Coefficient of Expansion of Liquid Helium II*

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The coefficient of thermal expansion of liquid helium II was measured from the λ point down to 0.85°K. Near the λ point the coefficient varies very rapidly with temperature and may tend to minus infinity at the λ point. The coefficient becomes positive below 1.15°K. The results are discussed in relation to Landau's theory and a value for $(\rho/\Delta)(\partial\Delta/\partial\rho)$ deduced.

1. INTRODUCTION

THE coefficient of thermal expansion of liquid helium under its saturated vapor pressure has not hitherto been measured directly, but it may be derived from the density measurements of Onnes and Boks[1] along the vapor pressure curve, or by extrapolating the density measurements obtained at higher pressures by Keesom and Keesom.[2] From these two investigations one obtains several points on a density *versus* temperature graph, and it is immediately obvious that there is a maximum near the λ point and the coefficient of expansion is positive above the λ point but negative from the λ point down to 1.2°K. However, there are too few points for the coefficient to be derived accurately and, in particular, the interesting region in the immediate vicinity of the λ point is inadequately covered, as the observations there are spaced 0.1°K apart.

Several recent investigations have indicated the need for more accurate values of the coefficient of expansion. It is important in Pippard's[3] theory of the attenuation of first sound near the λ point, and is also needed to derive certain parameters used in the Landau-Khalatni-kov[4] theories of the viscosity of the normal component and the attenuation of first and second sound. It is also essential to a proper discussion of the nature of the λ transition.[5-7] Moreover, it is now clear[8] that liquid helium is very similar to a Debye solid below 0.6°K and its coefficient of expansion should then be positive.[9] Using recent measurements of the variation of the velocity of first sound with pressure, it is even possible to predict the magnitude of this positive coefficient.[10]

The measurements described in the present paper originally extended down to 1.2°K, but the results suggested that the coefficient would change sign near 1°K and we were therefore encouraged to extend the measurements to lower temperatures. The coefficient did, in fact, become positive below 1.15°K.

2. METHOD

The dilatometer, shown in Fig. 1, was completely immersed in a bath of liquid helium. The copper chamber A was filled with liquid helium by condensing in pure gas through the Monel tube D until the meniscus stood in the glass capillary C, and a valve at the room temperature end of D was then closed so that A, C, and D formed a closed system. The procedure was to make a small measured change in bath temperature and to use a cathetometer to measure the resulting change in position of the meniscus in C. The temperature of the bath was deduced from its vapor pressure as read on a butyl phthalate manometer, using the 1949 scale.[11] In addition the carbon resistance thermometer R was used to detect changes in bath temperature as small as 10^{-5} °K and this made it possible to maintain the temperature steady to better than 5×10^{-5} °K by adjusting a fine needle valve in the pumping line. In a preliminary investigation, a differential oil manometer was used to show that the difference in vapor pressure between the liquid in A and in the bath was less than 0.2 mm of oil in the steady state, corresponding to a temperature difference of less than 0.0002 °K near the λ point. The temperature inside A was observed to follow a change of bath temperature within a time of the order of one minute.

Several disturbing effects resulted from changes in the mass of gas contained in the dead space above the meniscus in C. As the bath level fell the mean temperature of the Monel tube D slowly increased, the mass of gas contained in it slowly decreased and the meniscus drifted slowly upwards. It was difficult to make a satisfactory correction for this effect, so the dead space was reduced by inserting inside D a glass capillary with a bore of 0.1 mm, and the upward drift then became negligibly slow as long as the bath level was not allowed to fall below the bulb E_1. For a similar reason, the bath temperature was changed by adjusting a needle valve

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[1] H. Kamerlingh Onnes and J. D. A. Boks, Leiden Comm. No. 170b (1924).
[2] W. H. Keesom and A. P. Keesom, Physica 1, 128 (1933).
[3] A. B. Pippard, Phil. Mag. 42, 1209 (1951).
[4] I. M. Khalatnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) 23, 8 (1952).
[5] W. H. Keesom, *Helium* (Elsevier, Amsterdam, 1942), p. 255.
[6] K. R. Atkins and C. E. Chase, Proc. Phys. Soc. (London) A64, 833 (1951).
[7] K. R. Atkins, Phil. Mag. Suppl. 1, 169 (1952).
[8] Kramers, Wasscher, and Gorter, Physica 18, 329 (1952).
[9] L. Goldstein, Phys. Rev. 89, 597 (1953).
[10] K. R. Atkins and R. A. Stasior, Can. J. Phys. 31, 1156 (1953).
[11] H. van Dijk and D. Shoenberg, Nature 164, 151 (1949).

