Reaction $p+p\rightarrow\pi^{+}+d$*

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Absolute differential cross sections for the reaction $p+p\rightarrow\pi^{+}+d$ have been measured at incident proton energies (lab system) from 310 to 338 Mev and at meson angles (c.m.) from 30° to 90°. The two final particles were counted in coincidence. The data are well fitted by the phenomenological theoretical expression

$$4\pi\frac{d\sigma}{d\Omega}(\text{c.m.})=\alpha_{10}\eta+\beta_{10}\eta^{3}\frac{(x+\cos^{2}\theta)}{x+\frac{1}{3}},$$

with $x=0.082\pm0.034$, $\alpha_{10}=(0.138\pm0.015)\times10^{-27}$ cm², and $\beta_{10}=(1.01\pm0.08)\times10^{-27}$ cm²; θ=meson c.m. angle and $\eta=p_{\pi}$ (c.m.)$/m_{\pi}c$. By measuring α_{10} we have determined directly the amount of S-wave in this reaction.

Further measurements with polarized protons of ~315 Mev are presented. These confirm directly the presence of interfering S- and P-waves and give a result $|Q|=0.39\pm0.05$ for the asymmetry $(R-L)/(R+L)$ which would be obtained at 90° c.m. with 100 percent polarized protons. The polarized and unpolarized results are used together to determine the relative phases of the S- and P-wave mesons.

Using the recent Chicago measurement of the sign of the proton polarization, the sign we observe for meson production asymmetry shows that spin up protons produce more mesons to the right than to the left. The sign of this result, according to a theoretical prediction of B. T. Feld, provides strong evidence in favor of the Fermi type meson-nucleon interaction, and against the Yang type interaction.

I. INTRODUCTION

CARTWRIGHT *et al.*[1] were the first to observe positive mesons produced in proton-proton collisions. They used nuclear emulsions to detect the mesons. The presence of a pronounced peak at the high-energy end of the meson spectrum suggested that a large fraction of the production could be attributed to the reaction $p+p\rightarrow\pi^{+}+d$. That this was indeed the case was confirmed[2] by the detection of π-d coincidences in the early stages of the experiment here reported. We did not measure absolute cross sections at that time.

Since then we have reported[3–5] absolute differential cross sections at various angles and at proton energies from 324 to 338 Mev. The summary presented here includes new data extending to 310 Mev [(T_{π}) c.m. $=9.5$ Mev]. These data show for the first time, through the excitation function and angular distribution, the presence of S-wave mesons in the above reaction. Using the same techniques, we have made further measurements[6] with a polarized proton beam of ~315 Mev. These measurements establish directly the simultaneous presence of S- and P-wave mesons and give additional information on their relative phases. This information can be used to set conditions on the p-p scattering phase shifts at these energies, for the states involved.

We have fitted all the data by the method of least squares to the phenomenological theory of Watson and Brueckner[7] as presented in the notation of Rosenfeld.[8]

II. EXPERIMENTAL TECHNIQUE

A. Unpolarized Protons

Figure 1 shows a nonrelativistic velocity vector diagram of $p+p\rightarrow\pi^{+}+d$ and displays the essential features of the particle dynamics for 342-Mev incident protons. The maximum angle that the deuterons can make with respect to the incident protons is ~6° and corresponds

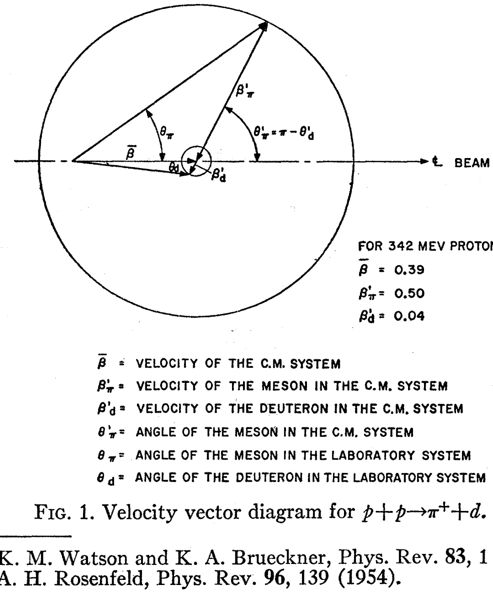

FIG. 1. Velocity vector diagram for $p+p\rightarrow\pi^{+}+d$.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

[1] Cartwright, Richman, Whitehead and Wilcox, Phys. Rev. **78**, 823 (1950).
[2] Crawford, Crowe, and Stevenson, Phys. Rev. **82**, 97 (1951).
[3] F. S. Crawford, Jr. and M. L. Stevenson, Phys. Rev. **91**, 468 (1953).
[4] Frank S. Crawford, Jr., University of California Radiation Laboratory Report UCRL-2187, April, 1953 (unpublished).
[5] M. Lynn Stevenson, University of California Radiation Laboratory Report UCRL-2188, April, 1953 (unpublished).
[6] F. S. Crawford, Jr. and M. L. Stevenson, Phys. Rev. **95**, 1112 (1954).
[7] K. M. Watson and K. A. Brueckner, Phys. Rev. **83**, 1 (1951).
[8] A. H. Rosenfeld, Phys. Rev. **96**, 139 (1954).

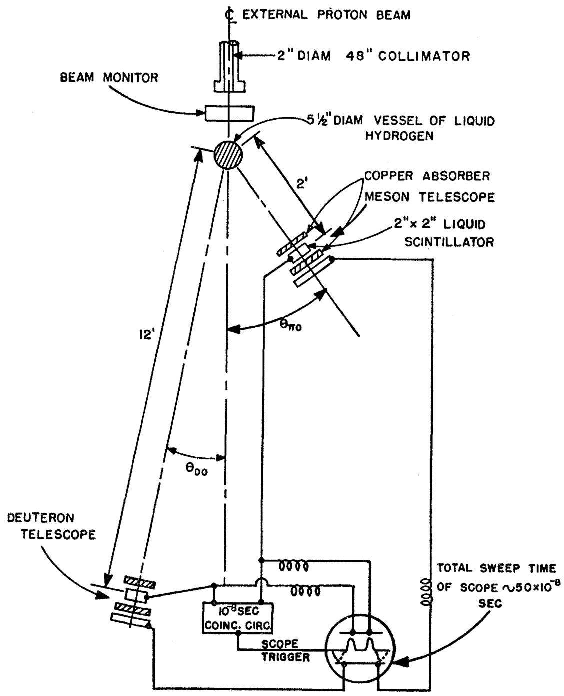

FIG. 2. Schematic diagram of $p+p\rightarrow\pi^{+}+d$ geometry and electronics.

to a c.m. angle of $\sim$90°. The meson lab angle is always equal to about one-half of the c.m. angle. These over-all features of the particle dynamics persist in general, over the entire range of proton energies investigated.

