Pb}/\lambda_{Pb})], \quad (3)$$

where E_{el}, E_{Pb} are the average kinetic energies of the showers. For the evaluation of (3), the spectrum of Barrett *et al.*[13] has been used which gives an exponent $\gamma=1.18$ in the energy region concerned. Although it is derived for the primary cosmic radiation, there are good reasons to believe that practically the same spectrum also applies to the proton component at mountain altitude.[9,14]

The results of the calculations, with λ_{Pb} taken to be 164 g/cm², are summarized in Table I. In spite of the low accuracy resulting from the comparatively small number of lead showers, it is seen that the uncorrected mean free paths are in good agreement with most of the older data, while those computed according to (3) yield values of transparencies lower than those usually accepted.

However, it should be borne in mind that, from the foregoing reasons, Eq. (2) is bound to lead to an overestimate of the transparency, or in other words, to its possible maximum value which would be correct only if the primary energies for light-element showers and for lead showers were the same. On the other hand, Eq. (3) gives a minimum transparency valid only if lead showers can still be properly described as pure nucleon-nucleon collisions without the interference

TABLE I. Mean free paths for shower production λ_{el} and transparencies t_{el} obtained from comparison with lead.

Element	Uncorrected [Eq. (2)] λ_{el} (g/cm²)	Uncorrected [Eq. (2)] t_{el} (%)	Corrected [Eq. (3)] λ_{el} (g/cm²)	Corrected [Eq. (3)] t_{el} (%)
Li	$69\pm^{11}_{8}$	$23\pm^{11}_{10}$	$57\pm^{8}_{7}$	$7\pm^{11}_{13}$
C	$90\pm^{13}_{10}$	$29\pm^{9}_{9}$	$75\pm^{11}_{9}$	$15\pm^{11}_{14}$
Al	$99\pm^{14}_{12}$	$16\pm^{10}_{11}$	$93\pm^{12}_{10}$	$11\pm^{10}_{11}$

of second-generation effects. Otherwise, the estimates of the primary energy obtained from the F plots is too low, and the energy bias is overcompensated. Now it is already clearly demonstrated by the graphs of Figs. 2 and 3, and will be further documented below, that particularly in lead showers, second-generation effects play a significant part, and consequently the results of Eq. (3) will underestimate the transparency.

The second method, consisting of a comparison of the rates in the three light elements, is therefore more promising since in this case the energy estimates are more reliable, and any remaining slight error is of negligible effect on the corrections made. However, a direct comparison of shower rates obtained at different periods extending over many months would be a dubious procedure and could easily introduce systematic errors. Hence it was decided to use, instead of the absolute rates, the ratios R_{Li}, R_C, and R_{Al} of the numbers of events starting in the light-element producer plates to the numbers of showers originating in all lead plates. By taking the lead showers as a standard of comparison only, no additional energy bias is introduced and the differences in the detection probabilities for showers starting in different plates likewise do not affect the validity of the procedure.

Then, writing $S(E_{el})$ for the respective primary intensities, one has evidently

$$R_{Li}:R_C:R_{Al}=S(E_{Li})[\exp(x_{Li}/\lambda_{Li})-1]:S(E_C)\cdot[\exp(x_C/\lambda_C)-1]:S(E_{Al})[\exp(x_{Al}/\lambda_{Al})-1]. \quad (4)$$

To evaluate (4), one may proceed directly to a determination of the mean free path in nuclear matter λ. With this parameter, the transparency t of a nucleus of radius r_n has the well-known approximate form

$$t(\lambda/r_n)=(\lambda^2/2r_n^2)\{1-(1-2r_n/\lambda)\exp(-2r_n/\lambda)\}. \quad (5)$$

For light nuclei, the more accurate numerical computations of R. W. Safford, reported by Rossi,[15] should be used.

Since by definition for any element, $\lambda_{el}=\lambda_{el}^0(1-t)$ where λ_{el}^0 is the geometrical mean free path, it follows that, for instance, for the two elements Li and Al the

[13] Barrett, Bollinger, Cocconi, Eisenberg, and Greisen, Revs. Modern Phys. **24**, 133 (1952).
[14] Sitte, Froehlich, and Nadelhaft, Phys. Rev. **97**, 166, (1955).
[15] B. Rossi, *High-Energy Particles* (Prentice-Hall Publications, Inc., New York, 1952), p. 360 ff.

