Space-Charge-Limited Currents in Solids

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Currents, far in excess of ohmic currents, can be drawn through thin, relatively perfect insulating crystals. These currents are the direct analog of space-charge-limited currents in a vacuum diode. In actual crystals, the space-charge-limited currents are less than their theoretical value for an ideal crystal by the ratio of free to trapped carriers. Space-charge-limited currents become, therefore, a simple tool for measuring the imperfections in crystals even in the range of one part in 10^{15}.

The presence of traps not only reduces the magnitude of space-charge-limited currents, but also is likely to distort the shape of the current-voltage curve from an ideal square law to a much higher power dependence on voltage. The particular shape can be used to determine the energy distribution of traps.

The presence of traps tends to uniformize the charge distribution between electrodes, to introduce a temperature dependence of the current, and to give rise to certain transient effects from which capture cross sections of traps may be computed.

Space-charge-limited currents offer another mechanism for electrical breakdown in insulators.

I. INTRODUCTION

THE solid state analog of space-charge-limited currents in a vacuum diode are the space-charge-limited currents in an insulator. This was clearly pointed out at least fifteen years ago as a simple consequence of the band theory of solids.[1]

While there have been many references, as in the work of Hilsch, Gudden, and Pohl,[2] to the transient effects of space charge in solids, there have not been until recently direct measurements of steady-state space-charge-limited currents.[3–6] The lack of such measurements is remarkable since simple theory allows amperes per square centimeter of space-charge-limited current to be passed through thin sheets of insulators. Two requirements, however, need to be fulfilled in order to observe space-charge-limited currents of significant magnitude: At least one of the two electrodes must take ohmic contact[5,7] to the insulator and the insulator must be relatively free from trapping defects. The concept of an ohmic contact to an insulator is perhaps not a common one and needs to be defined. An ohmic contact is used here to mean an electrode that supplies an excess or a reservoir of carriers ready to enter the insulator as needed. The virtual cathode formed in front of a thermionic emitter in a vacuum diode is a familiar example of an ohmic contact to the insulating vacuum space between cathode and anode.[8] The current through the vacuum diode or between electrodes in an insulating solid does not depend on the amount of excess carriers as long as there is an excess.

Figure 1 shows one example of an ohmic contact to an insulator obtained by the use of a metal whose work function is less than that of the insulator. The presence of the virtual cathode is evident in Fig. 1(b).

The requirement of relative freedom from traps will be made quantitative later. For the present, it is sufficient to point out that traps lower the drift mobility of carriers and thereby the magnitude of the space-charge-limited currents.[4] Trap densities of $10^{18}/cm^3$ (not unreasonable for the usual polycrystalline insulator) would be sufficient to reduce the space-charge-limited currents to almost unmeasurable values.

The measurements of space-charge-limited currents reported by Smith[9] are on relatively perfect insulating

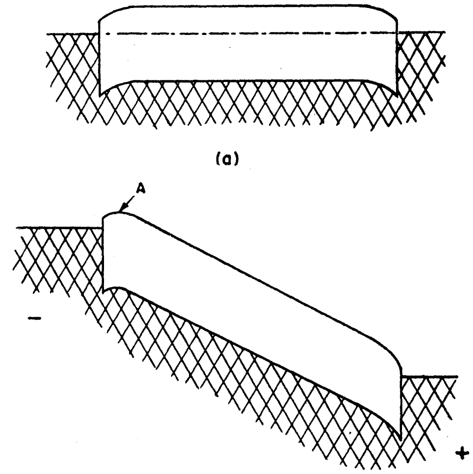

FIG. 1. (a) Ohmic contacts to an insulator at zero applied field. (b) Finite field applied to Fig. 1(a) showing virtual cathode at A.