Coincidence detection of the meson and deuteron was the most important feature of the experimental technique. For this purpose two scintillation counter telescopes were used, each of which consisted of two counters. (See Fig. 2.) Appropriate copper absorbers were placed between the two counters of each telescope so that the meson and deuteron could not enter the rear counters of their respective telescopes. The rear counters were used in anticoincidence to subtract penetrating background.

The sizes and positions of the counters varied from run to run. The most frequently used front meson counter was a liquid scintillator 2 by 2 by 1.5 inches, placed 24 inches from the target. This counter defined the solid angle. The deuteron counter was usually a liquid scintillator 3 by 3 by 0.5 inches, located $\sim$140 inches from the target. The deuteron telescope was moved horizontally and vertically in increments of one counter width until almost all the deuterons were detected. It was impractical to count all the deuterons within the distribution; therefore, in practice, deuterons were usually detected only at the positions shown in Fig. 3. The number not counted in the "corners" of the distribution usually represented a correction of $\sim$20 percent.

The fact that the deuteron counter was always at an angle less than $\sim$6° made necessary the use of a liquid-hydrogen target. A CH_2 target would have caused too many diffraction-scattered protons from the carbon to enter the deuteron telescope. Most of the data were obtained with a 1.00-g cm^{-2} cylindrical liquid-hydrogen target.

A time-of-flight technique was used to eliminate most of the background from the deuteron counter by preventing high-energy protons from giving accidental coincidences. The rf fine structure of the proton beam made this technique possible. The duration of the external scattered beam produced in one frequency modulation cycle is $\sim 20\times10^{-6}$ sec. Within this time there are about 300 equally spaced bursts of protons, separated by the cyclotron rf period of 6.0×10^{-8} sec. Each burst lasts $\sim 5\times10^{-9}$ sec. At the deuteron telescope position, there was a time-of-flight separation of $\sim 2\times10^{-8}$ sec between the deuterons of $\sim$120 Mev and full-energy protons scattered at small angles into the deuteron telescope. The 10^{-8}-sec coincidence circuit adequately resolved this time separation.

Figure 2 shows schematically the procedure by which the $p+p\rightarrow\pi^{+}+d$ event was identified. A coincidence between a pulse in the front meson counter and one in the front deuteron counter triggered a fast oscilloscope. The pulses from all four counters were displayed on the scope by means of appropriate delays in the counter cables, and were photographed on a continuously moving film. The presence of a pulse in the rear counter of either the meson or the deuteron telescope classified an event as "hard" and prevented it from being classi-

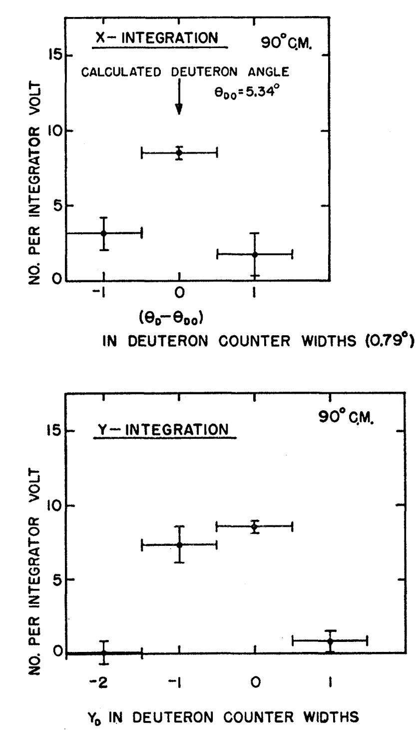

FIG. 3. Typical deuteron integration curves.

fied as a possible $p+p\rightarrow\pi^++d$ event. In order to determine the fraction of accidental coincidences among the "soft" events, in a given run, we inserted into the meson telescope an amount of cable delay equal to the time separation of 6.0×10^{-8} sec between rf pulses, and repeated the run. The number of resulting "soft accidental" traces was then subtracted from the total number of soft events to obtain the number of real meson-deuteron coincidences. The soft accidentals were usually ~10 to 20 percent of the soft reals.

Not shown in Fig. 2 are the auxiliary electronics, which provided us with data during the run, corresponding to the information that was recorded on the film.

We identified the process as $p+p\rightarrow\pi^++d$ essentially by determining the masses of the final products, after first showing, through the angular correlation, that a two-body process was involved. Identification was made by measuring the momenta and ranges of the final products. If the momentum of the incident proton is known, then a measurement of the angles of the final products determines their momenta uniquely. We determined the energy of the incident protons by measuring a Bragg curve, using the technique of Mather and Segrè.[9]

The experiment was performed at various beam energies from 310 to 340 Mev. To obtain a given energy the 340-Mev beam was degraded with appropriate beryllium absorbers. These were placed at Position A in Fig. 7 during the earlier runs and at Position B during the later runs.

Measurement of the meson and deuteron angles was accomplished by setting the meson telescope at the desired angle and then measuring the coincidence counting rate *vs* the deuteron telescope position, as is shown in Fig. 3. The width of the observed patterns agrees with that calculated from the finite meson counter width and the multiple Coulomb scattering of the meson and deuteron.

Typical range measurements of the particles in coincidence are shown in Fig. 4. Together with the angular correlation, the ranges determine the particle masses to be those of a meson and a deuteron. The absence of a step on the deuteron range curve shows that, within the errors, a single monoenergetic particle was detected. That is, the source of coincidences was the reaction $p+p\rightarrow\pi^++d$, with no detectable contribution from $p+p\rightarrow\pi^++n+p$. The shape of the deuteron range curve agrees with that calculated from the energy spread of the deuterons caused by the finite size of the target and the finite angular width of the defining meson counter. The arrows shown in Fig. 4 indicate the expected ranges of mesons and deuterons.

Further evidence that π mesons were detected came from the scope photographs taken during the running of meson range curves. When mesons were stopped in the front counter, the expected number of $\pi-\mu$ decays was observed. A pulse from the recoiling muon was observed on the trailing edge of the pulse from the stopped pion.

