Mean Free Path for Shower Production by High-Energy Pi Mesons*

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In order to investigate a possible difference in the mean free paths for shower production by protons and by π mesons of very high energy, an experiment was carried out at Echo Lake, Colorado (altitude 3260 m) in which the rate of production of penetrating showers by the atmospheric N component was studied as a function of the absorber thickness up to a total of 650-g/cm^2 Pb, or about four proton mean free paths. A difference in the cross sections would then result in the "filtering out" of the component with the shorter mean free path, and manifest itself in an attenuation curve which is the superposition of two exponentials rather than a single exponential curve. By splitting the top absorber into three sections and using three top trays instead of the usual one, with the rigid condition that only a single counter in each tray be discharged, losses due to the complete absorption of all secondaries of an interaction occurring in the top absorber were minimized and could be estimated from the results of the runs with smaller absorber thicknesses in which the lead absorber was distributed in various ways between and above the trays. Three further counter trays, placed in a lead pile, were used to select five types of showers of different multiplicity. The average energy of the primaries of these events were estimated to be about 5.3 Bev ("low-energy group"), 8.3 Bev ("medium-energy group"), and 16–20 Bev ("high-energy group"). Even after correcting for spurious events, the rates of the two latter groups—for which a substantial π-meson component can be expected—do not fit a straight line on a semilogarithmic plot, thus indicating that the mean free paths of the two components are not identical. Under the assumption of equal detection probability for the interactions of both types of primaries, the mean free path for shower production by π mesons of both energy groups is found to exceed 320 g/cm^2. It must be stressed that these results should not be interpreted as a proof for an interaction cross section smaller than the geometrical nuclear cross section, but rather as an indication that in collisions of very energetic π mesons, the number of π-meson secondaries emitted differs from that resulting from proton collisions, perhaps because of a larger probability for the production of other secondary particles.

I. INTRODUCTION

ALTHOUGH considerable effort has gone into the study of the interaction cross section of the energetic π mesons ejected in cosmic-ray nuclear interactions, no generally accepted picture has as yet evolved. It is probably generally conceded that, if the experimental techniques applied are very carefully designed so that collisions with small energy transfer (especially nuclear scattering) are not disregarded, the results of the measurements tend to approach the "geometrical" nuclear cross section. This is very clearly demonstrated in the summaries given, for instance, by Rochester and Rosser[1] and by Camerini *et al.*[2] However, it is of interest to note for the following considerations that in most experiments in which only energetic interactions were recorded—that is, when meson reproduction by mesons was demanded—a mean free path well in excess of the geometrical value was obtained. Thus, the cloud chamber investigations of Fretter[3] and of Brown and McKay[4] yielded mean free paths in lead of 750 g/cm^2 and 316 g/cm^2, respectively, and Piccioni's[5] earlier counter experiments gave for iron a value of 1200 g/cm^2. Perhaps the most noticeable exception is the recent cloud-chamber study of Duller and Walker[6] in which secondary mean free paths approaching the geometrical value with increasing particle energy were reported.

It should, however, be noted that in most of these experiments the particles were identified only as "shower secondaries"—and therefore were probably for the most part, but not exclusively, π mesons,—and that the energy of the particles was usually known only very roughly, but in most cases did not exceed an average value around 1 Bev. For energies around and below a few Bev, however, measurements of the interaction mean free path of nucleons have led to conflicting results, and the claim has frequently been made that in this energy region the cross section decreases sensitively with decreasing primary energy (e.g., Walker,[7] Sitte,[8] Boehmer and Bridge[9]). Although this conclusion has more recently been disputed (Froehlich *et al.*,[10] Sitte[11]), a geometrical interaction cross section can be considered as completely assured only for nucleons of several Bev energy. Thus it appeared of interest to study the cross section for collisions of π mesons of similarly high energies. This problem is also of importance for the propagation of the nucleonic cascade in the atmosphere: at energies around 10 Bev, decay and nuclear interactions become of comparable probability, in the lower parts of the atmosphere, if the mean free path has the

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1 G. D. Rochester and W. G. V. Rosser, Repts. Progr. Phys. **14**, 227 (1951).
2 Camerini, Lock, and Perkins in *Progress in Cosmic-Ray Physics*, edited by J. G. Wilson (North-Holland Publishing Company, Amsterdam, 1952), Chap. 1.
3 W. B. Fretter, Phys. Rev. **76**, 511 (1949).
4 W. W. Brown and A. S. McKay, Phys. Rev. **77**, 342 (1950).
5 O. Piccioni, Phys. Rev. **77**, 1 (1950) and Phys. Rev. **77**, 6 (1950).
6 N. M. Duller and W. D. Walker, Phys. Rev. **93**, 215 (1954).
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8 K. Sitte, Phys. Rev. **78**, 714 (1950).
9 H. W. Boehmer and H. S. Bridge, Phys. Rev. **85**, 863 (1952).
10 Froehlich, Harth, and Sitte, Phys. Rev. **87**, 504 (1952).
11 K. Sitte, Acta Phys. Austriaca **6**, 167 (1952).

geometrical value. These energetic π mesons may then play an important part in the development of the nuclear cascade.