[1] N. F. Mott and R. W. Gurney, *Electronic Processes in Ionic Crystals* (Oxford University Press, New York, 1940), p. 172.
[2] B. Gudden, *Lichtelectrische Erscheinungen* (Verlag Julius Springer, Berlin, 1928).
[3] P. K. Weimer and A. D. Cope, RCA Review **12**, 314 (1951).
[4] A. Rose, RCA Review **12**, 362 (1951).
[5] R. W. Smith and A. Rose, Phys. Rev. **92**, 857 (1953); A. Rose and R. W. Smith, Phys. Rev. **92**, 857 (1953).
[6] W. Shockley and R. C. Prim, Phys. Rev. **90**, 753 (1953); G. C. Dacey, Phys. Rev. **90**, 759 (1953).
[7] R. W. Smith, this issue [Phys. Rev. **97**, 1525 (1955)].
[8] L. S. Nergaard [RCA Rev. **13**, 464 (1952)] proposes a model of an oxide cathode in which the flow of current within the cathode coating itself, as well as in the vacuum just outside the cathode, may be space-charge-limited.
[9] R. W. Smith and A. Rose, preceding paper [Phys. Rev. **97**, 1531 (1955)].

crystals of CdS having ohmic contacts. Even so, there are a sufficient number of traps that the simple model for space-charge-limited currents needs to be modified as in the following analysis to take their effect into account. The traps not only reduce the magnitude of the space-charge-limited current, but also distort the shape of the current-voltage curve and add certain interesting and informative transient effects.

The analysis of space-charge-limited currents is carried out in terms of the following approximate but simple formalism. Let the space between two electrodes have a capacitance C. This is an approximating concept. In the case of plane parallel electrodes the capacitance is that between the two electrodes. The charge that can be accommodated in the interior space is

$$Q=CV, \tag{1}$$

where V is the applied voltage.

The space-charge-limited current is immediately given by

$$I=Q/T, \tag{2}$$

where T is the transit time of the charge Q between electrodes.

The well-known expressions for space-charge-limited currents in vacuum and in a trap-free insulator are readily derivable from Eq. (2). They are given here to clarify the formalism.

II. VACUUM DIODE

The space charge forced into the vacuum diode per cm^2 of plate area and for a plate separation of d cm is

$$Q=CV=(V/4\pi d)\times 10^{-12} \text{ coulomb}. \tag{3}$$

The transit time of the charge Q between plates is approximately

$$T=d/(6\times 10^{7}\times V^{\frac{1}{2}}) \text{ sec}. \tag{4}$$

The space-charge-limited current is, from (2), (3), and (4)

$$I=5\times 10^{-6}(V^{\frac{3}{2}}/d^2) \text{ amperes/cm}^2. \tag{5}$$

The accurate value of the coefficient is 2.3×10^{-6}.

III. TRAP-FREE INSULATOR

The space charge forced into an insulator per cm^2 of plate area is, from Eq. (1)

$$Q=(Vk/4\pi d)\times 10^{-12} \text{ coulomb}; \tag{6}$$

k is the dielectric constant of the insulator and d the electrode spacing. The transit time of the charge Q between electrodes is

$$T=d/E\mu=d^2/V\mu. \tag{7}$$

E is the electric field in the insulator and μ the drift mobility. From Eqs. (2), (6), and (7) the space-charge-limited current is

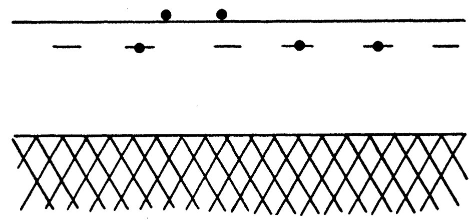

FIG. 2. Insulator having shallow traps in thermal equilibrium with electrons in the conduction band.

$$I=10^{-13}(V^2\mu k/d^3) \text{ amperes/cm}^2. \tag{8}$$

The accurate value of the coefficient is also 10^{-13}.