FIG. 1. The dilatometer. A, copper chamber. C, glass capillary. P_1 and P_2, platinum tubes sealed into C. D, Monel tube leading to a valve at room temperature. R, carbon resistance thermometer.

Figure 2 shows how the position of the meniscus varied with temperature under these circumstances. The difference between this curve and a similar curve with A in place was assumed to arise entirely from the expansion of the liquid in A. This procedure had the additional advantage that it eliminated the expansion of the liquid in the bulb E_2.

The peculiar shape of the curve in Fig. 2 requires comment. The initial fall of the meniscus between K and L was caused partly by the negative coefficient of expansion of the liquid in the bulb E_2 and partly by the evaporation of liquid into the dead space to increase the vapor pressure. The subsequent rise from L to M was much more pronounced before the dead space was reduced and it may be explained in the following way. Film flow up the outside of D cooled the dead space immediately above the surface of the bath, but, as the rate of film flow decreased on approaching the λ point, the volume of dead space cooled in this way decreased and some of the gas condensed out, producing the rise in the meniscus. The exact position of the λ point was determined with the aid of the following phenomena. The bath temperature was allowed to rise slowly and the reading of the carbon resistance thermometer was continually observed. Below the λ point the temperature was uniform throughout the bath and the temperature of the resistor followed the temperature at the surface, but as soon as the λ point was reached a vertical temperature gradient began to establish itself and the temperature in the vicinity of the thermometer rose rapidly. The bath was then cooled again and steadied down at a temperature about 5×10^{-5} °K below the λ point as determined by the above method. It was then observed that, even though the vapor pressure of the bath and the reading of the carbon resistance thermometer were both constant, there was a steady fall of the meniscus as shown by the portion MN of the curve in the inset to Fig. 2. Presumably, the good thermal conductivity of the liquid column in the capillary C had been destroyed so that, even though the temperature

in the pumping line rather than by varying the power in a heater immersed in the bath. The latter procedure varied the amount of gas flowing up the cryostat and hence disturbed the temperature distribution along the Monel tube D, producing an erratic variation in the mass of gas in the dead space. The dead space at room temperature was about 5 cm^3, but, being at a much higher temperature, contained only a small fraction of the total mass of gas and was unimportant as long as the room temperature was not allowed to change by more than 1°K per hour. After all these precautions had been taken the drift of the meniscus at a steady temperature was less than 0.1 mm per hour.

An unavoidable effect of a similar nature still remained. As the temperature was changed in order to make an expansion measurement, the change in vapor pressure resulted in a change in the mass of gas in the bulb E_1. The necessary correction was evaluated by making measurements with the copper chamber A removed and the end of the platinum tube P_2 sealed off.

FIG. 2. The variation of the meniscus position with temperature in the absence of the chamber A. Capillary 2.

of the liquid in E_2 remained constant, the liquid near the meniscus was slowly warmed by the influx of stray heat and evaporated into the dead space. Similar phenomena were observed with the copper chamber A in position and the onset of the downward drift of the meniscus was subsequently used to indicate that the liquid in A had reached the λ point.

Two corrections were applied to obtain the final results. As the meniscus rose in the capillary it displaced gas which condensed out and produced a small additional rise. The necessary correction was never greater than 1 percent and could be estimated with sufficient accuracy from the known density of the gas. Also, as the length of the liquid column in the capillary changed, the hydrostatic pressure on the liquid in A changed sufficiently to alter its density significantly, since liquid helium has a high compressibility.[12] For the finest capillary used this effect required a correction as high as $3\frac{1}{2}$ percent.

The measurements below 1.2°K were made in a special cryostat which has been described elsewhere.[13] In this case the glass capillary C was 60 cm long and extended up to liquid air temperatures. The temperature was measured by a carbon resistance thermometer which had been calibrated against the magnetic susceptibility of chrome alum.