Since for protons of similarly high energies practically every interaction results in meson production, i.e., in the production of a penetrating shower, it appeared plausible to approach the problem of π-meson interactions in the same way, by studying showers produced by these particles. π mesons whose energies are around or above 10 Bev have even at mountain altitude a reasonable chance to escape decay until they collide with a nucleus. Thus, the beam of energetic N particles incident from the air on a shower-detecting arrangement will contain a few percent of π mesons together with the more abundant nucleons, the exact ratio depending on energy and altitude and being determined by the ratio of the mean free paths for collision and for decay. It is then possible to demonstrate a difference between the properties of the two main components of the atmospheric N component, if such a difference exists, in the following way: Assume, for instance, that π mesons have an appreciably smaller cross section than nucleons. Then one can "filter out" these particles by observing shower production under a very thick absorber, since the attenuation of the two components will be greatly different. To quote an example: if, for instance, the incident beam is composed of 5 percent π mesons and 95 percent protons, and if the mean free path in lead of the former is 660 g/cm^2 or four times that of the protons, then the beam of N-particles emerging from an absorber of 660 g/cm^2 thickness will consist of an about equal number of protons and of π mesons, and a determination of the mean free path of this mixture will yield a value which is easily distinguished from that found for the unfiltered atmospheric N component. A simple calculation shows that this method will produce statistically significant results in a reasonable length of time if the ratio of the mean free paths of the two types of N particles is at least 1.5. The method is, therefore, adequate for at least a preliminary study of the shower production by π mesons of very high energies, for which no other source than the atmospheric N component is available.

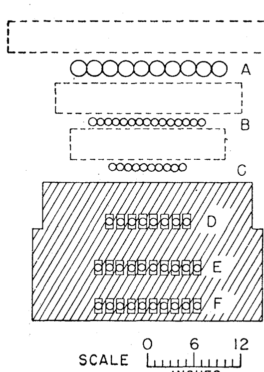

FIG. 1. Schematic diagram of the experimental arrangement.

II. THE EXPERIMENTAL METHOD

1. Description of the Arrangement

In a counter experiment designed to measure collision mean free paths, the customary procedure is to place a set of trays arranged to detect penetrating showers (in the following referred to as the "P set") under absorbers of various thicknesses, making sure that all events are rejected in which an interaction occurred already in the top absorber, regardless of possible subsequent collisions in the P set. Thus, for instance, if the interactions of charged N particles are being studied, the P set will be supplemented by a top tray which must be struck by exactly one particle, and all cases must be rejected in which more than one top counter is discharged. However, this method will obviously fail if, as in the present experiment, very thick absorbers must be used. Even if the P set is arranged to select showers of high multiplicity and penetration, one cannot exclude the possibility that some interactions had taken place in the top absorber, but had failed to trigger more than one counter of the top tray, either due to the absorption of the shower particles or due to their spread and the geometrical inefficiency of the counter tray.

This failure can be eliminated, or at least greatly diminished, by splitting the top absorber in three sections and placing individual "top trays" under every one of the absorber layers, demanding that in each of these three trays one and only one counter be discharged. In this way, none of the trays has to be covered by more than about 220 g/cm^2 although a total of three times that amount is used in this experiment, and absorbers of even more than 220 g/cm^2 have been considered safe in other experiments.

The experimental arrangement is shown schematically in Fig. 1. Of the three top trays A, B, and C, tray A consisted of ten counters of sensitive area 2 in.$\times$24 in., tray B of fifteen counters 1 in.$\times$20 in., and tray C of ten counters 1 in.$\times$16 in. Below this "top telescope," the P set begins with a production layer of 4-in. Pb+$\frac{1}{4}$-in. Fe, and contains three trays of counters: tray D with eight counters of 1 in.$\times$16 in., trays E and F each with ten counters of 1 in.$\times$20 in. The individual counters of these trays were separated by $\frac{1}{2}$-in. lead strips. The absorber between D and E was 4-in. Pb+$\frac{1}{4}$-in. Fe, and that between E and F 3-in. Pb+$\frac{1}{4}$-in. Fe. Heavy side shielding (8-in. Pb) protected the trays of the P set in all directions against soft particles traveling under a large angle.

The counter pulses, equalized by diode clippers, were fed into conventional integrating circuits and cathode followers, all located in the trays. The output pulses then were fed into a set of discriminators, and their shaped outputs were mixed to define the various events chosen for the records. While the selection in all cases was based on the passage of a single, unaccompanied particle through the three top trays A, B, and C, five different types of showers were continuously recorded. For the purpose of a simple description of the events, the following notation will be used throughout: A subscript n to the letter marking the tray (A_n) means that *exactly n* of its counters were struck; and a superscript m (B^m) denotes events in which *m or more B* counters were discharged. Furthermore, wherever possible the abbreviations "T" and "P" will be used for the top telescope (consisting of the three trays A, B, and C) and for the P set (of trays D, E, and F), respectively. Thus, the five types of shower events recorded can be written as:

$$T_1P^{221}=(A_1B_1C_1D^2E^2F^1);\quad T_1P^{222}=(A_1B_1C_1D^2E^2F^2),$$
$$T_1P^{321}=(A_1B_1C_1D^3E^2F^1);\quad T_1P^{332}=(A_1B_1C_1D^3E^3F^2),$$
$$T_1P^{333}=(A_1B_1C_1D^3E^3F^3).$$

The first group of events, consisting of a telescope particle, two or more particles at trays D and E, and at least one shower particle reaching tray F, comprises showers of comparatively low energy. The second and third, expected to be of about comparable frequency, form a medium-energy group, and the last two which require high multiplicities in several trays of the P set, will be mostly due to very energetic primaries. It is, therefore, mainly in the two latter groups that one may expect an effect if the π-meson component can be enriched by passing the N radiation through a thick absorber. A rough estimate of the primary energies of the three groups will be given in the Appendix.

2. Correction for Spurious Events

Since the rates of the large showers under very thick absorbers become exceedingly small, not only constant and careful checks on the stability of the arrangement is required, but also careful evaluation of all spurious effects. These can be divided into two groups: spurious events which give rise to counts without being due to interactions of N particles and therefore increase the rate of "showers" registered, and losses due to inefficiencies still remaining in spite of the precautions taken in the design of the arrangement. In this section, the effects of the first kind leading to spurious counts will be discussed.

