Experiments on the Behavior of an Ionized Gas in a Magnetic Field*

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(Received March 15, 1954; revised manuscript received June 24, 1954)

Measurements with probes in a plasma of helium ions and electrons in the afterglow of an induction-excited discharge in a toroidal tube with a toroidal magnetic field reveal an oscillatory probe current which is presumably indicative of fluctuations in the ion density. The space distribution of these fluctuations and the dependency of their amplitude and frequency on the pressure and magnetic field are consistent with the assumption that plasma waves, similar to the magnetohydrodynamic type described by Alfvèn, exist in this toroid. Measurements of electron density by a microwave method in another toroidal tube, which is in the form of a microwave resonant cavity, show that in this tube the degree of diffusion control in helium at low pressures is apparently orders of magnitude less than one would calculate from the classical theory of ambipolar diffusion of ions and electrons in a magnetic field. The measurements show a marked minimum in the diffusion coefficient as a function of magnetic field at about 600 gauss. An attempt is made to explain this minimum and the lack of diffusion control in terms of plasma waves. Experimental evidence that plasma waves exist in this toroid in a magnetic field has been seen in oscillations which appear on the rf envelope of the exciting pulse from the magnetron.

THE rate of diffusion of ions and electrons across a magnetic field has attracted some interest.[1–5] Probe measurements reported by Bohm[1] have indicated that the degree of control of diffusion of ions and electrons by a magnetic field is not nearly so great as predicted by the orthodox collision theory. He attributed this lack of control of ion diffusion by the magnetic field to a turbulence which is, in some manner, associated with plasma oscillations.

In the pursuance of a research project specified in a contract with the Geophysics Research Division of the Air Force Cambridge Research Center, measurements have been performed at Tufts College on the properties of an ionized plasma in the presence of a magnetic field.

PROBE MEASUREMENTS OF PLASMA PROPERTIES IN A MAGNETIC FIELD IN THE AFTERGLOW OF A PULSED DISCHARGE

The first experiment involved the measurement by probes of the properties of a plasma in a toroidal tube in a magnetic field during the "afterglow" period which follows the pulsed induction-excited discharge. The toroidal tube was constructed of two Corning glass U bends, $1\frac{1}{2}$ in. i.d., with two brass straight sections joining them in the race track tube diagrammed in Fig. 1. This tube was wound with No. 13 wire for the excitation of a toroidal dc magnetic field and was linked with two pulse transformer cores, the 10-turn primaries of which were excited by the circuit diagrammed in Fig. 1. Either electron current or positive-ion current to the probe could be measured during the period which followed the excitation pulse. The probe was connected to its resistor and amplifier only during an examining interval of about 1000 μsec which could be delayed at will to include any portion of the afterglow period under scrutiny. This arrangement prevented paralysis of the sensitive ampli-

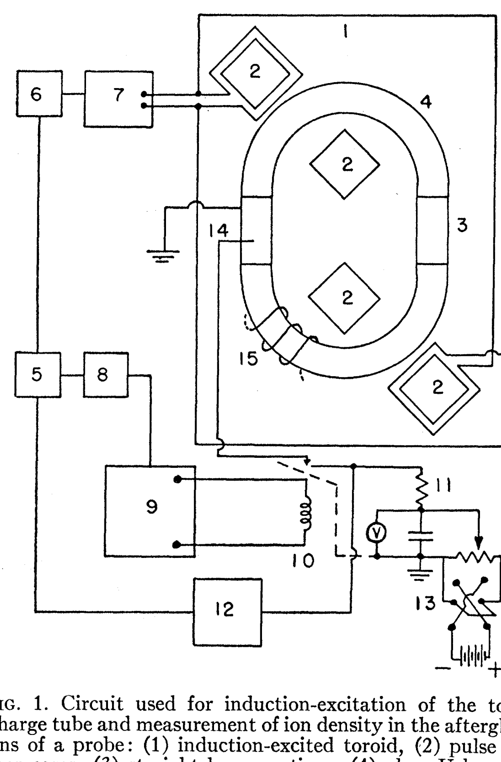


FIG. 1. Circuit used for induction-excitation of the toroidal discharge tube and measurement of ion density in the afterglow by means of a probe: (1) induction-excited toroid, (2) pulse transformer cores, (3) straight brass sections, (4) glass U bends, (5) master trigger source, (6) trigger amplifier, (7) pulse generator, 0.02 μf charged to 20 kv, (8) variable trigger delay, (9) pulse generator for relay coil, (10) Stevens-Arnold relay, (11) signal resistor for measuring positive-ion or electron current, (12) synchroscope, (13) dc voltage source for probe, (14) probe, and (15) coils for annular magnetic field.