relation holds:

$$R_{Li}/R_{Al}=S(E_{Li})\{\exp[x_{Li}/\lambda_{Li}^0(1-t_{Li})]-1\}/ S(E_{Al})\{\exp[x_{Al}/\lambda_{Al}^0(1-t_{Al})]-1\}, \quad (6)$$

or again using a power law for the energy spectrum,

$$(R_{Li}/R_{Al})(E_{Li}/E_{Al})^{\gamma}=\{\exp[x_{Li}/\lambda_{Li}^0(1-t_{Li})]-1\} \times\exp[x_{Al}/\lambda_{Al}^0(1-t_{Al})]-1\}=\Phi(\lambda). \quad (7)$$

An analogous expression can be written down for $(R_C/R_{Al})(E_C/E_{Al})$. Since the left-hand side of (7) is known from the experimental data, a graphical solution for the only unknown λ can now be obtained. The solutions computed from the two relations overlap in the region 2.30×10^{-13} cm $\leq\lambda\leq2.80\times10^{-13}$ cm; hence the value of $(2.55\pm0.25)\times10^{-13}$ cm will be used below. The collision mean free paths and transparencies of the light elements calculated with this value are summarized in Table II. They are considerably lower than those previously reported: a discrepancy which was to be expected in view of the overestimates of the transparency inherent in the earlier experiments.

2. Estimate of Second-Generation Effects

The importance of second-generation effects of the primary can now be estimated by calculating, with the mean free path in nuclear matter given above, the probability that the incident particle undergoes more than one collision inside the target nucleus. In order to do this, consider the probability P_1 that the primary will collide just once while traversing the nucleus. With the same simple model that leads to Eq. (5) one has evidently, writing b for the impact parameter and r_1 for the interaction radius,

$$r_n^2\pi P_1=2\pi/\lambda\int_0^{r_1+r_n}\int_0^{2(r_n^2-b^2)^{\frac{1}{2}}}\exp(-x/\lambda) \times\exp\{-[2(r_n^2-b^2)^{\frac{1}{2}}-x]/\lambda\}bdbdx, \quad (8)$$

which gives, for $r_1\ll r_n$,

$$P_1=(\lambda^2/r_n^2)[1-(1+2r_n/\lambda+2r_n^2/\lambda^2) \times\exp(-2r_n/\lambda)]. \quad (9)$$

For smaller nuclear radii r_n, numerical integration has to be carried out. Similarly, the probability $P_{\geq1}$ that the primary interacts *at least* once, is for $r_1\ll r_n$

$$P_{\geq1}=1-(\lambda^2/2r_n^2)\{1-(1+2r_n/\lambda)\exp(-2r_n/\lambda)\} \quad (10)$$

[corresponding to the transparency formula (5)]. For the more general cases, the transparency data given by Rossi[15] can be used to obtain $P_{\geq1}$.

The simplest criterion for the importance of the second-generation effects is, then, the value of the expression $(P_{\geq1}-P_1)/P_{\geq1}$ which measures the fraction of events among all interactions in which the primary collides more than once. Second-generation effects can be neglected if $(P_{\geq1}-P_1)/P_{\geq1}\ll1$, which for $r_1\ll r_n$ means

$$(P_{\geq1}-P_1)/P_{\geq1} =\frac{1-\dfrac{3\lambda^2}{2r_n}\left\{1-\left(1+\dfrac{\lambda}{r_n}+\dfrac{4\lambda^2}{3r_n^2}\right)\exp\left(-\dfrac{2r_n}{\lambda}\right)\right\}}{1-\dfrac{\lambda^2}{2r_n^2}\left\{1-\left(1+\dfrac{2r_n}{\lambda}\right)\exp\left(-\dfrac{2r_n}{\lambda}\right)\right\}}\ll1. \quad (11)$$

The quantity $(P_{\geq1}-P_1)/P_{\geq1}$ is plotted as a function of r_n/λ in Fig. 5, the approximate expressions (9) and (10) being used so that all elements can be represented on the same graph. Since only the ratio of the probabilities is needed, the results are not significantly modified if the more correct values for the P's are introduced. The relative probabilities for repeated interactions in the four nuclei Li, C, Al, and Pb are likewise indicated, based on the value for $\lambda=(2.55\pm0.25)\times10^{-13}$ cm as derived in sect. IV.1. It is seen that already for light elements the probability for a second collision is by no means very small, while for elements as heavy as lead it approaches certainty.