The apparatus proved unsuitable for measurements in the helium I region because of the disturbing effect of bubbles in the capillary C and the difficulty of obtaining a uniform temperature throughout the system.

TABLE I. 0.85°K to 2.05°K.

Temp. °K	α_s deg⁻¹ ×10⁻⁴	α_p deg⁻¹ ×10⁻⁴	α_p^ρ deg⁻¹ ×10⁻⁴	K_T cm² dyne⁻¹ ×10⁻⁸	$\gamma-1$ ×10⁻⁴
0.85	+4.70	+4.75	+4.75	1.20	1.46
0.90	4.70	4.77	4.77	1.20	1.65
0.95	4.45	4.58	4.58	1.20	1.32
1.00	3.78	3.97	3.97	1.20	0.86
1.05	3.00	3.29	3.29	1.21	0.46
1.10	1.62	2.00	2.00	1.21	0.13
1.15	−0.54	−0.01	−0.01	1.21	0.00
1.20	3.11	2.42	2.41	1.22	0.12
1.25	5.88	4.95	4.93	1.22	0.42
1.30	8.94	7.76	7.73	1.22	0.86
1.35	12.3	10.8	10.7	1.22	1.40
1.40	16.0	14.0	13.9	1.22	1.99
1.45	20.2	17.8	17.6	1.23	2.74
1.50	24.5	21.7	21.4	1.23	3.86
1.55	29.8	26.4	25.8	1.24	4.55
1.60	35.5	31.5	30.8	1.24	5.62
1.65	41.7	36.9	35.9	1.25	6.81
1.70	48.6	43.0	41.6	1.26	8.20
1.75	56.5	50.0	47.6	1.26	9.72
1.80	65.5	58.1	55.0	1.27	11.8
1.85	75.8	67.4	63.0	1.29	14.1
1.90	87.1	77.4	71.6	1.31	16.6
1.95	100.8	90.1	83.0	1.32	19.7
2.00	115.6	103.8	94.1	1.34	22.6
2.05	136.2	123.0	110	1.36	27.1

[12] We are grateful to Dr. A. B. Pippard for pointing out the importance of this correction.

[13] Atkins, Edwards, and Pullan, Rev. Sci. Instr. (to be published).

TABLE II. The vicinity of the λ point.

$T_\lambda-T$ °K	α_s deg⁻¹ ×10⁻⁴	α_p deg⁻¹ ×10⁻⁴	α_p^ρ deg⁻¹ ×10⁻⁴	K_T cm² dyne⁻¹ ×10⁻⁸	$\gamma-1$ ×10⁻⁴
0.001	−458	−440	−439	...	...
0.002	407	389	390	...	...
0.003	374	356	358	1.453	106.6
0.004	357	339	342	1.448	100.2
0.005	343	325	328	1.444	93.6
0.010	301	284	287	1.433	78.4
0.020	257	240	244	1.421	63.6
0.030	231	214	219	1.412	54.3
0.040	213	197	202	1.405	48.4
0.050	199	183	189	1.398	44.0
0.060	186	171	177	1.392	40.5
0.070	177	162	168	1.388	37.5
0.080	168	153	158	1.384	34.9

3. RESULTS

Three glass capillaries and two copper chambers were used. The internal diameters of the capillaries were: Capillary 1, 0.1016±0.0006 cm; Capillary 2, 0.0489±0.0002 cm; Capillary 3, 0.0465±0.0007 cm. Chamber 1 had a volume of 29.27±0.15 cm³ and Chamber 2 a volume of 14.68±0.07 cm³. All these figures include small corrections for the contraction between room temperature and liquid helium temperatures. Above 1.2°K measurements were made with three combinations, Capillary 1 and Chamber 1, Capillary 1 and Chamber 2, and Capillary 2 and Chamber 1, and all three sets of measurements agreed within the experimental error, suggesting that no important precaution or correction had been ignored. The temperature intervals were usually chosen so that the meniscus moved through a distance of the order of 1 cm and altogether more than 200 separate observations were made above 1.2°K. A curve was drawn through all these points and the resulting smoothed values of the expansion coefficient were estimated to have a random error of 1 percent. These smoothed values are collected in Tables I and II in the column headed α_s. The measurements below 1.2°K are also given in Table I and are shown graphically in Fig. 3. Although these latter measurements have a random error of 5 to 10 percent, they demonstrate convincingly that the coefficient changes sign near 1.15°K.