IV. INSULATOR WITH SHALLOW TRAPS

Let the insulator have only shallow traps (Fig. 2), that is, traps lying close enought to the conduction band to be in thermal equilibrium with electrons in the conduction band. The same expression for the space-charge-limited current will be obtained as in the case of the trap-free insulator. One need only insert for the drift mobility the product of the drift mobility for free carriers and the fraction of the total space-charge that is free. While the same total charge is forced into the insulator as in the case of the trap-free insulator, only a fraction of this charge is free. The drift mobility must be reduced by the same fraction. The value of this fraction is determined by the number and depth of traps and is *not dependent on the applied voltage.*[10] Accordingly, the space-charge-limited current has the same square-law dependence on voltage as in the simple trap-free model of Eq. (8).

Let the fraction of free charge be θ. The space-charge-limited current is then given by

$$I=10^{-13}[V^2(\mu_0\theta)k/d^3] \text{ amperes/cm}^2, \tag{9}$$

where μ_0 is the drift mobility of *free* carriers.

If there is a single level of shallow traps whose density is N_t/cm^3 and whose distance from the conduction band is E volts, the fraction θ is given at room temperature by the approximate relation

$$\theta=(N_c/N_t)e^{-E/kT}, \tag{10}$$

where $N_c=10^{19}$ at room temperature. For $N_t=10^{17}$ and $E=0.5$ volt, $\theta=10^{-7}$ and the space-charge-limited currents are sharply reduced.

V. INSULATOR WITH TRAPS DISTRIBUTED IN ENERGY

Consider, as shown in Fig. 3, an insulator in which the traps are distributed uniformly in energy below the conduction band. The prominent characteristics of this model are a consequence of the distribution of

[10] The electron temperature is assumed here to be the same as the crystal temperature.

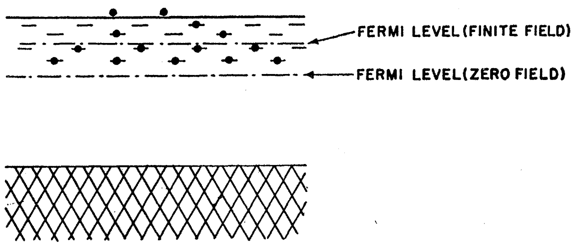

FIG. 3. Insulator having a distribution of traps in energy and showing the shift in Fermi level due to charge injected by an applied field.

traps in energy and not of the strict uniformity of distribution. For a given applied voltage, the charge, Q, forced into the insulator, is distributed in three major parts: free charge in the conduction band, trapped charge above the newly determined Fermi level, and trapped charge condensed in the states between the original Fermi level and the newly determined Fermi level. Since the condensed charge is likely to be very nearly the total charge, the new location of the Fermi level is given very closely by considering all of the charge Q to be condensed. With this approximation, the shift in Fermi level will be proportional to the space charge Q which is, in turn, proportional to the applied voltage V.

We can write for the free carrier density

$$n_c = N_c e^{-E_f/kT} e^{\Delta E/kT}. \qquad (11)$$

Here N_c is the number of states in the bottom kT slice of the conduction band, E_f is the original distance of the Fermi level from the conduction band and ΔE is the shift in position of the Fermi level owing to the condensed charge Q forced into the insulator by the applied voltage V. Also, from previous remarks:

$$\Delta E = Q/en_t d = VC/en_t d, \qquad (12)$$

where n_t is the number of traps per cm^3 per unit range in energy and e the electron charge. From Eqs. (11) and (12) the free carrier density is given by

$$n_c = N_c e^{-E_f/kT} e^{VC/n_t dekT} \qquad (13a)$$

$$= n_{c0} e^{\alpha V}, \qquad (13b)$$

where n_{c0} is the initial, thermal equilibrium concentration of free carriers and α is used for $C/n_t dekT$.