* The work reported here was sponsored by the Geophysics Research Division of the Air Force Cambridge Research Laboratories.

[1] A. Guthrie and R. K. Wakerling, *The Characteristics of Electrical Discharges in Magnetic Fields* (McGraw-Hill Book Company, Inc., New York, 1949), first edition, pp. 11 and 68.
[2] M. H. Johnson and E. O. Hulburt, Phys. Rev. **79**, 802 (1950).
[3] M. H. Johnson in *Proceedings of the Conference on Ionospheric Physics*, Vol. II, Part B, 1950 (unpublished).
[4] O. T. Fundingsland and G. E. Austin, Phys. Rev. **79**, 232 (1950).
[5] H. Alfvèn, *Cosmical Electrodynamics* (Oxford University Press, London, 1950), first edition, p. 56.

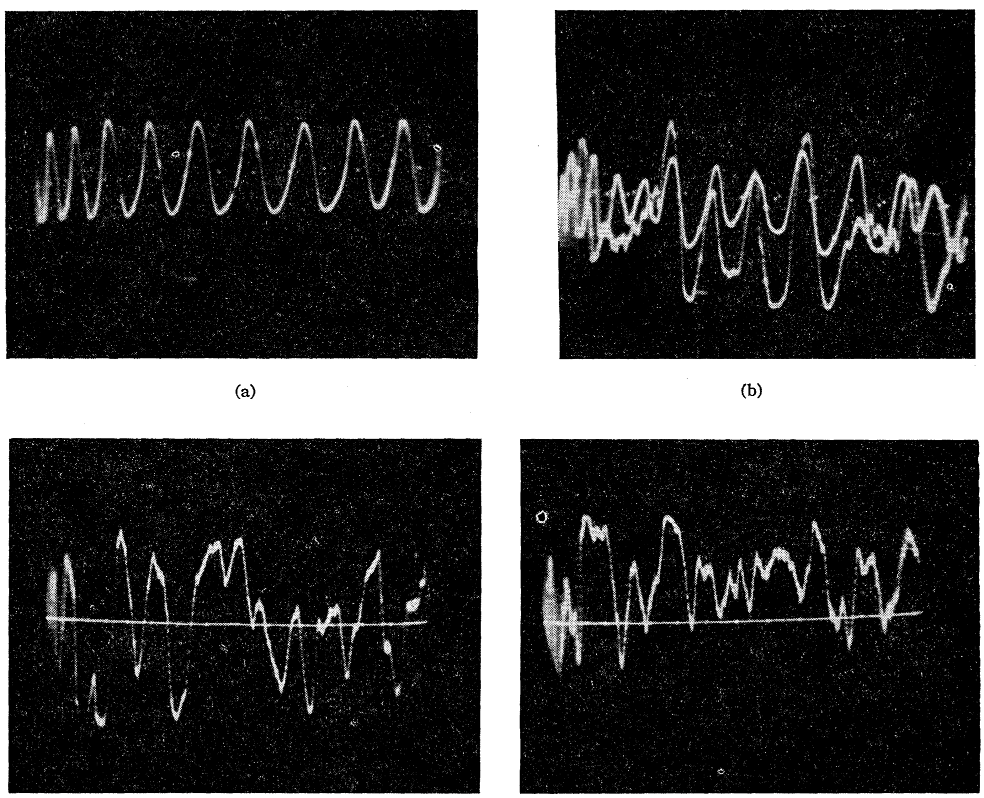
fier by the excitation pulse and minimized the "robbing" of the plasma by the probe. The dc toroidal magnetic field current was supplied by means of storage batteries for intervals of a few seconds during which measurements were made. Magnetic fields were obtainable up to about 800 gauss.