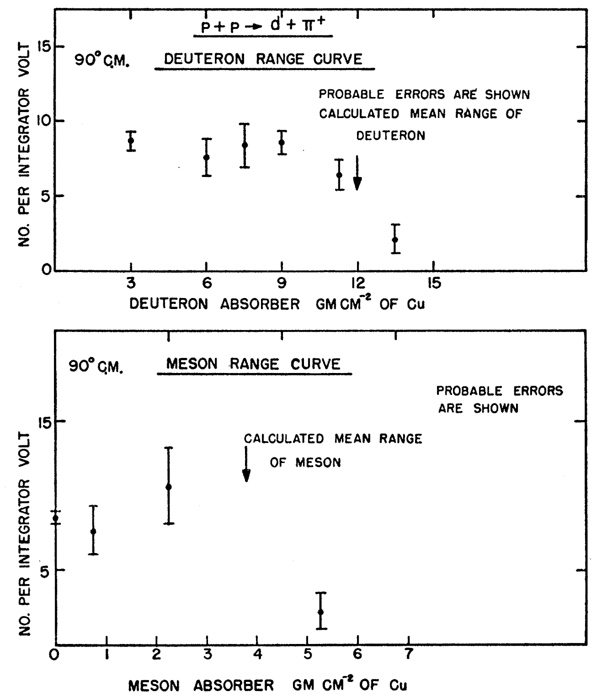

FIG. 4. Typical meson and deuteron range curves at 332 Mev.

At each angle, an absolute cross section was determined. Therefore, measurements were performed to insure that the reaction was detected with full efficiency. Pulse-height distributions for the meson and deuteron were obtained from the film data. A typical example is shown in Fig. 5. Only pulses larger than the "cutoff" value were accepted by the coincidence circuit. We estimate that less than 2 percent of the $\pi-d$ events were lost because of insufficient pulse height.

Time-delay measurements were made to insure that no events were lost because of misaligned cable delays. Figure 6 shows a typical time-of-flight plateau.

Apart from the deuteron integration correction, the $\pi-\mu$ decay in flight generally represented the largest correction made to the unpolarized beam data. This correction was usually approximately five percent. In Table I, the measurements at 312 and 325 Mev, which show relatively large correction factors, were made with a clearing magnetic field to eliminate background in part of a three-counter meson telescope. Because the path length of the mesons was longer than usual, a relatively large correction for $\pi-\mu$ decay in flight was necessary. These two are the only measurements in which a magnet was used, and which therefore differ from the general description given above.

Other small corrections made to the data include nuclear attenuation, Coulomb scattering in the walls of

[9] R. Mather and E. Segrè, Phys. Rev. 84, 191 (1951).

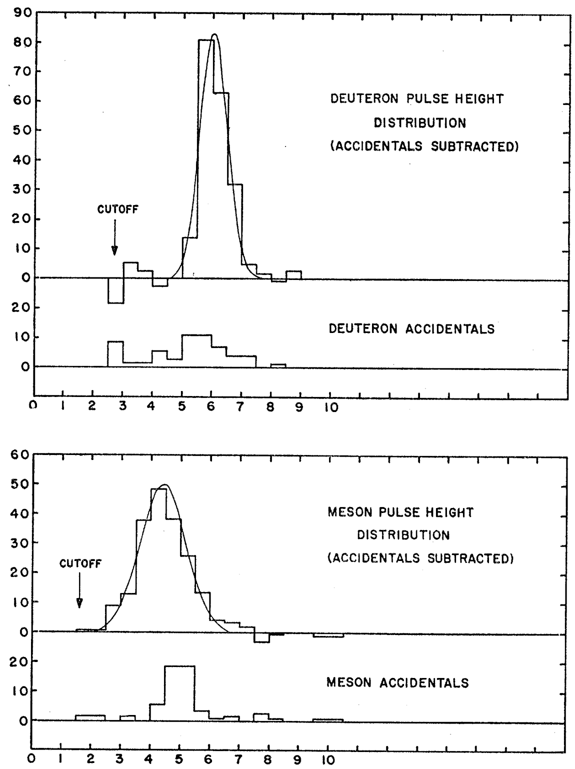

FIG. 5. Typical pulse-height distributions.

the liquid scintillators, finite aperture of the meson counter, finite beam and target, overlap error in the deuteron integration pattern, and loss of events by electronic dead time.

The statistical error of each cross section was compounded with an error of approximately three percent attributed to uncertainties in the systematic corrections. This was done in the usual manner by taking the square root of the sum of the squares of the errors. In all cases the compounding of this additional error resulted in a negligible increase in the error obtained from counting

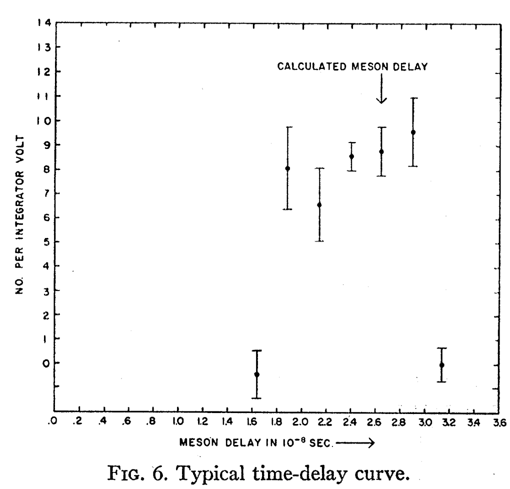

FIG. 6. Typical time-delay curve.

statistics alone. The last column of Table I tabulates all the over-all correction factors that were applied to the raw data.

In order to search for large hidden systematic errors in the over-all detection scheme, a measurement was made of the elastic p-p scattering cross section at 90° c.m. The aforementioned apparatus and technique were used. A value of (3.3 ± 0.3) mb sterad^{-1} was obtained, in good agreement with the results of Chamberlain *et al.*[10]

A more detailed account of the experimental procedure and apparatus used in the $p+p\rightarrow\pi^++d$ experiment with unpolarized protons is given in references 4 and 5.

B. Polarized Protons

With minor modifications, the techniques described in Sec. A were used with the 73 percent polarized proton beam[11] to measure the azimuthal asymmetry of $p+p\rightarrow\pi^++d$. Because of the low intensity of the polarized beam, a large meson solid angle was necessary. The meson counter was a plastic scintillator 5.75 in. by 5.75 in. by 0.25 in. It was placed 20 in. from a 0.5-g cm^{-2} slab-shaped liquid-hydrogen target. The deuteron telescope was made large enough to detect all the deuterons at one counter position, with allowances for multiple scattering.