Three types of events are the main contributors: (a) counts due to double or triple knock-on showers;[12] (b) counts due to random coincidences of two or more incident penetrating particles, all but one missing the top trays; and (c) nuclear interactions initiated by μ mesons. Their importance for the various groups of showers will be discussed separately.

(a) Knock-on showers affect the recorded rates in two ways. On the one hand, knock-on showers occurring in the P set above the three trays may be mistaken for penetrating events, thus increasing the number of showers registered. On the other hand, knock-on electrons originated in one of the three lead absorbers by the N particle traversing the top telescope and accompanying it to the level of one of the top trays, may reduce the apparent rate of traversals without interaction and hence the recorded shower rate. This effect is not negligible already for an arrangement using a single top tray (although it seems to have escaped the notice of most experimenters in this field), but it becomes particularly important in an experiment such as the present one where the passage of an unaccompanied particle is demanded at three trays.

To make the appropriate corrections, single and double knock-on rates for the individual trays (e.g., $T_1D^2E^1F^1$, $T_1D^3E^1F^1$, and also $A^2B_1C_1P^{221}$ and so on) were continuously recorded and checked. From the results the probabilities for knock-on accompaniment of a certain multiplicity were computed for all the trays. These figures, together with the "telescope rate" T_1P^{111} which was likewise continuously recorded, were then used to obtain the spurious contributions. The corrections for knock-on events in the top absorbers were calculated in the same way. Since three top trays were used, it is essential to consider also the possibility of knock-on events occurring at more than one tray.

It may be mentioned that spurious events in the P set have an appreciable effect only on the smallest showers (T_1P^{221}) at all absorber thicknesses, and on the events T_1P^{321} under the thickest absorber. In all other cases the corrections are quite negligible as a result of the experimental condition demanding multiplicity at three shielded trays.

(b) The most important contribution in this group comes from coincidences of a "telescope" meson traversing all top trays, with another meson that misses them. All possible combinations have been carefully analyzed, in Fig. 2 these spurious events contributing most to the shower types T_1P^{221} and T_1P^{222} are sketched. They include coincidences of the kind $(T_1P^{221})+(T_0P^{111})$ for the second shower group.

Most of the rates involved were likewise recorded during the experiment; others could be computed from the measured rates (like for instance the events T_0P^{111} since only T^0P^{111} was registered). The resolving times of the integrating circuits and of the coincidence system were measured both with a double-pulse generator and by observation of random coincidences at high counting rates. The rates of chance coincidences of the type shown in Fig. 2 could therefore be calculated. The contributions were small but not negligible for the shower groups T_1P^{221} and T_1P^{222}, especially in the shielded runs. All the other shower groups were practically unaffected since they all involve either knock-on events together with random coincidences, or triple coincidences.

[12] Under this heading are comprised all events recorded in the apparatus described as nonpenetrating secondary events produced by a single penetrating particle.

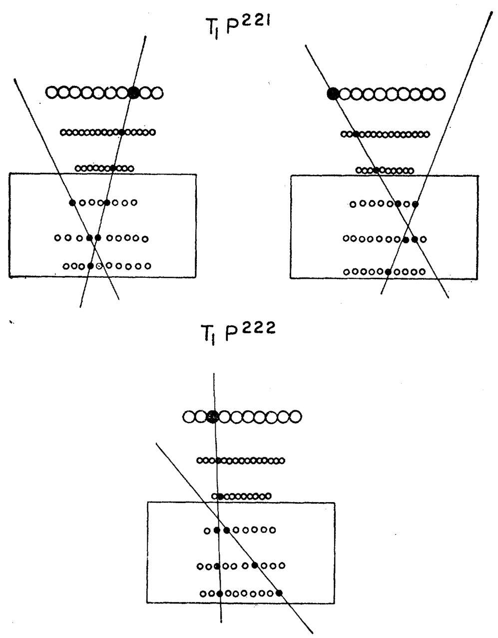

FIG. 2. Chance coincidences contributing spurious counts to the shower groups T_1P^{221} and T_1P^{222}.

(c) Corrections for nuclear interactions initiated by μ mesons can be calculated if one assumes that structure and composition of these showers, and hence the probability of their detection in the apparatus, do not differ significantly from those originated by N particles. Writing I_N and I_μ for the intensities of the N component and of the μ mesons, $p(\lambda_N)$ and $p(\lambda_\mu)$ for the probabilities that a shower is initiated in the production layer between trays C and D (λ_N and λ_μ are the interaction mean free paths of the two components), and $P(n)$ for the detection probability for a shower of n particles, one has for the numbers N_N, N_μ of showers due to the two kinds of primaries:

$$N_N = I_N p(\lambda_N) P(n), \qquad N_\mu = I_\mu p(\lambda_\mu) P(n); \tag{1}$$

therefore, since $p(\lambda_N) = (1 - e^{-\xi/\lambda_N})$, $p(\lambda_\mu) = (1 - e^{-\xi/\lambda_\mu})$, with ξ = thickness of the production layer,

$$(N_\mu/N_N) = (I_\mu/I_N)(1 - e^{-\xi/\lambda_\mu})/(1 - e^{-\xi/\lambda_N}). \tag{2}$$

The cross section for nuclear interactions of μ mesons has now been measured by various authors (e.g., George,[13] Amaldi *et al.*,[14] Barrett *et al.*[15]) and the values obtained agree reasonably well, considering the smallness of the effect and the arbitrariness of the definition of a shower. For events of size and penetration comparable to that of the present experiment, a cross section of 5–6$\times 10^{-30}$ cm^2 per nucleon, or about 1/10 000 of that of a high-energy nucleon, is probably correct. The contributions of μ-meson initiated showers have therefore been computed with this value, corresponding to $\lambda_\mu = 1.6\times 10^6$ g/cm^2 in lead, and for $(I_\mu/I_N) = 20$ in the unshielded run. In evaluating the intensity ratio under the various absorber thicknesses used, the attenuation of the N component was taken from the observed shower frequencies, and that of the μ mesons from their well-known range spectrum. The thickness ξ of the producing layer was 4-in. Pb+$\frac{1}{4}$-in. Fe.