The striking feature[6] of these probe measurements in both helium and nitrogen in a magnetic field is the fact that both electron current and positive-ion current are oscillatory in character. The measurements show (a) that these oscillations are absent in the absence of a magnetic field, (b) that their amplitude increases toward the periphery of the tube, (c) that their amplitude increases as the magnetic field and the pressure are reduced, and (d) that their frequency increases as the magnetic field and the pressure are increased. The authors have attempted to interpret[6] these oscillations in terms of plasma waves of a magneto-hydrodynamic type, i.e., a transverse wave with direction of propagation in the direction of the magnetic field.

Since this tube could not be baked, and hence the gas in this tube could not be purified to a high degree, no accurate measurements could be made upon the rate of diffusion loss of ions and electrons in the afterglow in a magnetic field. Attachment losses of electrons could easily obscure the diffusion loss under these conditions.

DETECTION OF OSCILLATIONS IN A DC CURRENT DISCHARGE IN A TOROID IN A MAGNETIC FIELD

A second experiment performed on the properties of a plasma in a magnetic field employed a large copper toroid with an inside diameter of about 2 in. and a large diameter of 3 ft. This toroid has a much more uniform magnetic field than that shown in Fig. 1. A dc discharge was excited in argon at pressures from 0.2 to 1 mm Hg by placing a constant potential difference of about 600 volts with a 3000-ohm resistor in series, across two probes which were located diametrically opposite each other around the toroid. A third probe, inserted into the



(a) (b) (c) (d)

FIG. 2. (a) Argon at 0.5 mm Hg, $H=350$ gauss; frequency of oscillations is 6500 cps. (b) Argon at 0.5 mm Hg, $H=460$ gauss, $H=530$ gauss. (c) Argon at 0.2 mm Hg, $H=530$ gauss. (d) Argon at 0.2 mm Hg, $H=690$ gauss.

Current wave forms picked up by a probe in a plasma produced in the toroid shown in Fig. 1 at the indicated values of pressure and magnetic field in argon gas. The gas is excited by a dc potential of 600 volts between two probes which are diametrically opposite across the toroid. These wave forms are attributed to plasma waves of the magneto-hydrodynamic type. Note increase in higher-frequency components with increase in the magnetic field. Time is the abscissa and time marker dots are 100 μsec apart.

[6] W. H. Bostick and M. A. Levine, Phys. Rev. 87, 671 (1950).

plasma in the toroid, was used to pick up the oscillations shown in Fig. 2, which reveal that the frequency of the oscillatory signals increases with the magnetic field value. This dependency of frequency on magnetic field is in the same direction as that found in the oscillations in probe current in the afterglow in the toroid shown in Fig. 1.

Later this tube, with nitrogen at a pressure of 10^{-4} to 10^{-2} mm Hg, was excited by means of a short microwave cavity placed at one station in the toroid. Under certain conditions it was possible to observe at the station diametrically opposite to the exciting station the light from the discharge excited by the microwave cavity. It was also possible under these conditions to pick up oscillations on the probes placed at three different stations around the toroid. Unfortunately, due to insufficient magnetron power the microwave cavity could be broken down at these relatively low pressures only over a small variation of magnetic-field values near gyroresonance at the microwave frequency for the electrons. The variation with magnetic field could therefore not be explored as it was with the dc discharge in this tube.

The gas in this large copper toroid could not be purified to a high degree and therefore it was impossible to devise, with this large tube, an experiment which would enable one to measure accurately the rate of diffusion of ions and electrons in a magnetic field.

MEASUREMENT OF DIFFUSION OF IONS AND ELECTRONS IN A MAGNETIC FIELD IN A MICROWAVE CAVITY RESONATOR IN THE FORM OF A TOROID

In order to measure accurately the rate of ambipolar diffusion of thermalized ions and electrons in a gas of high purity in a magnetic field the construction of a microwave coaxial cylindrical cavity with an annular magnetic field and associated equipment, (see Fig. 3), was planned by O. T. Fundingsland and G. E. Austin of the Geophysics Research Division. The construction of this cavity and equipment, and the subsequent measurements have been carried out in the laboratory at Tufts College. This tube was constructed of OFHC copper, baked on the pumps at 450°C, sealed off, wound with magnetic field coils, and resealed to the system without the admission of air. The system could be pumped to a pressure of 10^{-7} mm Hg, and the pressure rise rate of the tube shut off from the pumps was 1.4×10^{-6} mm/minute.