The quantity α_s measured in these experiments is the coefficient of expansion as both the temperature and pressure change along the vapor pressure curve. Various other quantities which may be derived from α_s are also included in Tables I and II. The coefficient of expansion at constant pressure, α_p, may be obtained from

$$\alpha_p = \alpha_s + \frac{\gamma}{\rho u_1{}^2}\left(\frac{dp}{dT}\right)_s, \qquad (1)$$

where $(dp/dT)_s$ is the slope of the vapor pressure curve.[11] For the velocity of first sound, u_1, we used the measurements of Atkins and Chase[6] increased by 0.8
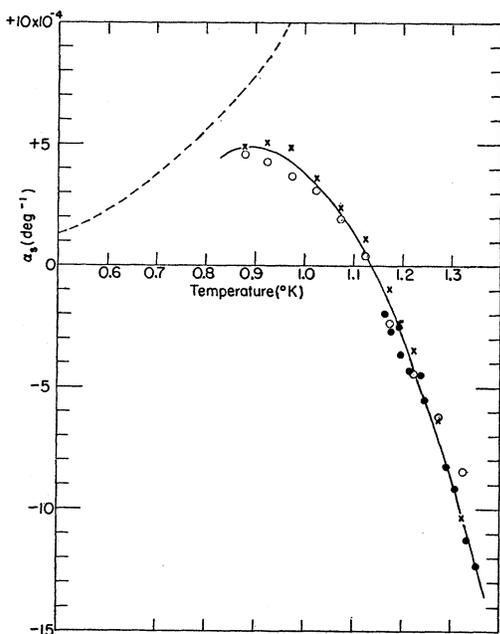
FIG. 3. The coefficient of expansion near 1°K. ○ Chamber 1, Capillary 3. ✕ Chamber 2, Capillary 3. ● Results obtained previously above 1.2°K. - - - - Phonon contribution. ——— Derived from straight line in Fig. 5.

percent as suggested by Chase.[14] γ is the ratio of the specific heats at constant pressure and constant volume and may be taken as 1.00 without introducing any significant error in the correcting term. The actual value of γ can then be deduced from

$$\gamma - 1 = T u_1^2 \alpha_p^2 / C_p. \qquad (2)$$

C_p, the specific heat at constant pressure, is equal to C_s, the measured specific heat under the saturated vapor pressure, to within 0.1 percent, but unfortunately the various measurements of C_s disagree by about 10 percent, and so there is a corresponding uncertainty in $\gamma - 1$. The isothermal compressibility, K_T, can be obtained from

$$K_T = \gamma / \rho u_1^2, \qquad (3)$$

with an error of the order of 2 percent arising mainly from u_1^2. The values quoted for all these quantities correspond to pressures and temperatures on the vapor pressure curve, but, from a theoretical point of view, it may be more correct to use values of the coefficient of expansion along a line of constant density, $\alpha_p{}^\rho$. To correct for small changes of density, one uses

$$\Delta \alpha_p = (\partial \alpha_p / \partial p)_T (\partial p / \partial \rho)_T \Delta \rho$$

$$= -\frac{1}{K_T} \left(\frac{\partial K_T}{\partial T} \right)_p \frac{\Delta \rho}{\rho}. \qquad (4)$$

The values of $\alpha_p{}^\rho$ in Table I correspond to the density

[14] C. E. Chase, Proc. Roy. Soc. (London) A220, 116 (1953).

at 0°K and zero pressure, while the values in Table II are corrected to the density on the vapor pressure curve at 2.180°K.