The resulting contributions of μ-meson-initiated showers are quite negligible for small absorber thicknesses, but become increasingly important under thick absorbers. Under 650 g/cm^2 Pb, for instance, these spurious contributions amount to 4.4 percent of all recorded interactions in the low-energy group, and to 5.6 percent of the recorded medium-energy showers and 6.5 percent of the most energetic group.

A summary of the fractional contributions of the various corrections is given in Table I.

III. RESULTS

1. Effect of Thick Absorbers

Like the commonly neglected knock-on correction, a possible absorption of all the secondaries of an event occurring in the top lead absorber, and thus preceding the interaction in the producing layer, must be very carefully watched. As it has been said, the splitting of the 650-g/cm^2 absorber minimized this effect, but it should not be *a priori* assumed that it eliminates it since nuclear interactions with small energy transfer preceding large showers are rare, but by no means

TABLE I. Fractional contributions (in percent) of the three main corrections to the observed shower rates: knock-on showers, chance coincidences, and μ-meson-initiated nuclear interactions.

Absorber thickness (g/cm^2 Pb)	Type of correction	Shower group T_1P^{221}	T_1P^{222}	T_1P^{321}	T_1P^{332}	T_1P^{333}
0	knock-on	−3.4	− 0.1	−0.8	⋯	⋯
	coincid.	−0.1	− 0.5	⋯	⋯	⋯
	μ mesons	⋯	⋯	⋯	⋯	⋯
198[a]	knock-on	+2.4	+10.3	+9.8	+10.4	+10.4
	coincid.	−1.7	− 0.9	⋯	⋯	⋯
	μ mesons	−0.8	− 1.0	−1.0	− 1.1	− 1.1
440	knock-on	−4.4	+10.2	+8.6	+10.4	+10.4
	coincid.	−4.2	− 3.6	⋯	⋯	⋯
	μ mesons	−2.7	− 3.7	−3.7	− 4.0	− 4.0
650[a]	knock-on	−9.2	+10.1	+8.3	+10.4	+10.4
	coincid.	−6.5	− 8.1	⋯	⋯	⋯
	μ mesons	−4.3	− 5.5	−5.5	− 6.4	− 6.4

[a] The values listed refer to the run with equal absorber distribution.

[13] E. P. George, *Progress in Cosmic Ray Physics*, edited by J. G. Wilson (North-Holland Publishing Company, Amsterdam, 1952), p. 395.

[14] Amaldi, Castagnoli, Gigli, and Sciuti, Nuovo cimento 9, 453 (1952).

[15] Barrett, Bollinger, Cocconi, Eisenberg, and Greisen, Revs. Modern Phys. 24, 133 (1952).

impossible (e.g., Froehlich *et al.*[10]). It was therefore decided to obtain quantitative evidence on the influence of the absorption effect by repeating the measurements at equal total absorber thickness in various distributions over the three shelves.

Normally, the obvious absorber arrangement would be equal shielding of the three top trays A, B, and C, so that the maximum absorber thickness over any shelf would be about 220 g/cm^2. To study the losses occurring in this thickness, the run with the smallest amount of absorber (198 g/cm^2) was carried out in three arrangements: equal distribution (66 g/cm^2 on each shelf), $\frac{2}{3}$ on top and no absorber above C, and the entire amount above A with both B and C unshielded. Similarly, in order to continue the absorption curve beyond the 200-g/cm^2 region the maximum absorber was likewise redistributed during part of the run, with 340-g/cm^2 shielding of A and approximately 90 g/cm^2 and 220 g/cm^2 above trays B and C, respectively.

If the absorption effect were negligible, the rates should remain unaffected by the redistribution. In fact, however, they changed quite noticeably. Thus, in the run with the 198-g/cm^2 absorber the shower rates, after correction for knock-ons, were the same as long as no shelf carried more than $\frac{2}{3}$ of the total, but they increased by 10–15 percent when all the absorber was placed on top of tray A. Similarly, the redistribution of the 650-g/cm^2 absorber led to an increase of the shower rates of several percent.

To evaluate the corrections for the shower rates, write $f(x)$ for the fraction of interactions in an absorber of thickness x whose secondaries are entirely stopped in x before reaching the tray immediately beneath the absorber, or else are emitted under so large an angle that they miss it. Thus $f(0)=0$, and it can be assumed that also for sufficiently small values of x the value of $f(x)$ remains negligible. The first control run with the redistributed 198-g/cm^2 absorber shows, for instance, that this is true at least up to $x=130$ g/cm^2. Generally, however, the number of nucleons emerging from the three layers x_1, x_2, x_3 either without having interacted, or after an interaction with sufficiently small energy transfer, will be (writing for the total absorber thickness $x=x_1+x_2+x_3$):

$$N=N_0\{e^{-x/\lambda}+f(x_1)(1-e^{-x_1/\lambda})e^{-(x_2+x_3)/\lambda}$$
$$+f(x_2)(1-e^{-x_2/\lambda})e^{-(x_1+x_3)/\lambda}$$
$$+f(x_3)(1-e^{-x_3/\lambda})e^{-(x_1+x_2)/\lambda}\}$$
$$=N_0e^{-x/\lambda}\{1+\sum_{i=1}^{3} f(x_1)(e^{x_i/\lambda}-1)\}. \quad (3)$$