The measurements performed with this apparatus concern the ambipolar diffusion of the thermalized He^+ ions and electrons in helium gas across a magnetic field in a toroidal tube. No provision was made for the insertion of probes in this apparatus.

Measurements of electron density in the microwave cavity shown in Fig. 3, by means of the frequency-shift

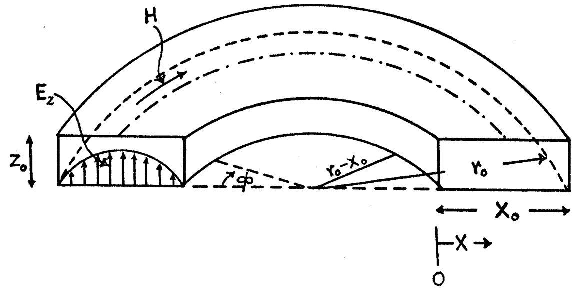


FIG. 3. Microwave cavity with an annular magnetic field used for measurement of diffusion of ions and electrons across a magnetic field. The electric field of the exciting signal is E_z while H is the annular magnetic field. Also, $r_0=6.00$ in. $=15.24$ cm; $x_0=2.00$ in. $=5.08$ cm; and $z_0=3.154$ cm.

method,[7] were performed first at one value of the annular magnetic field and over a range of pressures from 0.050 to 1.0 mm Hg. In this first set of experiments the frequency ($\omega/2\pi\cong 2930$ Mc/sec) of the fundamental mode of the cavity (no ϕ variation and one half-cycle variation in r) was used for measurement of electron density in the afterglow following breakdown of the gas in the cavity at gyroresonance for the electrons at the centerline of the cavity on the sixth mode at 3970 Mc/sec.

The second set of measurements utilized the fundamental mode of the cavity for breakdown of the gas, and the frequency-shift measurement of the cavity was performed with either the fifth or sixth mode. With this arrangement it was found that breakdown could be achieved with values of the magnetic field of about 160 gauss and greater. It was thus possible to measure diffusion as a function of magnetic field for various pressures.

The arrangement for the first set of measurements with the fundamental mode used as the probing mode will now be discussed to illustrate the method of determining electron density.

The electron density n at the centerline of the cavity can be obtained from the following calculations: the angular frequency shift $\Delta\omega$ is given by

$$\Delta\omega=-\tfrac{1}{2}\epsilon_0\int\sigma_i E^2dv\Big/\int E^2dv, \qquad (1)$$

where σ_i is the imaginary part of the complex conductivity perpendicular to the magnetic field, E is the electric field of the unperturbed cavity, ϵ_0 is 8.85×10^{-12} farads/meter, and $\int(\quad)dv$ represents an integral over the volume of the cavity.

Now[8]

$$\sigma_i=-e^2n\omega/[m(\omega^2-\omega_g^2)], \qquad (2)$$

[7] M. A. Biondi and S. C. Brown, Research Laboratory of Electronics, Massachusetts Institute of Technology, Technical Report No. 34, Feb. 26, 1947 (unpublished).

[8] B. Lax at the Conference on Gaseous Electronics, October, 1951, e7 (unpublished).

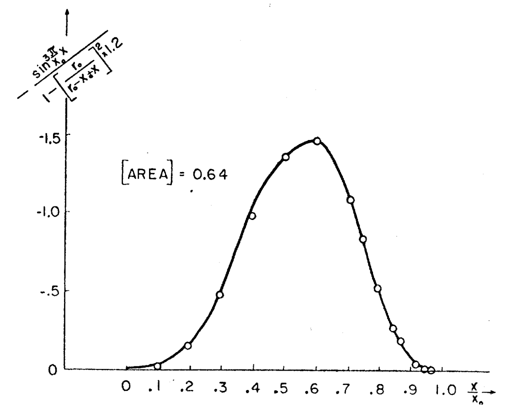


FIG. 4. A plot of the integrand of Eq. (6) so that $\Delta\omega$ in Eq. (7) can be evaluated from the area under the curve.

where ω_g is the gyroresonance frequency for electrons, m is the mass of the electron in kg, n is the number of electrons/meter,[3] and e is 1.6×10^{-19} coulomb.