Accidental coincidences with the polarized beam were substantially fewer than with the unpolarized beam at the same beam intensity, primarily because the time spread of the polarized beam was approximately ten times that of the unpolarized beam. The polarized beam differed in another important respect from the unpolarized beam, in that its energy spread was much greater than that of the unpolarized beam. Therefore, since the measurements were made near meson threshold, precautions had to be taken to insure that no mesons were lost in the target because of insufficient energy. In order to obtain mesons with sufficient energy to avoid such losses, the meson telescope was placed at an angle corresponding to 69° in the c.m. system rather than at 90°, the expected angle of maximum asymmetry.

It was believed that there would be less likelihood of introducing false asymmetries if, throughout the measurement, the $p+p\rightarrow\pi^++d$ reaction were detected with full efficiency. Meson-range plateaus, time-delay plateaus, and pulse-height plateaus were obtained to check this point.

In order to exclude the "unbound" reaction $p+p\rightarrow\pi^++p+n$, an absorber, of two-thirds of the deuteron range in copper, was placed in front of the deuteron telescope. In addition, the deuteron counter was made only as large as was necessary to include all the deuterons, since protons from the unbound reaction, which

[10] Chamberlain, Segrè, and Wiegand, Phys. Rev. **83**, 923 (1951).
[11] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **93**, 1430 (1954).

TABLE I. Cross-section data for $p+p\rightarrow\pi^++d$ with unpolarized protons.

Date	T_p Mev	T_π (c.m.) Mev	$\eta=\frac{p_\pi \text{ (c.m.)}}{m_\pi c}$	θ (c.m.) degrees	$4\pi\frac{d\sigma}{d\Omega}$ (c.m.) experimental mb/steradian	$4\pi\frac{d\sigma}{d\Omega}$ (c.m.) least squares mb/steradian	Over-all correction factor
1–7–'53	338	21.5	0.577	30	0.449 ±0.052	0.470	1.12
5–23–'54	338.6	21.7	0.577	57	0.239 ±0.014	0.249	1.09
1–7, 2–10–'53	338	21.5	0.577	88	0.126 ±0.021	0.119	1.05
10–17, 11–19–'52	332	19.0	0.540	30	0.423 ±0.031	0.395	1.18
8–9, 10–15–'52	332	19.0	0.540	60	0.189 ±0.020	0.202	1.12
10–16, 17, 11–17–18–'52	332	19.0	0.540	89	0.125 ±0.0093	0.106	1.13
4–1–'54	325	16.0	0.493	79	0.1014±0.0045	0.103	1.27
1–8, 2–12–'53	324	15.5	0.484	89	0.0816±0.0126	0.090	1.20
4–1–'54	312	10.4	0.392	90	0.0597±0.0037	0.066	1.45
5–23–'54	310	9.52	0.377	55	0.151 ±0.028	0.107	1.16
5–21–'54	310	9.52	0.377	69	0.0819±0.0040	0.080	1.15
5–25–'54	310	9.51	0.377	69	0.0869±0.0044	0.080	1.15

might not be excluded by the copper absorber, are expected to have a larger angular spread than the deuterons.

We will use the terms "left" and "right" to refer to the left and right sides of the incident beam as viewed by an observer looking in the direction of motion of the beam. For instance, the polarized beam is produced by a "left" scatter, as is shown in Fig. 7. The details of production of the polarized beam can be found in reference 11.

Absolute differential cross sections were measured on the left and right sides of the beam. During each asymmetry measurement approximately 10 left-right cycles were made. On each side of the beam, measurements were made on hydrogen-plus-container and on the container alone. The approximate times required were 45 minutes and 15 minutes respectively. Thus, the time per left-right cycle was approximately two hours.

During the polarized beam experiments, real coincidences were observed from the empty container, although container effects had never been observed in previous $p+p\rightarrow\pi^++d$ measurements with unpolarized protons. The effect was found to be characteristic of the target. This target was used for the first time during the asymmetry measurements. The coincidences from the "empty" styrofoam container were found to be $p+p\rightarrow\pi^++d$ events produced when the beam passed through a long path of hydrogen vapor which was in equilibrium with the liquid hydrogen of the full container. There was a vapor effect as long as there was liquid hydrogen in the "full" target. The vapor effect disappeared when the targets were truly empty. The hydrogen vapor contribution was ~10 percent of that of the liquid hydrogen and gave the same asymmetry. This is an important point, because some of the asymmetry measurements did not include empty-container data.

A continuous monitor of the meson telescope was maintained by recording separately the coincidences between front and rear meson counters. These events were due mainly to protons from p-p collisions. In this way there was provided a high-counting-rate monitor of the meson detection efficiency, the reproducibility of the meson telescope angle, and the beam polarization. The same value was obtained for the p-p asymmetry as has been observed by Chamberlain *et al.*[11] A similar monitoring procedure was used with the deuteron telescope.

The following checks were made to search for instrumental sources of asymmetry. The counters were shown to be insensitive to stray magnetic fields, and to be uniform in sensitivity. Large intentional misalignments of the counter telescopes and of the liquid-hydrogen target were made. In this way it was demonstrated that only gross misalignments could have given a false asymmetry.

A final over-all check was made by replacing the polarized beam with the ordinary "scattered" beam, which has been shown[11] to be unpolarized. All other conditions, including beam energy, electronics, counter alignment, etc., remained the same. The result, in each case, was the vanishing of the asymmetry to within statistics. See Fig. 8.

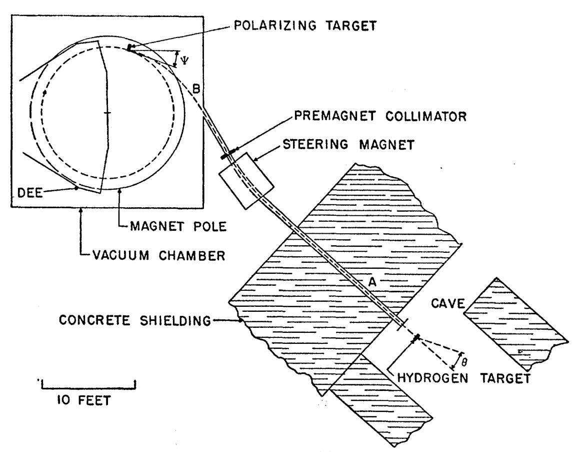

FIG. 7. Over-all geometry of the polarized proton beam.