In particular, the observed increase of the 198-g/cm^2 run when the entire absorber was placed on top of the arrangement yields values of $f(198)$ between 0.033 and 0.066 for the various shower groups. Similarly, the values of $f(x)$ corresponding to the thicknesses used in the 650-g/cm^2 run can then be obtained. Starting with an extrapolated value for the equal-absorber distribution one can compute the value of $f(x)$ for $x=340$ g/cm^2, and check the accuracy of the first choice of $f(220)$ from the plot of $f(x)$ made with this value. If necessary, an improved correction $f(220)$ is determined, and the procedure is repeated until a satisfactory smooth curve $f(x)$ is obtained. A mean free path $\lambda=164$ g/cm^2 was assumed throughout; in view of the evidence presented below favoring a long mean free path for the production of large showers this may seem incorrect, but since the criterion for the rejection by the top trays is very much less stringent than that for shower selection, it is likely that even the smallest, elastic interactions must be included so that the use of the geometrical cross section appears justified.

The results of this calculation are summarized in Fig. 3 which shows the fractions $f(x)$ of undetected showers as a function of the absorber thickness x. The corrections to be applied to the recorded rates are now easily computed with the help of Eq. (3).

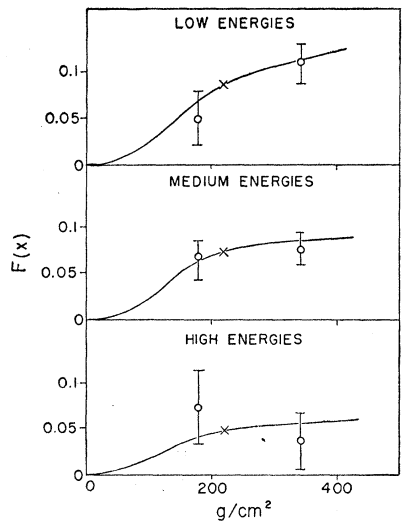

FIG. 3. Fractions $f(x)$ of showers originating in the top absorber, and not detected by the top trays, as a function of the absorber thickness x. [Crosses indicate the extrapolated values $f(220)$.]

TABLE II. Shower rates in counts/hour, corrected for spurious events and for the effect of the thick absorbers, for the various groups of showers recorded. (The errors include the statistical standard uncertainties, and the effects of the uncertainties in the corrections.)

Events \ Absorber thickness (g/cm^2 Pb)	0	198	440	650
T_1P^{221}	8.43±0.15	2.99 ±0.07	0.768±0.026	0.301 ±0.016
T_1P^{222}	5.06±0.12	1.47 ±0.048	0.391±0.019	0.160 ±0.008
T_1P^{321}	5.12±0.12	1.56 ±0.048	0.385±0.011	0.142 ±0.011
T_1P^{332}	2.42±0.08	0.624±0.029	0.151±0.011	0.063 ±0.0047
T_1P^{333}	1.69±0.08	0.373±0.022	0.107±0.009	0.0408±0.0037

2. Attenuating Curves

A summary of the shower rates, corrected for spurious events and for the effect of the thick absorbers, is given in Table II,[16] and is shown in a semilogarithmic plot in Fig. 4. The curves demonstrate clearly that the rates are not attenuated according to a simple exponential law, and that on the whole the deviations from the expected straight lines become more striking with increasing shower energy.

It should be noted that this nonlinearity may appear enhanced by the fact that the initial slope of the curves also increases with the average primary energy, and for the events T_1P^{333}, for which an average initiating energy of about 20 Bev was estimated, corresponds to a mean free path of only (135 ± 12)-g/cm^2 Pb: significantly below the "geometrical" value of 164 g/cm^2. This, at the first moment, dismaying result may, however, not be unreasonable. If, for example, the commonly neglected correction for the effect of thick absorbers is omitted, an initial slope consistent with the geometric mean free path of 164 g/cm^2 is found. However, since the above correction increases with increasing absorber thickness, by omitting the correction the deviation from linearity of the curves becomes larger yet. Furthermore, in reviewing the data obtained in earlier work (e.g., Walker,[7] Boehmer and Bridge,[9] and Gottlieb[17]) one finds with a high consistency mean free paths shorter than those corresponding to the geometrical nuclear cross section, although in many cases including that value within the limits of the experimental errors; but there seems to be no experiment in which a longer mean free path was observed. This accumulated evidence should perhaps not be considered as purely accidental, and it may well be that at very high primary energies the true interaction cross section exceeds that usually designated as "geometrical." In any case, even if the curves were drawn with an initial slope of 164 g/cm^2, they would still be far from linear; the over-all attenuations between 0 and 650 g/cm^2 for the five groups of showers corresponds to mean free paths still significantly in excess of that value.

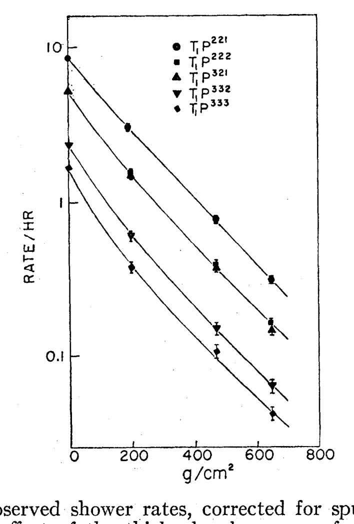

Fig. 4. Observed shower rates, corrected for spurious events and for the effect of the thick absorbers, as a function of the absorber thickness.