If the cavity, see Fig. 3, is approximated by a straight rectangular box of width x_0, height z_0, and length L (equal to the mean circumference of the toroid), the variation of E with x may be approximated by

$$E=E_0\sin(\pi x/x_0), \tag{3}$$

and the variation of n with x may be approximated by

$$n=n_0\sin(\pi x/x_0). \tag{4}$$

With the use of Eqs. (1), (2), (3), and (4) there results

$$\Delta\omega=\frac{e^2n_0\omega}{2\epsilon_0 m}\int_0^{x_0}\frac{\sin^3(\pi x/x_0)}{\omega^2-\omega_g^2}dx\Big/\int_0^{x_0}\sin^2(\pi x/x_0)dx. \tag{5}$$

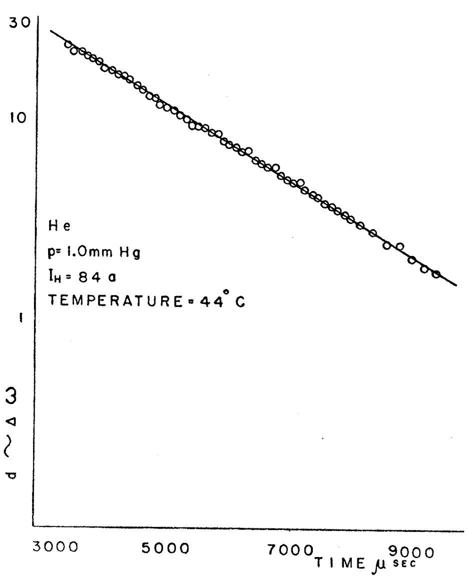


FIG. 5. Experimental plot of the values of d, the divisions on a Sperry wave meter ($\sim\Delta\omega$), which are proportional to the electron densities in the afterglow in a toroid with an annular magnetic field.

The integral in the denominator of Eq. (5) is equal to $x_0/2$ and $\omega_g^2=\omega_{g0}^2r_0^2/(r_0-x_0+x)^2$, where ω_{g0} is the gyroresonant frequency at the magnetic field corresponding to the position r_0 in Fig. 3. Thus Eq. (5) can be written

$$\Delta\omega=\frac{e^2n_0}{\epsilon_0 m\omega x_0}\int_0^{x_0}\frac{\sin^3(\pi x/x_0)}{1-[\omega_{g0}^2r_0^2/\omega^2(r_0-x_0+x)^2]}dx. \tag{6}$$

Now the total number of turns of the magnetic field coil is 1037, the mean circumference of the toroid is 79.8 cm, and the number of turns per cm at the mean radius r_m is 13.0.

The magnetic field at the mean radius (the center line) is then (16.32) I gauss, where I is the magnetic

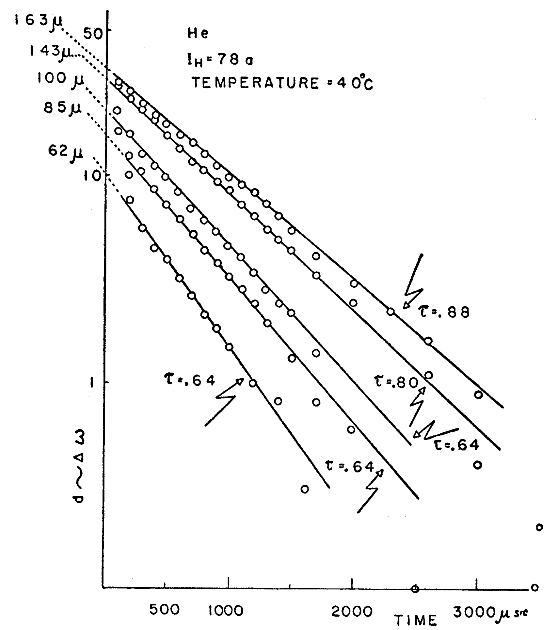


FIG. 6. Typical experimental plots of values of $d(\sim\Delta\omega)$, which are proportional to electron density, in a toroid with an annular magnetic field, at various pressures.

field current measured in amperes. Most of the first set of data have been taken at $I=84.0$ amperes which corresponds to a magnetic field at the mean radius of $H(r_m)=1370$ gauss. The value of H at r_0 is then $H(r_0)=1141$ gauss. The measured value of the wavelength λ used for gyroresonance breakdown is $\lambda=7.564$ cm which corresponds to $f=3970$ Mc/sec. The calculated value of gyroresonance, $H(r_m)=1370$ gauss, is

$$f(r_m)=\omega(r_m)/2\pi=3825\text{ Mc/sec},$$

which means that the position where gyroresonance breakdown was produced in the cavity was at a radius of $3825r_m/3970=0.965r_m$, which is approximately equal to r_m.