4. DISCUSSION

4.1 The λ Transition

Ehrenfest's[5,15] treatment of the λ point as a second-order transition assumes that the Gibb's free energy and its first-order derivatives are continuous, but that there are finite discontinuities in the second-order derivatives C_p, α_p, and K_T. He then deduces that the slope of the λ line is

$$\left(\frac{dp}{dT} \right)_\lambda = \rho (C_p{}^{II} - C_p{}^{I}) / T (\alpha_p{}^{II} - \alpha_p{}^{I}) \qquad (5)$$

$$= (\alpha_p{}^{II} - \alpha_p{}^{I}) / (K_T{}^{II} - K_T{}^{I}), \qquad (6)$$

where X^{II} is obtained by extrapolating the quantity X to the λ point on the liquid helium II side and X^I is obtained by a similar extrapolation on the liquid helium I side. Figure 4 shows the behavior of $\alpha_p{}^\rho$ in the neighborhood of the λ point and it is obviously varying so rapidly that it is impossible to make a satisfactory extrapolation in order to deduce $\alpha_p{}^{II}$. The same difficulty is experienced with K_T (Table II) and C_p.[16] We must therefore abandon the attempt to verify the Ehrenfest relations on the basis of present experimental evidence.

Tisza,[17] however, has presented an alternative treatment of λ transitions which assumes that some of the
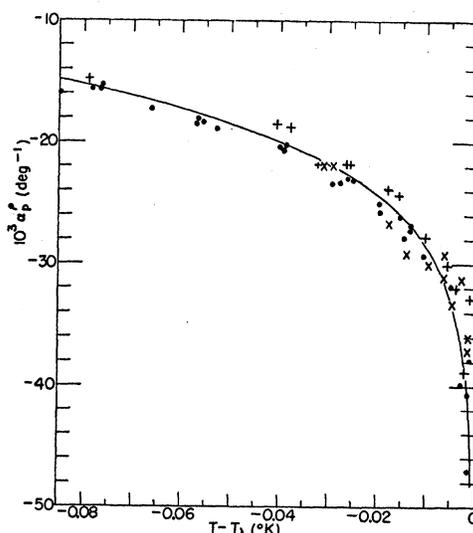
FIG. 4. The vicinity of the λ point. ● Chamber 1, Capillary 1. ✕ Chamber 2, Capillary 1. + Chamber 1, Capillary 2. ——— $\alpha_p{}^\rho$ $= 0.0008 + 0.0148 \log(T_\lambda - T)$.

[15] P. Ehrenfest, Proc. Roy. Acad. Amsterdam 36, 153 (1933).
[16] W. H. Keesom and A. P. Keesom, Physica 2, 557 (1935).
[17] L. Tisza, in Phase Transformations in Solids, edited by R. Smoluchowski (John Wiley and Sons, Inc., New York, 1951), p. 1.

second-order derivatives of the Gibb's free energy tend to infinity at the transition temperature. This is consistent with the behavior of C_p[16] and K_T[6] and is now seen to be equally consistent with the behavior of α_p (Fig. 4). In fact, between 2.1°K and the λ point, the coefficient of expansion may be represented by

$$\alpha_p{}^\rho = +0.0008 + 0.0148 \log(T_\lambda - T), \qquad (7)$$

which tends to minus infinity at the λ point (full curve in Fig. 4). The form of Eq. (7) is of particular interest because Onsager[18] has discussed a two-dimensional model of a ferromagnetic and has shown that the specific heat in the vicinity of the Curie point is $C_p = A \log(T_c - T) + B$. In the present case it is relevant to enquire whether there should not be additional terms representing, for example, the contribution of the phonons, but these terms would probably vary slowly in the vicinity of the λ point.

4.2 The Phonon Contribution

The coefficient of expansion at constant pressure is

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{V}\left(\frac{\partial S}{\partial p}\right)_T. \qquad (8)$$

In terms of Landau's theory,[19,20] the entropy S may be written as the sum of roton and phonon contributions, so that

$$\alpha_p = -\frac{1}{V}\left(\frac{\partial S_{ph}}{\partial p}\right)_T - \frac{1}{V}\left(\frac{\partial S_r}{\partial p}\right)_T$$

$$= \alpha_{ph} + \alpha_r. \qquad (9)$$

As

$$S_{ph} = \frac{16\pi^5 k^4 T^3}{45 h^3 c^3 \rho}, \qquad (10)$$

the phonon contribution to the coefficient of expansion is

$$\alpha_{ph} = \frac{16\pi^5 k^4 T^3}{15 h^3 c^3}\left(\frac{1}{c}\frac{\partial c}{\partial p} + \frac{1}{3}K_T\right). \qquad (11)$$