It must therefore be assumed that the attenuation curves are the result of the superposition of at least two curves. It is not possible to explain the observed curvatures in terms of a known background radiation superimposed on the N radiation. Mu mesons would be the only known particles capable of giving such a background. However, the effect of spurious contribution of mu mesons has been carefully evaluated. In all cases of importance—with the exception of mu-meson-initiated nuclear events—their effect was shown to be largest for the lowest-multiplicity shower events. The magnitude of the correction for mu-meson-initiated nuclear showers is small, and even assuming a reasonable error in the estimate of its effect could in no way explain the magnitude of the observed curvature in the high-multiplicity events. Furthermore, the attenuation and the relative abundance of a background radiation capable of explaining the observed effect does not fit anything that can be expected from the mu meson component.

It must, therefore, be concluded that the observed attenuation curves are the result of the superposition of curves belonging to different components of the N radiation, and that at least one of them has a mean free path for shower production which is considerably longer than that of the proton component.

3. Barometer Effect

Since the value of the mean free path affects the branching ratio between nuclear interaction and decay—a larger value obviously favoring the latter—a consequence of a longer mean free path of one of the constituents of the N component that might be detectable, is a changed barometer effect of the "filtered" radiation emerging from a thick absorber. The N component, thus enriched in the less interacting particles, should show a reduced barometer coefficient, and a simple calculation shows that the magnitude of the effect is just big enough to make its detection possible, under favorable circumstances, in an experiment of the kind reported here. It was, therefore, decided to analyze the data of the run with the heaviest lead shielding with respect to pressure variation, and to determine the barometer coefficient of the high-energy group. Pressure records were obtained with each rate reading,[18] and the barometer coefficient was calculated according to

[16] The errors quoted include the statistical standard uncertainties, and the effects of the uncertainties in the corrections.

[17] M. B. Gottlieb, Phys. Rev. 82, 349 (1951).

[18] We are indebted to Dr. H. S. Bridge and Mr. E. Todd for supplying us with copies of their continuous pressure records taken with a Bendix Aviation Corporation Micro-Barograph accurate to 0.01 inch of Hg.

standard procedure.[19] Unfortunately, earlier indications of a reduced barometer effect were not borne out by the final analysis which gave for the high-energy group a coefficient of $-(10.8\pm3.0)$ percent, consistent both with the N component value of -11.7 percent and with the expected smaller value for the "enriched" mixture (8–9 percent).

IV. DISCUSSION AND CONCLUSIONS

1. Mean Free Path for Shower Production

For a quantitative analysis of the data presented above, the following basic assumptions will now be made:

(1) After correction for spurious effects, all the recorded showers are due either to proton or to π meson primaries of high energy. Other possible components of the N radiation can be neglected.

(2) In the high-energy region concerned, showers produced by those two kinds of primaries do not differ appreciably in their structure and composition. The detection probability P for the two contributing types of showers is, therefore, the same.

With these two assumptions, the rate R_t of showers observed under an absorber t can be written as follows:

$$R_t=N_0P\{\alpha(1-e^{-\xi/\Lambda})\cdot e^{-t/\Lambda}+(1-\alpha)\cdot(1-e^{-\xi/\lambda})\cdot e^{-t/\lambda}\}, \quad (4)$$

where N_0 stands for the total number of N particles incident from the air, α for the fraction of π mesons among them, ξ for the thickness of the production layer, Λ and λ for the mean free path for shower production by π mesons and by protons, respectively. Writing A_t for the attenuation ratio $A_t=R_t/R_0$ (R_0=rate at $t=0$), one has

$$A_t=\frac{\alpha(1-e^{-\xi/\Lambda})e^{-t/\Lambda}+(1-\alpha)(1-e^{-\xi/\lambda})e^{-t/\lambda}}{\alpha(1-e^{-\xi/\Lambda})+(1-\alpha)\cdot(1-e^{-\xi/\lambda})}. \quad (5)$$

A general method for evaluating such an equation for Λ and a test for consistency with the assumption that the attenuation of the observed rate with increasing absorber thickness consists of the superposition of the exponential attenuation of two components with different mean free paths, is to subtract the contribution of one of the components and demand that a semi-logarithmic plot of the remainder *vs* the absorber thickness is a straight line. Letting

$$\beta=\frac{\alpha(1-e^{-\xi/\Lambda})}{\alpha(1-e^{-\xi/\Lambda})+(1-\alpha)(1-e^{-\xi/\lambda})},$$

then

$$A_t=\beta e^{-t/\Lambda}+(1-\beta)e^{-t/\lambda}, \quad (6)$$

where β represents the fraction of the rate R_0 which is π meson initiated. First from the fact that under the smallest absorber thickness $A_{t1}=198$ g/cm^2 the N-component beam still consists predominantly of protons, it is a good approximation to set $A_{t1}=A_1=\exp(-t_1/\lambda)$ or generally $\exp(-t/\lambda)=(A_1)^{t/t_1}$. Now if the experimental data are consistent with the superposition of two exponential functions, one should be able to adjust the parameter β so that, employing the experimental data for the A's a plot of the function $G(\beta,t)=\ln\{A_t-(1-\beta)(A_1)^{t/t_1}\}$ *vs* t is a straight line. The negative reciprocal of the slope of this graph is, of course, Λ.