The calculated gyrofrequency at $r=r_0$ is 3190 Mc/sec. The frequency $\omega/2\pi$ of the probing signal is approximately constant and is 2934 Mc/sec. Figure 4 gives a plot of the integrand of Eq. (6) with the assumption that $(\omega_{g0}/\omega)^2$ may be approximated by the constant value of 1.2. The abscissa in Fig. 4 is x/x_0 instead of x so that the integral of Eq. (6) is actually x_0 times the area under the curve.

The area is approximately equal to 0.64 and is negative, so that $\Delta\omega$ should theoretically come out to be negative, which it actually does experimentally.

Then

$$\begin{aligned} n_0 &= (8.75\times 10^6)\Delta\omega \text{ electrons/m}^3 \\ &= 8.75\Delta\omega \text{ electrons/cm}^3. \end{aligned} \tag{7}$$

The minimum measurable $\Delta\omega/2\pi$ is 2×10^5 cps. Hence, the minimum measurable n_0 is 1.1×10^7 electrons/cm³. On the Sperry wave meter employed one small division equals 0.04 Mc/sec. Therefore

$$n_0 = (2.2\times 10^6)d \text{ electrons/cm}^3, \tag{8}$$

where d is the number of small divisions difference on the wave meter corresponding to $\Delta\omega$.

The experiments performed involve the measurement of the frequency shift as indicated by d on the wave meter. The values of $\log d$ are then plotted against time, and the slope of the resulting straight line is proportional to the ambipolar diffusion coefficient D_{aH} at the particular values of pressure and magnetic field employed. The time taken for the value of d to decrease by the factor $1/e$ is the ambipolar diffusion time constant τ_{aH}.

Typical sets of experimental data yielding a plot of $\log d$ *vs* t are shown in Figs. 5 and 6. Each set of data yields a value of τ_{aH}.

The quantity τ_{aH} measured in the magnetic field can be related to the effective ambipolar diffusion coefficient D_{aH} for the cavity whose characteristic diffusion length is Λ, by

$$\tau_{aH} = \Lambda^2/D_{aH}, \tag{9}$$

where $1/\Lambda^2 = \pi^2/z_0^2 + \pi^2/x_0^2$, or $\Lambda^2 = 0.675$ cm².

As already explained, the first set of experimental conditions involved the values of magnetic field of $H = 1370$ gauss ($I = 84$ amperes) and $H = 1275$ gauss ($I = 78$ amperes) at the mean radius of the toroid. Breakdown was produced at gyroresonance with the sixth mode of the cavity and the fundamental mode was used for the probing signal. A summary plot of the values of τ_{aH}, thus obtained at various pressures, is given in Fig. 7.

The second set of experimental conditions, as already explained, involved breakdown of the gas by the fundamental mode of the cavity and probing with the fifth and sixth modes. A variety of magnetic fields could be employed under these conditions. A summary plot of the experimental values of τ_{aH}, thus obtained, *vs* $I(\sim H)$ for various pressures is shown in Fig. 8. The oscilloscope display of the rf envelope of the 2000-μsec excitation pulse from the magnetron reflected from the cavity showed strikingly the presence of oscillations whose frequency increased and whose amplitude decreased as the magnetic field increased. The authors believe that

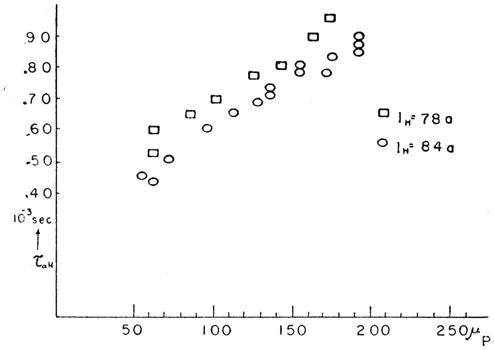


FIG. 7. Experimental values of the electron-density decay time τ_{aH} as a function of pressure in microns for two different magnetic fields.

these oscillations may have been produced by plasma waves of the magneto-hydrodynamic type already postulated in the interpretation of the probe measurements.