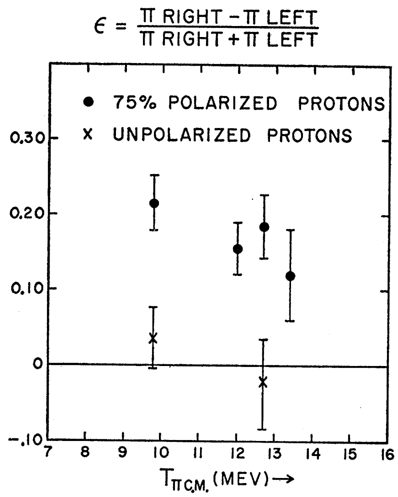

FIG. 8. $p+p\rightarrow\pi^{+}+d$ asymmetries obtained with polarized and unpolarized protons.

Those polarized beam measurements for which a blank-target subtraction was performed were used to obtain differential cross sections for unpolarized protons. These cross sections were obtained from the average of the left and right yields. The corrections described in the previous section were applied. The statistical errors of these cross-section measurements were approximately three percent. An error of five percent was assigned to cover uncertainties in the systematic corrections. These runs are included in the unpolarized beam data of Table I.

III. RESULTS WITH UNPOLARIZED PROTONS

The twelve absolute differential cross sections which comprise all of our data are presented in the sixth column of Table I, together with their rms errors. These values were fitted by the method of least squares to the phenomenological theory of Watson and Brueckner,[7] which predicts that near meson threshold, in Rosenfeld's[8] notation,

$$4\pi\frac{d\sigma\ (\text{c.m.})}{d\Omega}=\alpha_{10}\eta+\beta_{10}\eta^{3}\frac{(x+\cos^{2}\theta)}{x+\frac{1}{3}}. \qquad (1)$$

Here $\eta=p_{\pi}\ (\text{c.m.})/m_{\pi}c$, and $\alpha_{10}\eta$ and $\beta_{10}\eta^{3}$ are S- and P-wave contributions, respectively, to the total cross section; $x+\cos^{2}\theta$ is the c.m. P-wave angular distribution.[12] In order to perform a least-squares analysis easily, one needs an expression linear in the parameters. We therefore rewrite the above as

$$4\pi\frac{d\sigma\ (\text{c.m.})}{d\Omega}=a_{1}\eta+a_{2}\eta^{3}+a_{3}\eta^{3}\cos^{2}\theta.^{13} \qquad (2)$$

Table II presents the least-squares results for the a_i, their rms errors δa_i, and the correlation errors $\delta a_i\delta a_j$.[14] Inserting these values for a_i into Eq. (2), we have calculated the least-squares value of $4\pi d\sigma/d\Omega$ corresponding to each of the twelve experimental cross sections, and have listed these in column 7 of Table I, for comparison with experimental values in column 6. In order to facilitate comparison of the goodness-of-fit of the data

TABLE II. The constants a_i in mb (10^{-27} cm^2), and correlation errors $\delta a_i\delta a_j$ in (mb)^2, resulting from a least-squares fit of the data in Table I to Eq. (2).

$a_1=0.138\pm0.015$	$\delta a_1\delta a_2=-11.2\times10^{-4}$
$a_2=0.200\pm0.078$	$\delta a_1\delta a_3=3.83\times10^{-4}$
$a_3=2.44\pm0.17$	$\delta a_2\delta a_3=-42.3\times10^{-4}$

to (2) [or (1)], we have divided (2) by η^3 and plotted the result against η^{-2} and $\cos^2\theta$, in Fig. 9, so as to exhibit a plane surface in three dimensions. The experimental points are plotted there for comparison.

We can rewrite the least-square results in Rosenfeld's

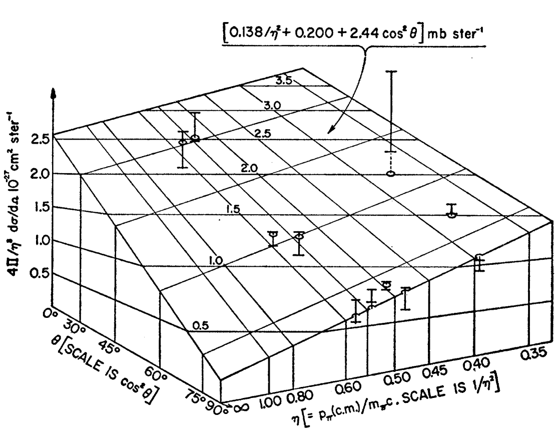

FIG. 9. Comparison of experimental points with the least-squares solution for the surface $4\pi d\sigma/d\Omega=a_1\eta+a_2\eta^3+a_3\eta^3\cos^2\theta$. [Eq. (2).]

[12] The first and second subscripts in α_{10} and β_{10} refer to the total isotopic spin of the two nucleons in the initial and final states, respectively.

[13] In all our previous cross section publications (see references 3 and 4) we believed that it was necessary to incorporate the relative excitation data at 0° of Schulz [A. G. Schulz, University of California Radiation Laboratory Report UCRL-1756, 1951 (unpublished)] in order to find the energy dependence of the angular distribution. If one inspects Eq. (2), however, he sees that a_1 and a_2 can be determined by two 90° (c.m.) points at different energies, after which a_3 can be determined by an angular distribution at any energy. Since Schulz measured only a relative excitation, then, in the spirit of Eq. (1) or (2), he measured essentially $a_1/(a_2+a_3)$. Analysis of his data gives $a_1/(a_2+a_3)=(4.8\pm4.6)\times10^{-2}$. Our data alone yield $a_1/(a_2+a_3)=(5.2\pm0.9)\times10^{-2}$, so that we no longer incorporate Schulz's data into ours. Our more accurate value is obtained mainly from our excitation points at 90° (c.m.), where the ratio of the S-wave term, a_1, to P-wave terms is greatly enhanced over that at 0°.