The density of trapped carriers is very nearly equal to the total density of injected electrons, or

$$\text{density of trapped carriers} = Q/de = VC/de. \qquad (14)$$

The fractional value of free charge is, from Eqs. (13) and (14),

$$\theta = (en_{c0}d/VC)e^{\alpha V}. \qquad (15)$$

θ is no longer a constant as in the previous case of shallow traps, but depends exponentially on the applied voltage. From Eqs. (9) and (15) the space-charge-limited current becomes:

$$I = 10^{-13}(V\mu_0 k/d^2)(en_{c0}/C)e^{\alpha V}. \qquad (16)$$

What is significant in Eq. (16) is that, owing to the distribution of traps in energy, the space-charge-limited current now increases exponentially with voltage compared with the square law dependence on voltage obtained in the trap-free and in the shallow-trap models. The exponential dependence is a consequence of the assumption of a *uniform* distribution of traps. If the uniform distribution of traps is replaced by one that decreases with distance from the conduction band, the exponential is replaced by a high power function of the voltage.

In particular, let the steepness of the trap distribution be approximated by a characteristic temperature T_c such that

$$n_t \propto e^{-E/kT_c}, \qquad (17)$$

where E is measured from the bottom of the conduction band. Small values of T_c lead to trap distributions varying rapidly with energy, while large values of T_c approximate a slowly varying trap distribution. The voltage dependence (see Appendix I) of space-charge-limited current is (for $T_c > T$)

$$I \propto V^{(T_c/T)+1}. \qquad (18)$$

For $T_c < T$, this reduces to the case of shallow traps where the exponent of V is 2.

VI. TRAP DISTRIBUTION FROM I VS V CURVE

One can expect to work backwards from an experimentally determined current-voltage curve to obtain the energy distribution of traps. Equation (18), for example, gives the trap distribution for experimental curves for which the current increases as a power of the voltage. For a current-voltage curve of arbitrary form, and for currents increasing faster than V^2, the following analysis may be made. From Eq. (9) one may write

$$I = \text{constant}\, V e^{\Delta E/kT}, \qquad (19)$$

and

$$\frac{dI}{dV} = \frac{I}{V}\left(1 + \frac{V}{kT}\frac{d(\Delta E)}{dV}\right). \qquad (20)$$

The solution of Eq. (20) for $d\Delta E/dV$ is

$$\frac{d\Delta E}{dV} = \left(\frac{V}{I}\frac{dI}{dV} - 1\right)\frac{kT}{V}. \qquad (21)$$

Since the charge condensed in traps is

$$Q = VC,$$

Eq. (21) may be rewritten as

$$C\frac{d\Delta E}{dQ} = \left(\frac{V}{I}\frac{dI}{dV} - 1\right)\frac{kT}{V}, \qquad (22)$$

or

$$\frac{dQ}{d\Delta E}=\frac{CV}{kT}\left(\frac{V}{I}\frac{dI}{dV}-1\right)^{-1}. \tag{23}$$

In Eq. (23), $dQ/ed\Delta E$ is the number of traps per unit energy range in the volume of specimen under test. A simple operational interpretation of Eq. (23) is the following. If one increases the applied voltage by an amount ΔV sufficient to double the current, the number of electron charges forced into the insulator is $\Delta VC/e$. This number is also the number of traps in a range kT near the Fermi level.

VII. COMPARISON OF SPACE-CHARGE-LIMITED CURRENT WITH PHOTOCONDUCTIVE CURRENT

The last two sections have shown how the form and magnitude of the trap distribution may be computed from the space-charge-limited current-voltage curve.

TABLE I. The energy distribution and density of traps derived from data on photoconductivity and from data on space-charge-limited currents.[a]

	Space-charge-limited currents	Photoconductivity
Form of current curve	$I \propto V^{(T_c+T)/T}$	$I \propto F^{T_c/(T+T_c)}$
Trap density in range of kT near the Fermi level	$\Delta Q/e$	$(\tau_0/\tau)n_c$

[a] Notes: T_c defines the trap distribution by Eq. (17). F is the number of optical excitations per second. ΔQ is the charge forced into the insulator when the voltage is increased by an amount sufficient to double the current. τ_0 is the observed response time of the photoconductor to interrupted light. τ is the lifetime of a free carrier in the conduction band. n_c is the density of free carriers at which τ_0 is measured. The Fermi level is defined by the relation: $n_c=N_ce^{-E_f/kT}=10^{19}e^{-E_f/kT}$.

