Thermalization of Positrons in Metals

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(Received September 15, 1954)

It is shown that a thermalization time of 3×10^{-12} sec follows from the assumption that the interaction between a positron and a conduction electron can be approximated by an exponentially screened Coulomb potential.

KNOWLEDGE of the thermalization time of a positron in a metal is pertinent to the interpretation of measurements of the angular correlation[1,2] of annihilation radiation and of the time distribution of annihilation.[3] Garwin[4] has pointed out that the estimate of 3×10^{-10} sec for thermalization in gold, made in Appendix I of reference 1, is too long because collisions between the positron and the conduction electrons are ignored; including this effect he estimated 10^{-14} sec for the thermalization time. Using a method similar to that of Sec. 5 of reference 5, we shall remove some of the crude approximations underlying Garwin's estimate and show that it is too short.

The conduction electrons are treated as a free-electron gas at the absolute zero of temperature. The Exclusion Principle is assumed not to apply to a system consisting of a positron and an electron. Transitions from an initial state consisting of a positron and an electron with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively, to a final state $(\mathbf{k}_1',\mathbf{k}_2')$ occur because of the interaction $-e^2r^{-1}\exp(-qr)$. The number of transitions suffered per second by the positron is $t^{-1}P(t)$, $P(t)$ being given as a multiple integral in Sec. 5.[5] To find the energy lost per second, we alter the integrand to include a factor representing the energy loss in a given transition, namely $\hbar^2(2m)^{-1}(k_1^2-k_1'^2)$. After a number of elementary integrations an expression for R, the rate of loss of energy by the positron, can be found. We now suppress subscripts and write k and E for positron wave number and energy; also, for k/q we put x.

$$R=(3\pi)^{-1}(me^4\hbar^{-3})EF(k/q)$$

$$F(x)=\frac{5}{x^2}-\frac{56}{3}+\left(4-\frac{6}{x^2}\right)\log(1+4x^2)+\frac{1}{x}\left(18-\frac{5}{2x^2}\right)\tan^{-1}2x \quad (1)$$

$$=\frac{32}{35}x^4\left(1-\frac{20}{9}x^2+\cdots\right) \quad \text{for} \quad |x|\ll\frac{1}{2}.$$

The definition of $F(x)$ given is valid for $E\leq E_F$, E_F being the Fermi energy of the free-electron gas. Note that R is independent of E_F for $E\leq E_F$. A classical mechanical computation of R would yield a result proportional to the density of free electrons. In the wave mechanical problem this proportionality would become $R\propto E_F^{\frac{3}{2}}$. The difference between the two results is caused by the action of the Pauli exclusion principle in prohibiting transitions of conduction electrons to states already occupied. One would expect to find $R\propto E_F^{\frac{3}{2}}$ for $E\geq E_F$.

Note that R approaches infinity as q goes to zero. This limit corresponds to using an unshielded Coulomb field as the perturbation and is equivalent to ignoring the polarization of the free electrons about any particular electron. The divergence of R is similar to the well-known divergence of the total cross-section for Coulomb scattering. We shall assume that the value of q appropriate for positron-electron interaction is the same as that found suitable for electron-electron interaction. There is evidence, both experimental and theoretical, that q is roughly equal to 10^8 cm^{-1}. Landsberg[6] found that he could explain the low-energy tail on the L emission line of metallic sodium with a value of q of 1.21×10^8 cm^{-1}. For values of q of roughly the same size Wolhfarth[7] was able to get agreement between calculated and measured electronic specific heats. Although the method of calculating q used by Lee-Whiting[5] for a fast electron travelling in a free-electron gas is not strictly applicable to the present problem, it does give a value of q for sodium in good agreement with Landsberg's empirical estimate. More recently Bohm and Pines[8] have shown that the short-range interaction between two members of a free-electron gas can indeed be approximated by the exponentially screened Coulomb potential. The long-range interaction is expressed in terms of oscillations of the whole gas, the quantum energy for which is too large to permit them to be important in the present problem. Bohm and Pines give them to be important in the present problem. Bohm and Pines give the screening parameter in the form βk_F, k_F being the wave number of an electron on the Fermi surface. They find that β lies between 0.5 and 0.75 for metals, sodium having the value 0.68. We take for convenience $q=1.02\times10^8$

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cm^{-1}, this being the value of k_F corresponding to a Fermi energy of 4 ev. Because the dependence of R on q is given roughly by $R \propto q^{-4}$, the uncertainty in the choice of q does lead to considerable uncertainty in R. Comparison of the theoretical and empirical values of q for sodium enables one to say that, for sodium at least, our estimate of R is not in error by more than a factor of 5.