The velocity, c, of the phonons was assumed to be the same as the velocity of first sound extrapolated to 0°K, or 239 ± 2 m/sec.[6,14] With the assistance of Dr. Gotlieb and Dr. Chung of the University of Toronto Computation Center, the quantity $\partial c/\partial p$ was evaluated by fitting the power series $c = c_0 + a_1 p + a_2 p^2 + \cdots$ to the data of Atkins and Stasior[10] at 1.21°K. The results were $a_1 = (7.64 \pm 0.13) \times 10^{-4}$ cm^3 sec^{-1} dyne^{-1} and $a_2 = (-1.63 \pm 0.12) \times 10^{-11}$ cm^5 sec^{-1} dyne^{-2}. Equation (11) then reduces to[21]

$$\alpha_{ph} = (+1.08 \pm 0.04) \times 10^{-3} T^3 \text{ deg}^{-1}. \qquad (12)$$

[18] L. Onsager, Phys. Rev. 65, 117 (1944).
[19] L. Landau, J. Phys. (U.S.S.R.) 5, 71 (1941).
[20] L. Landau, J. Phys. (U.S.S.R.) 11, 91 (1947).
[21] This equation differs slightly from an earlier estimate (reference 10) because of the more accurate evaluation of $\partial c/\partial p$.

This is plotted as the dashed curve in Fig. 3. Below 0.6°K the entropy is almost entirely due to the phonons[8] and the total coefficient of expansion probably tends asymptotically to Eq. (12) at sufficiently low temperatures.

In the next section, we shall assume that Eq. (12) represents the phonon contribution up to 2.0°K. This ignores the variation of c and $\partial c/\partial p$ with temperature and introduces uncertainties of the order of 20 percent. The Debye temperature is 19.8°K and departures from the T^3 law therefore amount to only 2 percent at 2.0°K and have been neglected. Differences between the actual spectrum of the normal modes and the Debye spectrum might also be important but cannot be evaluated at present. These considerations are relevant because, even at 2.0°K, α_{ph} has the same order of magnitude as α_r.

4.3 The Roton Contribution

According to Landau, the roton entropy is

$$S_r = \frac{2\mu^{\frac{3}{2}} k^{\frac{3}{2}} p_0^2 \Delta}{(2\pi)^{\frac{3}{2}} \rho T^{\frac{1}{2}} h^3}(1 + 3kT/\Delta) \exp(-\Delta/kT). \qquad (13)$$

Differentiating with respect to the pressure, one obtains

$$\alpha_r = -\frac{2k^{\frac{1}{2}}\mu^{\frac{1}{2}} p_0^2 \Delta}{(2\pi)^{\frac{3}{2}} \rho T^{\frac{1}{2}} h^3 u_1{}^2}\left(1 + \frac{3kT}{2\Delta}\right) \exp(-\Delta/kT)$$

$$\times\left[\frac{1}{2}\frac{\rho\partial\mu}{\mu\partial\rho} + \frac{2\rho\partial p_0}{p_0\partial\rho} + \frac{\rho\partial\Delta}{\Delta\partial\rho} - 1 - \frac{\rho\partial\Delta}{\Delta\partial\rho}\frac{\Delta}{kT}\right.$$

$$\left. \times\left\{1 + \frac{\frac{3}{2}(kT/\Delta)^2}{1 + 3kT/2\Delta}\right\}\right] \qquad (14)$$
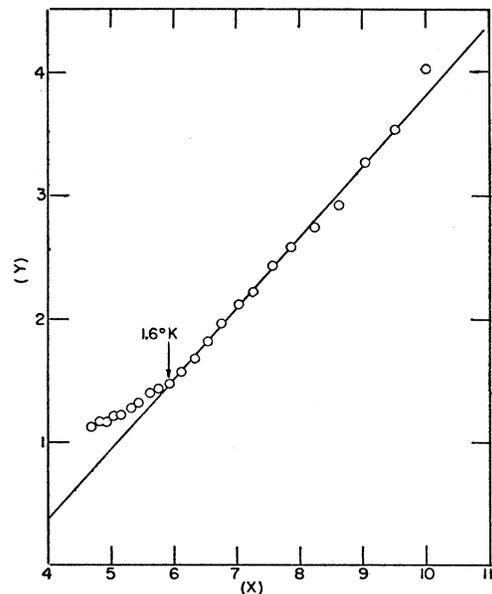
FIG. 5. Test of Landau's theory.