In reference 4, it was argued that the same information on trap distribution could be obtained from data on the form of the photocurrent-light curve and from data on the ratio of lifetime to observed time constant.

The results of the two analyses are summarized in Table I.

The characteristic temperature, computed from the space-charge-limited currents, should be more reliable than the characteristic temperature computed from the photoconductive currents. In the analysis of the latter an implicit assumption was made that all of the traps had the same capture cross section for electrons. The validity of this assumption is under study and must, in any event, be tested for each new crystal. There are some observations that require the presence of more than one type of trap and such mixtures can alter the form of the current-light curve. The form of the space-charge-limited current-voltage curve on the other hand should not be dependent on the capture cross section of the traps. Reasonable agreement between the two independent methods of measuring trap distributions is reported by Smith and Rose[9] and by Bube.[11]

[11] R. H. Bube and S. M. Thomsen, J. Chem. Phys. 23, 15 (1955).

VIII. FIELD AND CHARGE DISTRIBUTION BETWEEN ELECTRODES

For the simple case of trap-free insulator and plane parallel electrodes, the following relations for current, field, and charge distribution are known:

$$I \propto V^2 \tag{24}$$

$$E \propto x^{\frac{1}{2}} \tag{25}$$

$$\rho \propto x^{-\frac{1}{2}}, \tag{26}$$

where E is the electric field, x the distance from the cathode (for electron injection), and ρ the space charge density.

In Appendix II it is shown that, in general, when traps are present and when

$$I \propto V^{n+1}, \quad n \geq 1, \tag{27}$$

the field and charge distributions take on the forms

$$E \propto x^{n/(n+1)}, \tag{28}$$

$$\rho \propto x^{-1/(n+1)}. \tag{29}$$

For large values of n, the space charge density approaches a uniform distribution over most of the distance between cathode and anode. The free charge density, however, must always vary as the reciprocal of the field in order to keep the divergence of the current zero. Since the free charge is usually a negligible part of the total charge, it may undergo large variations without having significant effect on the distribution of the total charge.

The relative uniformity of charge density between cathode and anode leads one to expect only small or negligible currents when these electrodes are shorted together. The space charge flowing out of the insulator tends to flow out equally at both ends. Smith[9] has observed the short circuit current to be negligibly small. This is to be contrasted with the relatively large short-circuit reverse currents obtained from dielectric absorption effects as in some glasses.

IX. TRANSIENT EFFECTS

The following observation on space-charge-limited currents in CdS crystals is reported by Smith.[9] A sudden increase in voltage causes the current to transiently increase to very high values. In a matter of seconds or minutes the current subsides to a much smaller stationary value. The interpretation is that the sudden increase in voltage forced a corresponding increase of charge in the *conduction band*. In the course of seconds, most of this free charge settles into traps and one observes the rapid decay of current. The time required for the transient current to subside is a direct measure of the capture cross section of traps for free electrons.

If the space-charge-limited current has attained a stationary value at a given voltage it is found[9] that lowering the voltage from this value may cause the

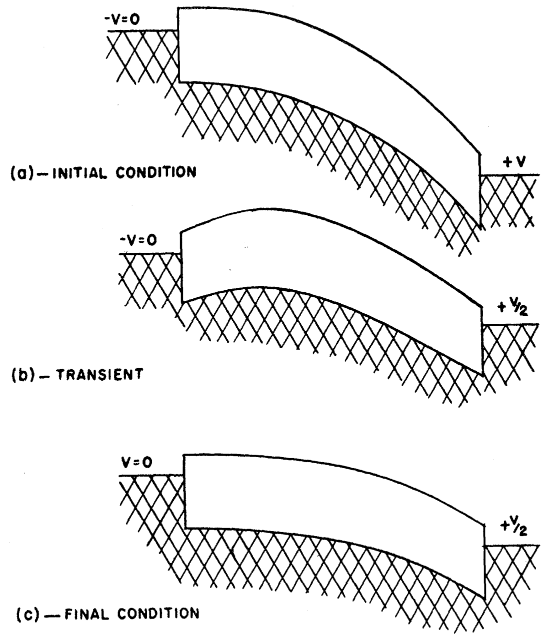

FIG. 4. Series of potential patterns showing the transient effects when the applied voltage is reduced.