[14] The use of the correlation errors $\delta a_i\delta a_j$, $i\neq j$, may be illustrated by finding the least-squares rms error for $x=a_2/a_3$. Under the usual assumption of "small" errors so that differentials may be used, we differentiate, square, average, and take the square root to obtain

$$\delta x=\pm[a_3^{-2}(\delta a_2)^2+a_2^2a_3^{-4}(\delta a_3)^2-2a_2a_3^{-3}\delta a_2\delta a_3]^{\frac{1}{2}}=\pm0.034.$$

notation, Eq. (1), to obtain

$$\alpha_{10}=a_1 \quad =0.138\pm0.015 \text{ millibarns},$$
$$\beta_{10}=a_2+\tfrac{1}{3}a_3=1.01\pm0.08 \text{ millibarns},$$
$$x=a_2/a_3 \quad =0.082\pm0.034,$$
$$\eta_c=(a_1/a_2)^{\frac{1}{2}}=0.83\pm0.21.$$

IV. DISCUSSION OF UNPOLARIZED BEAM RESULTS

These experiments have exhibited directly for the first time, by measurement of α_{10}, the presence of S-wave mesons in $p+p\rightarrow\pi^++d$. It is interesting to note that a simple theoretical estimate,[15] which considers the S-wave mesons to be due to a nucleon recoil correction to dominant P-wave interaction, predicts that α_{10}/β_{10} should be of the order of the mass ratio $m_\pi/m_{(\text{nucleon})}$. Our experimental result of $\alpha_{10}/\beta_{10}=0.137\pm0.025$ substantiates this.

Another indirect predication of α_{10} has been made by Brueckner, Serber, and Watson.[16] They relate $\gamma+p\rightarrow\pi^++n$ near threshold through the π^- to π^+ photoproduction ratio in deuterium to $\gamma+n\rightarrow\pi^-+p$; through detailed balancing to $\pi^-+p\rightarrow\gamma+n$; via calculation to $\pi^-+d\rightarrow\gamma+2n$; through Panofsky's[17] branching ratio for π^- capture in deuterium to $\pi^-+d\rightarrow2n$; through detailed balancing to $n+n\rightarrow\pi^-+d$, which finally, by the assumption of charge symmetry, may be replaced by $p+p\rightarrow\pi^++d$. Using the results of Bernardini *et al.*[18] for $\gamma+p\rightarrow\pi^++n$ near threshold, and for the π^- to π^+ photoproduction ratio in deuterium, one obtains $\alpha_{10}=(0.14\pm0.05)$ mb. This is in notably good agreement with our directly measured value of (0.138 ± 0.015) mb. Within the rather large error in the predicted value, which is presumed to cover the various uncertainties in the several steps, and provided that the assumption of charge symmetry is regarded as the least certain element in the above calculation, then we may regard the good agreement between our directly measured value and the predicted value as a confirmation of the assumption of charge symmetry.

TABLE III. $p+p\rightarrow\pi^++d$ transitions that conserve total angular momentum and parity, for meson angular momenta $l<2$. The indicated angular dependences hold if a single transition is present (no interference).

Initial p-p state	Relative transition amplitude	Final π state	Dependence on meson momentum and angle
$^1S_0$	$r_0=\|r_0\| \exp i\tau_0$	P	$\eta^3\times$const
$^3P_1$	$r_1=\|r_1\| \exp i\tau_1$	S	$\eta\times$const
$^1D_2$	$r_2=1$	P	$\eta^3\times(\frac{1}{3}+\cos^2\theta)$

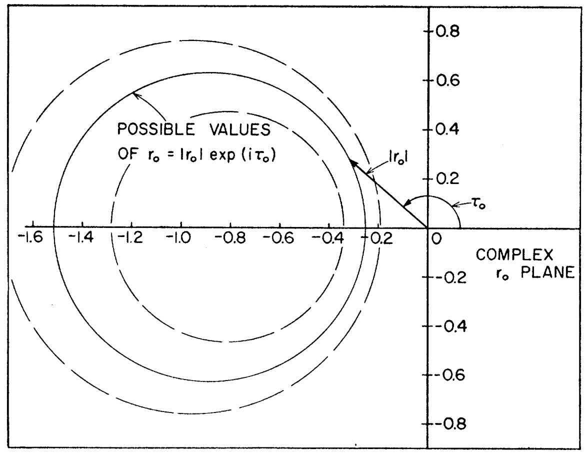

FIG. 10. Possible values of r_0 consistent with the experimental value of the P-wave angular distribution parameter, $x=0.082\pm0.034$. See Table III. The inner and outer dotted circles correspond to the errors ∓0.034, respectively.

The P-wave angular distribution is $x+\cos^2\theta$. If only the $^1D_2$ p-p states contributed to P-wave mesons, we would have $x=\frac{1}{3}$, as indicated in Table III. The measured value is $x=0.08\pm0.03$, which shows, therefore, that $^1S_0$ protons also contribute to P-wave meson production. The relative phase τ_0 between the $^1S_0$ and $^1D_2$ p-p contributions is unknown. Therefore the measurement of x serves only to put limits on the $^1S_0$ contribution. For the measured value of x these limits are given by $0.25<|r_0|<1.51$, as shown in Fig. 10. The value of τ_0 is 180° at both limits, and reaches a minimum of $\sim$133° at $|r_0|\sim0.60$. Arguments have been given[19,20] which predict predominance of the $^1D_2$ over the $^1S_0$ p-p contributions. Roughly, these arguments

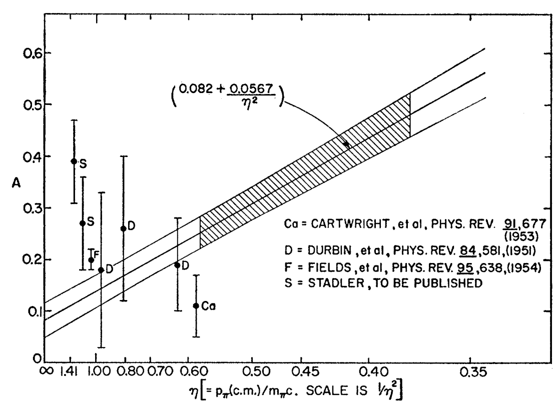

FIG. 11. Comparison of $p+p\rightarrow\pi^++d$ angular distribution parameter A obtained in this paper with results of other observers at higher energies. The curve and its rms error band were obtained from a least-squares fit of the data (not shown) of this paper alone. The cross hatching indicates the range of meson momenta covered in this paper.

[15] M. Gell-Mann and K. M. Watson, Ann. Rev. Nuclear Sci. 4, 219 (1954).
[16] Brueckner, Serber, and Watson, Phys. Rev. **81**, 575 (1951).
[17] Panofsky, Aamodt, and Hadley, Phys. Rev. **89**, 565 (1951).
[18] Goldwasser, Beneventano, Lee, Stoppini, Hanson, and Bernardini, Proceedings of the Fourth Annual Rochester Conference on High Energy Nuclear Physics (University of Rochester, Rochester, 1954).
[19] K. A. Brueckner and K. M. Watson, Phys. Rev. **81**, 923 (1952).
[20] Aitken, Mahmoud, Henley, Ruderman, and Watson, Phys. Rev. **93**, 1349 (1954).

point out that $^1D_2$ protons can produce the pion and one of the final nucleons of the deuteron in the strongly interacting state of total angular momentum $\frac{3}{2}$ and isotopic spin $\frac{3}{2}$, whereas the $^1S_0$ protons cannot. On the basis of this argument, one could take $|r_0|\sim 0.25$ and $\tau_0\sim 180°$.