In order to be able to put upper and lower limits on the value of Λ thus obtained, the following procedure was adopted: For each β_i chosen, two points of the function $G(\beta_i,t)$ were plotted at each absorber thickness employing the values $A\pm$ one standard statistical error, as given in Table II. Now, remembering that at the point $t=0$ no statistical error appears in $G(\beta,0)$, the above procedure then immediately indicates whether for a particular β_i chosen, the points $G(\beta_i,t)$ may be connected by a straight line, and the upper and lower limit of its slope should such a straight line exist. Regions of interest were investigated taking β in steps of 0.001.

Combined averages of T_1P^{222} and T_1P^{321} were used to determine Λ for the medium-energy group and T_1P^{332} and T_1P^{333} for the high-energy group. It was found that straight lines exist for the graph of $G(\beta,t)$ *versus* t only for values of β between 3 and 11 percent for the high-energy group. Similarily in the medium-energy case only values of β between 2 and 7 percent would lead to straight lines. In the latter case, where the contribution from π mesons is expected to be already small, the comparatively large experimental errors permitted the determination of only a lower limit for Λ; the upper limit yielding the trivial solutions $\Lambda=\infty$. The solutions for Λ thus obtained are:

High-energy group:

$$\Lambda_{\min}=320 \text{ g/cm}^2,$$
$$\Lambda_{\text{Av}}=425 \text{ g/cm}^2,$$
$$\Lambda_{\max}=900 \text{ g/cm}^2;$$

Medium-energy group:

$$\Lambda_{\min}=328 \text{ g/cm}^2,$$
$$\Lambda_{\text{Av}}=425 \text{ g/cm}^2.$$

Λ_{Av} was obtained using the mean value of the A's for which a straight line could only be obtained for $\beta=9$ percent and 5 percent for the high- and medium-energy groups respectively. The lower limit of Λ was found for $\beta=11$ percent and 7 percent respectively, while the upper value of Λ comes from $\beta=4$ percent in the case of the high-energy group.

For the low energy events T_1P^{221}, it is seen from the graphs of Fig. 4 that no appreciable π meson component appears to be present in the incident beam.

[19] E.g., L. Janossy and G. D. Rochester, Proc. Roy. Soc. (London) A181, 186 (1944); L. Janossy, *Cosmic Rays* (Clarendon Press, Oxford, 1948).

Accordingly, the data have not been analyzed for π meson mean free path.

2. Summary

The fact that even the minimum mean free paths compatible with the data obtained in this experiment are still considerably larger than those corresponding to a geometrical cross section, requires some attention. It is certainly in disagreement with the conclusions of Duller and Walker[6] who find mean free paths for secondary interactions ranging from (1700_{-500}^{+1200}) for 1-Bev particles, to the near-geometrical value of (180_{-45}^{+85}) at $\gtrsim$4-Bev initiating energy. If the present results are taken at their face value, it would thus appear that with further increasing energy, the cross section for shower production begins to decrease again.

Before accepting such a conclusion, one must carefully examine its basis. As it has been emphasized before, the experiment reported here is designed to study the production of showers of high multiplicity and high penetration, and hence from its results the cross section for this process is obtained, but not that for interaction with a nucleus nor even for "shower production." The identification of the two cross sections: "interaction" and "production of large showers," may appear plausible because of its validity for proton collisions, but it is certainly not necessarily correct. One can easily conceive of at least two ways in which the characteristic features of the collisions of the two kinds of particles may differ: first, although the maximum energy transfer in a collision between a π meson of about 10 Bev and a nucleon is practically the same as that in a collision between two particles of nucleonic mass, the average number of secondaries produced and the multiplicity distribution may differ, with a bias in favor of smaller showers for π meson primaries. Secondly, energetic π mesons may dissipate an appreciable fraction of their energy—significantly more than protons—in the production of particles other than secondary π mesons. This second possibility has support in the results of Schein[20] who observed the production of hyperons of the V_1^0 type by π mesons of only 230 Mev. It may well be that this process becomes quite frequent for π mesons of 10 Bev and higher energies, and that, therefore, in these collisions less dense showers are originated. In both cases, the second basic assumption for the evaluation as presented in IV.1—that of equal detection probability for proton showers and for π-meson showers—would not hold, or better it would apply only to an arbitrary selection of the π-meson interactions.

In conclusion, it can therefore be said that the results of this experiment should be interpreted as a proof for fundamental differences between the characteristics of the interactions of the various components of the N radiation at very high energies. As the mean free path for the production of large penetrating showers, which is geometrical for protons, is larger than 328 g/cm^2 and 320 g/cm^2, respectively, for π mesons of average energy around 8 Bev and 15–20 Bev. But the present experiment does not yield evidence for the nature of these differences; a study especially designed for this purpose would have to supplement it.

TABLE III. Contributions (in counts/hour) of various primary energy intervals to the groups of showers recorded.

Events / E_p (Bev)	T_1P^{221}	T_1P^{222}	T_1P^{321}	T_1P^{332}	T_1P^{333}
1	0.60	0.14	0.12	...	...
1– 2	2.27	0.82	0.80	0.05	...
2– 4	2.93	1.32	1.21	0.14	0.01
4– 6	1.83	1.22	1.20	0.35	0.10
6–10	1.26	1.00	0.85	0.47	0.23
10–20	0.89	0.88	0.78	0.71	0.66
>20	0.45	0.44	0.86	0.84	0.82
Total (R_{cal})	10.23	5.82	5.82	2.56	1.82
R_{exp}	8.43±0.15	5.06±0.12	5.12±0.12	2.42±0.08	1.69±0.08

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VI. APPENDIX: AVERAGE ENERGIES OF THE SHOWERS

The two main obstacles in an attempt to estimate the average primary energy from the shower rate are uncertainties (a) in the flux of the N component, and (b) in the determination of the detection probability.