current to "undershoot" its new stationary value. The interpretation here is shown in Fig. 4. Figure 4(a) shows the stationary potential distribution at an applied voltage V_1. When the voltage is lowered to $\frac{1}{2}V_1$, it requires some time for the trapped space charge forced into the crystal at V_1 to be thermally released. Before this trapped charge is thermally released, a space-charge barrier is presented to the cathode, as shown in Fig. 4(b). *This is not a virtual cathode type of barrier.* It actually *suppresses* the entrance of electrons from the cathode into the insulator. As time goes on, the trapped charge is thermally released and the potential distribution arrives at the new stationary value shown in Fig. 4(c). If the thermal release of carriers is sufficiently slow (traps of small capture cross section) the current can "undershoot" its final value. If the thermal release is fast there will be no "undershoot" but actually an "overshoot."

X. TEMPERATURE DEPENDENCE

The injection of space charge into an insulator converts it into a semiconductor of increasing conductivity with increasing voltage. At any given voltage the current should vary with temperature as would any semiconductor having the same conductivity. (This does not mean that the temperature variation of conductivity is determined only by the conductivity. As in any semiconductor the trap distribution governs the temperature dependence.) An increase in temperature does not alter the total amount of space charge, but does increase the fraction of this space charge in the conduction band.

Equation (18) indicates that lowering the temperature should make the current-voltage curve steeper. For very steep curves, the effect of lowering the temperature should be one of shifting the current-voltage curve along the voltage axis toward higher voltages. To match the same current the Fermi level must be closer to the conduction band at lower temperatures and this requires higher voltages according to Eq. (12).

XI. TRANSITION FROM OHMIC TO SPACE-CHARGE-LIMITED CURRENTS

Space-charge-limited currents increase as the square or as some higher power of the voltage. Ohmic currents increase linearly with the voltage. One would expect, therefore, that for any finite conductivity, there would be a range of voltages near zero for which the ohmic currents would predominate. For voltages higher than some critical voltage, space-charge-limited currents would predominate. The critical voltage at which this transition from ohmic to space-charge-limited behavior takes place should increase as the normal *volume-generated* conductivity increases. Results of this character are clearly reported in reference 9 where the critical voltage is varied by shining light on a CdS crystal.

What has just been described should certainly take place if the ohmic and space-charge-limited currents were in parallel, physically separate paths. When the two types of current occupy the same physical volume, the transition from one current to the other is likely to be somewhat more involved because the potential distributions are different for the two types of current. There will then be a competition between the two processes to establish their appropriate potential distribution. It would appear, however, from qualitative arguments that the mechanism that introduced the larger density of free carriers would control the potential distribution. Accordingly, higher volume generated carrier densities mean that a higher voltage is required before the injected space-charge-carrier densities predominate and determine the character of the current-voltage curve.

It is interesting that even in the range of voltage where the ohmic currents predominate in the steady state, the space-charge-limited currents may determine the transient behavior. This follows from the fact that when the voltage is increased there is a transient high density of space-charge-carriers in the conduction band —a density that may exceed that of the volume generated carriers. As these space-charge-carriers become trapped, their density falls below that of the volume-generated carriers and the latter lead to steady-state ohmic currents.

XII. TOOL FOR MEASURING CRYSTAL DEFECTS

As already outlined in an earlier section, the number and energy distribution of traps can be deduced from the current-voltage curve for space-charge-limited currents. What needs to be emphasized here is that these currents constitute an unusual tool for measuring defect structure—a tool that becomes particularly effective in the range of low concentration of defects.