The c.m. angular distribution, including both S- and P-mesons, is given by $A+\cos^2\theta$. From Eq. (2),

$$A=(a_1\eta+a_2\eta^3)/a_3\eta^3=0.082+0.056\eta^{-2}.$$

This curve, with its rms least-square error band, is plotted in Fig. 11, and its extrapolation is compared there with results obtained at higher energies by other workers. We see that the experimental points for $\eta>1$ tend to lie above the extrapolated curve. This may represent the breakdown of Eq. (1), which is only assumed to hold "near threshold," where, approximately, $\eta<1$. For instance, meson momenta higher than P-states may no longer be negligible.

The total cross section obtained by integration of Eq. (1) is given by

$$\sigma_T=\alpha_{10}\eta+\beta_{10}\eta^3=(0.138\eta+1.01\eta^3)\text{ mb}.$$

This curve is plotted in Fig. 12, where it is compared with the experimental results of other observers. Except for the lowest energy result of Durbin *et al.* at $\eta=0.625$, and perhaps the result of Cartwright *et al.* at $\eta=0.585$, we see that there is very good agreement between the predictions of Eq. (1), as fitted by our data, and other measurements extending up to cross sections an order of magnitude larger than those we have measured. Thus, the phenomenological theory's assumption of

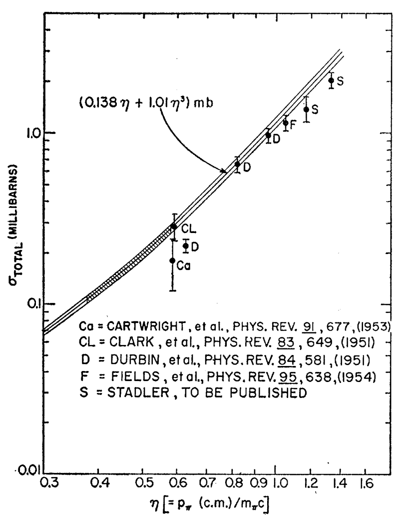

FIG. 12. Comparison of the total $p+p\rightarrow\pi^++d$ cross section obtained in this paper with results of other observers at higher energies. The curve and its rms error band were obtained from a least-squares fit of the data (not shown) of this paper alone. The cross hatching indicates the range of meson momenta covered in this paper. The experimental points shown for Clark *et al.*, Durbin *et al.*, and Stadler were obtained by detailed balancing from the inverse reaction $\pi^++d\rightarrow p+p$.

negligible energy dependence of α_{10}, β_{10}, and x is born out by experiment, at least for β_{10}, the dominant term in the total cross section for $\eta\sim 1$. We note that the experimental points, for $\eta>1$, tend to lie below the curve. This could indicate the beginning of the breakdown of applicability of Eq. (1). For instance, D-wave mesons may be not insignificant for $\eta>1$. In addition, it should be remembered that $\gamma+p\rightarrow\pi^0+p$,[21] $\gamma+p\rightarrow\pi^++n$,[22] and π-nucleon scattering[23] all go through maxima at T_π (c.m.)$\sim$125 Mev, or $\eta\sim 1.6$, and we should presumably expect a similar behavior for $p+p\rightarrow\pi^++d$. Thus, the tendency noted could be due to an approaching maximum near $\eta\sim 1.6$. Of course, the data shown in Fig. 12 barely suggest this as a possibility.

V. RESULTS WITH POLARIZED PROTONS

The raw data of the asymmetry measurements[6] on $p+p\rightarrow\pi^++d$ made with polarized protons near 315 Mev are shown in Fig. 8. Marshak and Messiah[24] have derived

$$\epsilon=(R-L)/(R+L)=PQA\sin\theta/(A+\cos^2\theta) \qquad (3)$$

for the asymmetry produced by interference between the S- and P-wave mesons. Here P is the beam polarization, $A+\cos^2\theta$ is the c.m. angular distribution, and Q is the quantity of interest in the theory. In Rosenfeld's notation,

$$Q=\frac{\eta_c\eta\sqrt{2}}{\eta_c^2+\eta^2}\sin(\psi-\tau_1). \qquad (4)$$

We note that Q, and therefore the asymmetry, disappears in the absence of S waves ($\eta_c=(a_1/a_2)^{\frac{1}{2}}=0$), or of P waves ($\eta_c=\infty$), and at certain values of the relative phase angle ψ and τ_1; $|Q|$ cannot exceed 0.70; ψ and τ_1 are given by $r_0+\surd\frac{1}{2}=|r_0+\surd\frac{1}{2}|\exp(i\psi)$ and $r_1=|r_1|\exp(i\tau_1)$. The terms r_0 and r_1 are the complex transition amplitudes for $\pi-d$ production from $^1S_0$ and $^3P_1$ p-p states, respectively, to P- and S-wave meson states, respectively, as is shown in Table III; r_0 and r_1 are defined relative to the amplitude r_2, which describes the transition from the $^1D_2$ p-p state to the P-wave meson state; r_2 is taken as unity; r_0 is written alternately as $r_0=|r_0|\exp(i\tau_0)$.

Taking $|P|=0.73$[25] (the sign of P is at present unknown), $A=(a_1\eta+a_2\eta^3)/a_3\eta^3$ from Eq. (2) and Table II, and averaging the angular dependence over the angular aperture centered at $\theta=69°$, we calculate from Eq. (3) a measured value of $|Q|$ for each of the experimental runs shown in Fig. 8. Because these values of $|Q|$ are all equal within the errors, we assume negligible varia-

[21] Walker, Oakley, and Tollestrup, Phys. Rev. **89**, 1301 (1953).
[22] Backer, Keck, Peterson, Teasdale, Tollestrup, Walker, and Worlock, Phys. Rev. **92**, 1090(A) (1953).
[23] Ashkin, Blaser, Feines, Gorwan, and Stern, Phys. Rev. **93**, 1129 (1954).
[24] R. E. Marshak and A. M. L. Messiah, Nuovo cimento **11**, 337 (1954).
[25] Chamberlain, Donaldson, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **95**, 850 (1954).

tion with energy in the region measured and average the results to obtain

$$|Q|=0.39\pm0.05$$

at an average $T_p=314$ Mev, $\eta=0.41$, (T_π) c.m.$=11.3$ Mev. Inserting $\eta_c=0.83$, $\eta=0.41$, and $|Q|=0.39\pm0.05$ into Eq. (4), we obtain

$$|\sin(\psi-\tau_1)|=0.70\pm0.14.^{26} \qquad (5)$$

P and $\sin(\psi-\tau_1)$ have the same sign.