By an approximate integration of (1) it is easily shown that the positron energy falls from 4 ev to 1 ev in about 3×10^{-15} sec, from 1 ev to 0.1 ev in about 2×10^{-13} sec and from 0.1 ev to 0.025 ev in about 3×10^{-12} sec. Since positrons are observed[3] to annihilate in metals with a lifetime of about 10^{-10} sec, most of them must be thermalized before annihilation.

The incompatibility of the fundamental assumptions of the time-dependent perturbation method found in reference 5 does not occur in this calculation.

PHYSICAL REVIEW VOLUME 97, NUMBER 6 MARCH 15, 1955

Electromagnetic Effects of Spin Wave Resonance in Ferromagnetic Metals

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(Received November 15, 1954)

It was shown experimentally by Rado and Weertman that under suitable conditions there is an observable effect of exchange interactions on the ferromagnetic resonance in metals. The present paper provides an electromagnetic theory of this "spin wave resonance" experiment and satisfactorily explains the exchange shift as well as the width and shape of the absorption line. A combined solution is obtained of Maxwell's equations and the equation of motion of the magnetization vector **M**, the latter equation including the exchange term due to the nonuniform orientation of **M** in the skin depth. It is shown that the triple refraction caused by the exchange effect necessitates the introduction of new boundary conditions. The final result, which is checked numerically and by an approximate calculation, is an expression for the measurable surface impedance and the "equivalent isotropic permeability" derived therefrom. This result is discussed and generalized, the properties of thermal spin waves in metals are briefly considered, and previous theories of exchange effects in ferromagnetic resonance are shown to be inadequate.

I. INTRODUCTION

IT was recently shown by Rado and Weertman[1,2] (to be referred to as RW) that under suitable conditions the effects of exchange interactions on the ferromagnetic resonance of metals can be observed experimentally. Such effects had not been observed previously but their physical basis has long been known. In the skin depth of a ferromagnetic metal the orientation of the magnetization vector **M** is not uniform so that the effective exchange field is not parallel to **M**. Thus there exists an exchange torque which is, in principle, capable of modifying the motion of **M** and the nature of the ferromagnetic resonance. Following RW, we refer occasionally to such a modified ferromagnetic resonance as "spin wave resonance."

The available theories of exchange effects in ferromagnetic resonance are inadequate in two respects. First, they do not predict satisfactorily under what conditions such effects might actually be observable, so that RW had to choose their experimental conditions largely on the basis of physical considerations. Second, these theories do not provide a reliable quantitative description of the exchange effects, so that their use offers at most a qualitative guidance in the interpretation of the RW experiments.

The present work, which we reported briefly at an earlier date,[3] is an attempt to eliminate the theoretical inadequacies mentioned above by giving a consistent description of spin wave resonance on the basis of electromagnetic theory. Such a description should make it possible to account for the position, width, and shape of the resonance line, as well as to evaluate the important exchange factor A and the spectroscopic splitting factor g from the experimental results.

Basically, the electromagnetic problem treated in the present paper involves a combined solution of the equation of motion of **M** (the so-called "spin wave equation") and Maxwell's equations, the solution being required to satisfy two sets of boundary conditions. The first set represents the usual continuity conditions on the tangential components of **E** and **H**, and the second set represents some new conditions that are imposed by the (semi-classically described) exchange effect on certain derivatives of **M**. Specifically, we consider a ferromagnetic metal, possessing a conductivity σ and a saturation magnetization M_s, which is exposed to a saturating static magnetic field of magnitude H_z and to a microwave field of circular frequency ω. The measurable quantity we calculate is the surface

[1] G. T. Rado and J. R. Weertman, Phys. Rev. **94**, 1386 (1954); the symbol μ^2 appearing in the next to last paragraph of this reference is a misprint and should read μ_2.
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