The effect of traps is generally to reduce the observed space-charge-limited currents below their theoretical value for a trap-free crystal. The measure of this reduction is the ratio of free to trapped carriers. Thus, the observed currents should approach those for a perfect crystal when the number of free carriers matches or exceeds the number of traps. The more perfect the crystal, the lower the field at which this occurs.

The density of electron charges forced into a crystal d centimeters thick, having plane parallel electrodes, may be written approximately as $10^7(V/d^2)$ electron charges/cm^3 for an assumed dielectric constant of 10. This means that for a millimeter thick crystal having 10^{10} traps/cm^3, the space-charge-limited currents should approach their theoretical values at ten volts. At this voltage the current will be about 10 microamperes/cm^2 for a mobility of 100 cm^2/volt-sec. The measurement of trap densities of only one part in 10^{15} becomes then a simple current-voltage measurement at low voltages and easily measurable currents.

XIII. CONCERNING BREAKDOWN IN INSULATORS

When the voltage across an insulator is increased steadily the power dissipation in the insulator is finally increased to the point where the insulator "burns out" or is said to "break down." If enough carriers are normally present in the insulator, the breakdown is a relatively slow and gradual process in which the increased voltage at first leads to an increased temperature. More carriers are generated at the higher temperature and the approach to breakdown becomes more rapid. This process is known as "thermal breakdown" and may be followed reversibly to values close to actual breakdown. A second process[12] that has received the major share of theoretical attention is a fast, electronic process known as "intrinsic breakdown." Here a critical electric field may be observed at which the carrier density is precipitously increased by a collision ionization and resulting avalanching process or by field emission from the filled band. Even though the actual breakdown field has a sharply defined value, there is reason to expect in this model also that the prebreakdown currents will increase faster than linearly with voltage.

The present discussion adds a third mechanism for increasing the carrier density in insulators and must be considered in analyzing breakdown data. This mechanism of space-charge-limited currents becomes more significant as the crystallinity of the insulator improves. It may be distinguished from intrinsic breakdown by the fact that the breakdown field should increase approximately linearly with electrode spacing.

[12] H. Fröhlich and J. H. Simpson, *Advances in Electronics* (Academic Press, New York, 1950), Vol. 2, p. 185.

A rough estimate of the contribution of space-charge-limited currents may be made as follows. Let the intrinsic breakdown field strength be known. From this value of field and the known geometry of the specimen a value for the space-charge density in the insulator may be computed. This value of space charge density, converted to carrier density, must be comparable with the trap density in order that space-charge-limited currents be significant. For example, at a field of 10^5 volts/cm in an insulator 10^{-3} cm thick, the number of electron charges per cm^3 forced into the insulator would be 10^{16}. Trap densities less than this value would allow space-charge-limited currents to be significant; trap densities greater than this value would tend to suppress the space-charge-limited currents.

APPENDIX

I. CURRENT-VOLTAGE CURVE FOR EXPONENTIAL TRAP DISTRIBUTIONS

The trap density per unit energy range is defined by

$$n_t = Ae^{-E/kT_c}, \tag{30}$$

where E is the energy measured from the bottom of the conduction band and T_c is a characteristic temperature greater than the temperature at which the currents are measured. The condensed charge forced into the insulator is

$$Q = VC. \tag{31}$$

This condensed charge raises the Fermi level by an amount ΔE defined by the relation

$$\int_{E_f-\Delta E}^{E_f} n_t dE = \frac{Q}{e} = \frac{VC}{e}, \tag{32}$$

or

$$\int_{E_f-\Delta E}^{E_f} Ae^{-E/kT_c} dE = \frac{VC}{e}. \tag{33}$$

The solution of Eq. (33), neglecting the upper limit of integration, is of the form

$$\Delta E = kT_c(K + \ln V), \tag{34}$$

where K contains the temperature but not the voltage. The ratio of free to trapped charge is [see Eq. (11)]