De Carvalho *et al.*[27] have looked for an asymmetry in meson production while examining sources of asymmetric background in elastic *p-p* scattering with 439-Mev polarized protons. Their measurement included about 78 percent of the "unbound" reaction $p+p\rightarrow\pi^{+}+p+n$, which is about equal in magnitude to $p+p\rightarrow\pi^{+}+d$ at Chicago energies.[8] They obtained essentially a null result of $2\epsilon=-0.07\pm0.085$.

Fields *et al.*[28] have measured the asymmetry in $p+p\rightarrow\pi^{+}+d$ using polarized protons of 415 Mev. They obtain $|Q|=0.45\pm0.08$, with (T_π) c.m.$=55$ Mev, $\eta=0.97$. The sign of Q agrees with our value. If we assume, as does the phenomenological theory, negligible energy variation in the parameters α_{10}, β_{10}, and x and in the relative phases τ_0 (or ψ) and τ_1, then we can use our results and Eq. (4) to calculate a predicted value of $|Q|$ at the Carnegie Tech. energy. Using $\eta_c=0.83\pm0.21$ from our unpolarized beam results, and our value of $|Q|=0.39\pm0.05$ at $\eta=0.41$, we would predict an increase in $|Q|$ by a factor 1.25 ± 0.24 in going to $\eta=0.97$, to yield $|Q|=0.49\pm0.10$. This value is clearly consistent with the result of Fields *et al.* The agreement would seem to indicate that the relative *S*- and *P*-wave phase angles, as well as the parameters of Eq. (1), are indeed energy-insensitive, as the phenomenological theory assumes.

Gell-Mann and Watson[15] have shown that, near meson threshold, τ_0 and τ_1 are related to the *p-p* scattering phase shifts for the corresponding states by the relations

$$\begin{aligned}\tau_0&=\alpha(^1S_0)-\alpha(^1D_2)+n\pi,\\ \tau_1&=\alpha(^3P_1)-\alpha(^1D_2)+(n'+\tfrac{1}{2})\pi\end{aligned}, \qquad (6)$$

where the α's are the *p-p* scattering phase shifts in the indicated states, and n and n' are integers. Thus, the result, Eq. (5), of our polarized-beam experiment can be used to put conditions on *p-p* scattering phase shifts calculated near 315 Mev.

In addition, it should be possible to calculate $\sin(\psi-\tau_1)$ from meson theories. If the sign alone of $\sin(\psi-\tau_1)$ were to be calculated from meson theory, then one could predict the sign of the proton beam polarization P. Presumably, at present, such a prediction could not be considered to be conclusive. In the event, however, that the sign of P becomes determined through a more easily understood process, then the sign of $\sin(\psi-\tau_1)$ will be known and can be compared with a meson-theoretical calculation.†

ACKNOWLEDGMENTS

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[26] This supersedes our previously published (see reference 6) value, $|\sin(\psi-\tau_1)|=0.61\pm0.10$, which resulted from using the value $\eta_0=0.62\pm0.16$ instead of our present value, $\eta_0=0.83\pm0.21$.

[27] de Carvalho, Heiberg, Marshall, and Marshall, Phys. Rev. **94**, 1796 (1954).

[28] Fields, Fox, Kane, Stallwood, and Sutton, Phys. Rev. **96**, 812 (1954).

† *Note added in proof.*—J. Marshall and L. Marshall [Leona Marshall, Bull. Am. Phys. Soc. **29**, 18 (1954)] have measured the sign of the polarization P of the 440-Mev polarized proton beam of the Chicago cyclotron by degrading the energy of the protons to 10 Mev and scattering them from helium. Their result predicts that the Berkeley polarized protons (produced by a left scatter) have their spin up, i.e. $P>0$. The sign of our observed meson asymmetry, $(R-L)>0$, with Eqs. (3) and (4), then, says that $\sin(\psi-\tau_1)>0$.

B. T. Feld [Nuovo cimento, **12**, 425 (1954)] has discussed the polarization of the protons in $\pi^{+}+d\rightarrow p+p$. On the basis of reasonable assumptions derived from a resonance model of the meson-nucleon interaction, he has shown that the Fermi and Yang type meson-nucleon interactions predict opposite signs for the polarization. In particular, he shows that if the Fermi type interaction is the correct one, protons produced to the left (as seen by the incident meson) will be polarized with their spins down. (The complementary protons to the right are polarized spin up.) The Yang solution predicts the opposite sign of polarization, with a slightly different magnitude.

If we consider the time reversal of a $\pi^{+}+d\rightarrow p+p$ collision, first averaging over the polarization of (either) one of the protons so that it can represent an unpolarized target, we see that an excess of spin down protons produced to the left, in $\pi^{+}+d\rightarrow p+p$, corresponds to an excess of mesons produced to the right (as seen by the incident proton) by spin up protons in $p+p\rightarrow\pi^{+}+d$. (The proton spins, as well as velocities, are reversed when the time is reversed.) The fact that we do indeed observe an excess of mesons produced to the right by spin up protons provides strong evidence in favor of the Fermi type interaction.

We emphasize that, as shown by Fig. 10, the present $p+p\rightarrow\pi^{+}+d$ data alone, i.e., without additional arguments such as those of Feld, are relatively unbiased towards either the Fermi ($|r_0|\sim0.25$) or the Yang ($|r_0|\sim1.5$) type interactions. Preliminary indications are that combination of the $p+p\rightarrow\pi^{+}+d$ data with the elastic $p-p$ scattering phase shifts of Segrè, *et al.* (to be published), through the Gell-Mann and Watson relation, Eq. (6), will favor one or the other set of solutions, independent of meson theory. In addition, using Eq. (6), the Fermi and Yang interactions predict quite different magnitudes for the polarization of the deuteron in $p+p\rightarrow\pi^{+}+d$ production with unpolarized protons.