This result gains importance in connection with the increasing evidence from various recent experiments[12,14,16] that even high-energy collisions are not completely inelastic, but that the primary retains a sizeable fraction—perhaps 20–25 percent of its initial energy. Its second interaction inside the target nucleus should, therefore, contribute a fair share to the total meson production. Assuming that the incident 25-Bev particle continues, after the collision, with a kinetic energy of 5 Bev, a check on the average multiplicities in the four nuclei Li, C, Al, and Pb has been made on the basis of the Fermi theory.[17] In order to get numerical agreement, its results had to be renormalized so that the primary nucleon-nucleon collision produces about 3.4 charged secondaries. The total meson production by a 25-Bev primary in these four elements is, then, about 3.9, 4.1, 4.5, and 4.8 charged shower particles respectively, and it is seen that, by taking into account also the increase in the number of shower protons, the

TABLE II. Mean free path for shower production λ_{el} and transparency t_{el} obtained from comparison of the shower rates of the light elements.

Element	λ_{el} (g/cm²)	t_{el} (%)
Li	62±5	15±7
C	72±3.5	12±4
Al	88±3.5	6±3

[16] Kaplon, Klose, Ritson, and Walker, Phys. Rev. **91**, 1537 (1953).

[17] E. Fermi, Progr. Theoret. Phys. **5**, 570 (1950); Phys. Rev. **81**, 863 (1951).

differences in the average numbers of shower particles observed in this experiment can well be explained by second-generation collisions of the primary only.

Consider now the intranuclear interactions of the shower secondaries: the number of these collisions increases strongly between lithium ($r_n = 2.65\times 10^{-13}$ cm) and aluminum ($r_n = 4.14\times 10^{-13}$ cm), and since the average energy of the secondaries is of the order of 2 Bev, their contribution to the production of shower particles should likewise be appreciable in the bigger nuclei, Al and Pb, if the mean free path for shower production in nuclear matter were the same for π mesons as for protons. The fact that the observed differences can be accounted for on the basis of primary interactions only, must therefore be interpreted as demonstrating that intranuclear shower production by π mesons of around 2 Bev can occur only with a mean free path considerably above that found for protons, 2.55×10^{-13} cm. This is in agreement with the data on λ_{sh} derived from the interactions in the lead plates (Sec. III.2): the value of $\lambda_{sh} = 380$ g/cm^2 corresponds to a mean free path in nuclear matter of about 18.5×10^{-13} cm,[15] and with this value even the heavier nuclei become highly transparent to the shower secondaries.

3. Conclusions

In summarizing the results, the following conclusions can be drawn:

(1) The suspected systematic errors in previous experiments which led to an overestimate of the transparency, appear to be real and significant. By using a method less sensitive to these errors, transparencies somewhat lower than those generally quoted are found: (15 ± 7) percent for Li, (12 ± 4) percent for C, and (6 ± 3) percent for Al. They correspond to a mean free path in nuclear matter of $(2.55\pm 0.25)\times 10^{-13}$ cm.

(2) Second-generation collisions of the primaries are of importance even for light nuclei, and contribute to the meson production in the intranuclear cascade. The average numbers of shower particles found in this experiment lead to a multiplicity of about 3.4 for a nucleon-nucleon collision at 25-Bev primary energy, and the differences between the various elements can be explained by the contributions from the subsequent interactions of the primary in the target nucleus.

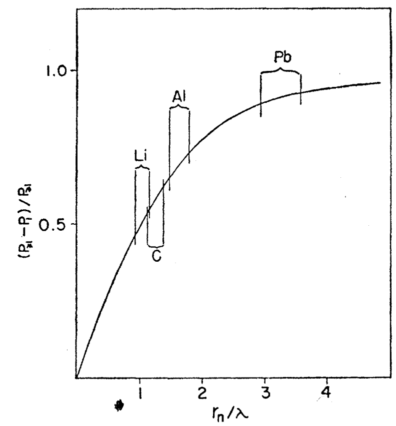

FIG. 5. $(P_{\geq 1}-P_1)/P_{\geq 1}$, the fraction of events in which the primary undergoes more than one collision, as a function of r_n/λ.

(3) It follows from the preceding that the contribution of the shower secondaries to the intranuclear cascade in light or medium-heavy nuclei is insignificant. Consequently, their mean free path in nuclear matter must be longer than that of the primary. This is in agreement with the observations on secondary shower production in the lead plates below the producers. The mean free paths for shower production derived from them correspond to a mean free path in nuclear matter of about 7 times the value for the primaries.

(4) Since in light elements the scattering mean free path of the secondaries is of the same order as that for shower production, these nuclei are also highly transparent with respect to scattering. It can therefore be expected that the "F-plot" method of energy determination will yield reliable results at least for not too heavy nuclei.

V. ACKNOWLEDGMENTS

The experimental work was carried out at the facilities of the Inter-University High Altitude Laboratories at Echo Lake, Colorado, and the authors are obliged to Professors Byron E. Cohn and Mario Iona, Jr., of the University of Denver for their cooperation.