$$\theta = en_{c0}e^{\Delta E/kT}/VC. \tag{35}$$

If Eq. (34) is used for ΔE,

$$\theta = \text{constant}\exp[(T_c/T)\ln V]/V \tag{36}$$

$$= \text{constant}\, V^{(T_c/T)-1}. \tag{37}$$

This value for θ is now inserted in Eq. (9) to give

$$I \propto V^{(T_c/T)+1}. \tag{38}$$

II. FIELD AND CHARGE DISTRIBUTION BETWEEN CATHODE AND ANODE

A solution is sought for the usual pair of equations for one-dimensional space-charge-limited flow:

$$\frac{dE}{dx}=\frac{4\pi\rho}{k}=\frac{4\pi}{k}(\rho_f+\rho_t), \tag{39}$$

$$I=\rho_f\mu E, \tag{40}$$

subject to the boundary condition $E=0$ at $x=0$. ρ is the space charge density and is composed of a part, ρ_f, in the conduction band and a part, ρ_t, in traps. Let $\rho_f \ll \rho_t$ so that it may be neglected in Eq. (39). Also, from Eq. (37) let

$$\rho_f=A\rho_t^n, \tag{41}$$

where

$$n=T_c/T. \tag{42}$$

Equation (39) may be rewritten, using Eqs. (40) and (41), in the form

$$\frac{dE}{dx}=\frac{4\pi}{k}\left(\frac{\rho_f}{A}\right)^{1/n}=\frac{4\pi}{k}\left(\frac{I}{\mu EA}\right)^{1/n}=BE^{-1/n}, \tag{43}$$

where $B=(4\pi/k)(I/\mu A)^{1/n}$.

The solution of Eq. (43) satisfying the boundary condition is

$$E=[(n+1)/n]Bx^{n/(n+1)}. \tag{44}$$

As $n\rightarrow\infty$, $E\rightarrow Bx$. If $n=1$, the usual form for a trap-free (as well as a shallow trap) model is obtained, namely $E\propto x^{\frac{1}{2}}$.

The distribution of trapped space charge using Eq. (39) is

$$\rho_t=\frac{k}{4\pi}\frac{dE}{dx}$$

$$=\frac{k}{4\pi}\frac{n+1}{n}Bx^{-1/(1+n)}. \tag{45}$$

Again this reduces to the familiar $x^{-\frac{1}{2}}$ form when $n=1$, *but approaches a constant for large n.*

The uniform distribution of space charge, at large n, means that when the electrodes are shorted only a vanishingly small net current will flow as the space charge leaves the insulator. The space charge will flow out almost symmetrically at both ends of the insulator.

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Equation of State of Metals from Shock Wave Measurements*

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Shock wave pressure magnitudes from about 150 to 500 kilobars have been attained for metals by using high explosives. A photographic technique for the nearly simultaneous determination of shock and free surface velocities is presented, and measurements for aluminum, copper, and zinc are given.

Expressions are derived which permit the calculation of pressure-compression points from measured velocity pairs. Consequent Hugoniot curves are presented, probable errors for which are 1 to 2 percent in compression for a given pressure. Finally, the known Hugoniot curves are employed in a calculation which determines temperatures and isotherms.

I. INTRODUCTION

WHEN a detonation wave interacts with an explosive-metal interface, a compression wave is transmitted into the metal. In the ordinary case this disturbance is a shock wave separating a compressed state from the undisturbed metal. The pressures attained behind such shock waves are typically in the range 150 to 500 kilobars (1 kilobar$=10^9$ dynes/cm^2 $=986.9$ atmospheres). The associated problem of determining pressure-compression data from shock wave measurements is the subject of the present investigation. Such data serve to supplement and extend the wealth of static pressure-compression data which exist for pressures up to 100 kilobars.[1]

Two basic assumptions are employed throughout the present considerations. First, since shock pressures are several hundred times yield points of the materials involved, an ordinary "fluid" type equation of state is assumed, i.e., a functional relationship (unspecified) between P, V, and T is assumed to be an adequate representation of the metal. This assumption precludes the explicit treatment of effects arising from the material rigidity which, however, are felt to play a

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[1] See P. W. Bridgman, Revs. Modern Phys. 18, 1–93 (1946) for a general review.