As to the first, both theoretical arguments (e.g., Messel and Ritson,[21] Clemental and Ferrari[22]) and direct observation[10] have now established that at mountain altitude, the spectrum of the N component resembles, in the energy region of about 10 Bev, very closely that of the primary cosmic radiation, and its intensity can be obtained in a good approximation from the primary flux under the assumption of exponential attenuation with a mean free path of 117 g/cm^2. Accordingly, the calculations were based on the energy spectrum of Singer,[23] normalized to an integral directional intensity of 0.85×10^{-4} particles/cm^2 sec sterad.

As to the second, the main difficulty lies in the fact that data on the characteristics of showers in lead, like

[20] M. Schein, *Proceedings of the Third Annual Rochester Conference on High Energy Physics* (Interscience Publishing Company, New York, 1953).

[21] H. Messel and D. M. Ritson, Proc. Phys. Soc. (London) **A63**, 1359 (1950).

[22] E. Clemental and F. Ferrari, Nuovo cimento **9**, 572 (1952).

[23] S. F. Singer, Phys. Rev. **80**, 47 (1950).

multiplicity distribution, range of secondaries etc., are available only up to primary energies of 10 Bev,[10] so that extrapolations had to be used. The method of calculation was that previously applied:[8,10] given the (differential) energy spectrum $s(E)$ and the multiplicity distribution $M(E)$, one computes first for each multiplicity the average numbers m, m', m'' of shower particles present at the levels of the trays D, E, and F, taking into account cascade multiplication as well as absorption and geometrical losses. From these figures, the probability $P(n,n',n'')$ is then obtained that this shower will discharge n or more, n' or more, and n'' or more counters of the three shower-detecting trays, and consequently the contribution of the energy interval (E,dE) to the events of the type $T_1P^{n,n',n''}$ is

$$dR_{n,\,n',\,n''}=s(E)\sum_{\mathrm{M}}M(E)P(n,n',n'')dE.$$

Integration over E yields the "calculated rates" which may serve as a check on the reliability of the method, and similarly, the average primary energies are now directly computed. Table III summarizes the contributions R_{cal} in counts/hour for the various shower groups. In the last row, the observes rates R_{exp} of the unshielded run are added for comparison, and it is seen that the agreement is surprisingly good: in no case are the deviations larger than 20 percent.

The average energies computed from this distribution are:

$$\begin{aligned}\text{Events } T_1P^{221}&:\langle E_p\rangle= 5.3 \text{ Bev},\\ T_1P^{222}&:\langle E_p\rangle= 7.9 \text{ Bev},\\ T_1P^{321}&:\langle E_p\rangle= 8.7 \text{ Bev},\\ T_1P^{332}&:\langle E_p\rangle=16.4 \text{ Bev},\\ T_1P^{333}&:\langle E_p\rangle=20.4 \text{ Bev}.\end{aligned}$$

If the agreement between all the observed and calculated rates is not to be considered as wholly fortuitous, these energy values should likewise be accurate within about 20 percent.

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Penetrating Showers in Light and Heavy Elements*

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A large cloud chamber fitted with one thick "producer plate" of light materials and with eight $\frac{1}{4}$-in. lead plates was operated at Echo Lake, Colorado (altitude 3260 m) to record penetrating showers originating in three light elements (Li, C, Al) and in lead. The average primary energies, estimated with the "F-plot" method of Duller and Walker, were between 20 and 25 Bev. From a comparison of the shower rates in the producer plates and in the top lead plate, upper and lower limits for the transparencies of the light elements are obtained, the latter if correction is made for the differences in the primary energies. More reliable values, however, are derived from a direct comparison of the rates in the three light elements. This yields a value of $\lambda=(2.55\pm0.25)\times10^{-13}$ cm for the mean free path in nuclear matter, and accordingly transparencies of (15±7) percent, (12±4) percent, and (6±3) percent, respectively for Li, C, and Al. It is shown that second-generation effects are not negligible even in light nuclei, and that they can account for the observed differences in the average multiplicities of showers initiated in the four elements studied. At a primary energy of 25 Bev, the average multiplicity of a nucleon-nucleon collision is about 3.4. The contribution of the π mesons to the intranuclear cascade is very small. This agrees well with the observed mean free path for shower production in the lead plates by secondary particles, which varies from (380±35) g/cm² for secondaries from light elements to (475±70) g/cm² for secondaries from lead. Since nuclear scattering in the lead plates was observed for secondaries from the light elements and from lead with mean free paths of (370±35) g/cm² and (300±36) g/cm² respectively, the total interaction mean free path in lead, comprising both scattering and secondary showers, is very nearly the same for secondaries emerging from light and heavy elements—about (190±25) g/cm²—and is also close to the geometrical value.

I. INTRODUCTION

MEASUREMENTS of the cross section for shower production as a function of the atomic weight of the target nucleus are of interest for a variety of reasons, but in particular because they provide the most direct means of studying the problem of the "mean free path of nucleons in nuclear matter," and that of the significance of second-generation effects in collisions of a nucleon with a nucleus (the intranuclear cascade). From the latter effect, evidence can also be derived on the interaction mean free path of π mesons in nuclear matter.

Since Cocconi's early paper on this subject,[1] it has become customary in high-energy interactions to use the concept of nuclear transparency as a quantitative expression of the deviation of the collision cross section from the geometrical nuclear cross section. Of the numerous papers on this subject, the experiments of Rolloson[2] and Froman *et al.*[3] with water may be men-

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[1] G. Cocconi, Phys. Rev. **75**, 1075 (1949).
[2] G. W. Rolloson, Phys. Rev. **87**, 71 (1952).
[3] Froman, Kenney, and Regener, Phys. Rev. **91**, 707 (1953).