FIG. 6. The increase in density caused by the rotons, $\delta\rho_r$, plotted against the fraction of normal component, ρ_n/ρ.

or, alternatively,

$$-\frac{\alpha_r u_1{}^2}{S_r}=a+b\frac{\Delta}{kT}\left\{1+\frac{\frac{3}{2}(kT/\Delta)^2}{1+3kT/2\Delta}\right\}, \qquad (15)$$

where

$$a=\frac{1}{2}\frac{\rho}{\mu}\frac{\partial u}{\partial\rho}+2\frac{\rho}{p_0}\frac{\partial p_0}{\partial\rho}+\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}-1,$$

$$b=-\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}.$$

In Fig. 5, $Y=-\alpha_r u_1{}^2/S_r$ has been plotted against

$$X=\frac{\Delta}{kT}\left\{1+\frac{\frac{3}{2}(kT/\Delta)^2}{1+3kT/2\Delta}\right\}.$$

α_r was deduced by subtracting the values of α_{ph} given by Eq. (12) from the values of $\alpha_p{}^\rho$ in Table I, the coefficient of expansion along a line of constant density being preferred since it is then more likely that Δ, μ, and p_0 are independent of temperature. Khalatnikov's[4] value of $\Delta/k=8.9°$K was used, and S_r was calculated from Eq. (13) with μ and p_0 chosen so that the total entropy at 1.6°K was 0.314 joule g^{-1} deg^{-1} in agreement with

some recent measurements of Hercus and Wilks.[22] Below 1.6°K a straight line can be fitted to the points in Fig. 5, yielding

$$\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}=-0.57\pm0.06, \qquad (16)$$

$$\frac{1}{2}\frac{\rho}{\mu}\frac{\partial\mu}{\partial\rho}+2\frac{\rho}{p_0}\frac{\partial p_0}{\partial\rho}+\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}=-0.95\pm0.2. \qquad (17)$$

Above 1.6°K the theory might be expected to break down because, as the number of rotons and phonons increases, the interactions between them become more important. Khalatnikov[4] quotes $(\rho/\Delta)(\partial\Delta/\partial\rho)\sim-\frac{1}{3}$ but gives no details of his calculations.

It will be seen that the negative coefficient of expansion just below the λ point is a consequence of the fact that $(\rho/\Delta)(\partial\Delta/\partial\rho)$ is negative and the resulting negative value of α_r is numerically greater than the positive value of α_{ph}, although not overwhelmingly so. However, α_r decreases exponentially with falling temperature [Eq. (14)] whereas α_{ph} varies only as T^3 [Eq. (12)], and so, at a sufficiently low temperature, α_{ph} begins to predominate and the total coefficient of expansion becomes positive. Theories, such as that of Feynman,[23] which give an atomistic explanation of the nature of rotons could be tested by comparing their predictions of $(\rho/\Delta)(\partial\Delta/\partial\rho)$, $(\rho/\mu)(\partial\mu/\partial\rho)$, and $(\rho/p_0)(\partial p_0/\partial\rho)$ with Eqs. (16) and (17).

Integrating Eq. (9) with respect to the temperature, the density is obtained in the form

$$\rho=\rho_0+\delta\rho_{ph}+\delta\rho_r. \qquad (18)$$

Figure 6 demonstrates that $\delta\rho_r$ is very nearly linearly proportional to ρ_n/ρ, the fraction of normal component. It is difficult to say whether this has any fundamental significance, but it is interesting because the approximate proportionality between S_r and ρ_n/ρ has long been recognized.[24–26]

[22] We are grateful to Dr. J. Wilks for information on his entropy measurements in advance of publication. As his values are 10 percent higher than previous ones, there is this amount of uncertainty in S_r.

[23] R. P. Feynman, Phys. Rev. 91, 1291, 1301 (1953).

[24] F. London, Revs. Modern Phys. 17, 310 (1945).

[25] L. Tisza, Phys. Rev. 72, 838 (1947).

[26] Gorter, Kasteleijn, and Mellink, Physica 16, 113 (1950).