ATTEMPT AT AN INTERPRETATION OF THE EXPERIMENTAL DATA

Figure 9 shows cross sections of both the microwave toroid and a circular cross section tube such as the induction-excited toroid. Let it be assumed that the ion

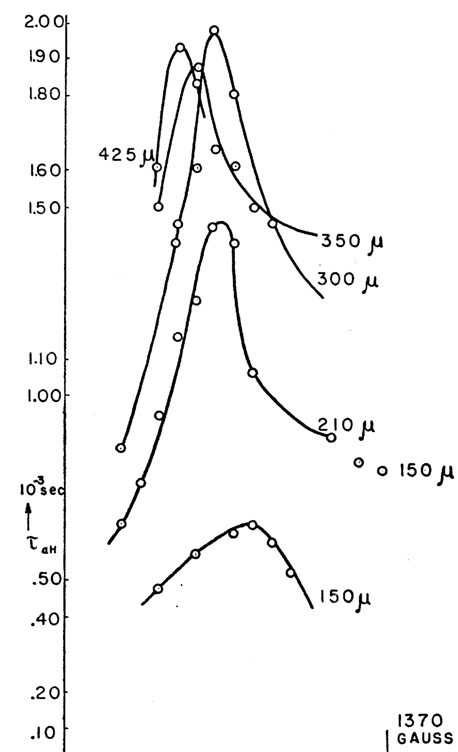


FIG. 8. Results of measurements of diffusion of He ions in helium gas in a toroid with an annular magnetic field. Ordinates are the decay times τ_{aH} which are proportional to $1/D_{aH}$, where D_{aH} is the ambipolar diffusion coefficient in a magnetic field. Abscissa is the magnetic field currents.

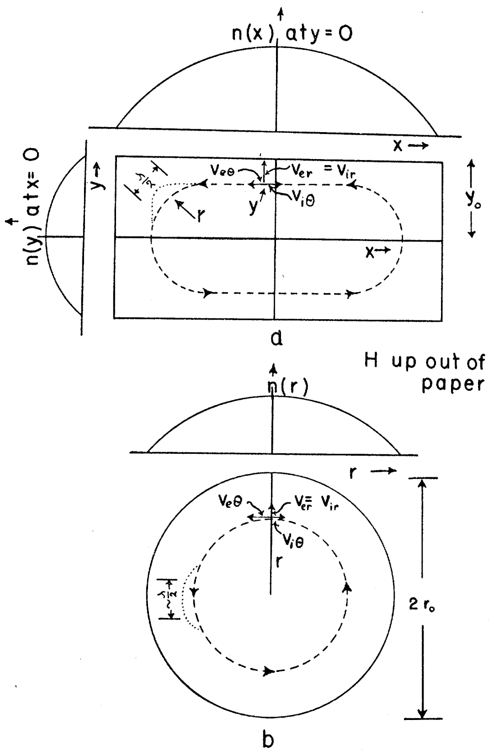


FIG. 9. Cross sections of rectangular and circular toroids showing the calculated direction of circulation of ions and electrons and the estimated ion density distributions.

density distributions are, to a first approximation, as diagramed in Fig. 9. We now apply the orthodox collision and ambipolar diffusion theory to compute the values of D_{aH} and τ_{aH} for Table I. The radial or outward velocities v_{er} and v_{ir} of the electrons and ions, respectively, are equal in sign and magnitude,[9] i.e.,

$$v_r = v_{er} = v_{ir} = 2kT(\text{grad}n)/\{[\nu_i m_i(1+\omega_i^2/\nu_i^2) + \nu_e m_e(1+\omega_e^2/\nu_e^2)]n\}. \quad (10)$$

The subscript i refers to the positive ions, e to the electrons. The quantity ω is the angular gyrofrequency, ν the collision frequency, n the ion density, and m the mass, k is Boltzmann's constant, and T, the absolute temperature of the ions and electrons, is assumed to be constant and thermalized by the neutral gas atoms in our microwave experiment.

In the absence of a magnetic field the ambipolar diffusion coefficient D_a is related to the radial velocity of ions, v_r, by

$$v_r = D_a(\text{grad}n)/n = 2kT(\text{grad}n)/(\nu_i m_i n),$$

or

$$D_a = 2kT/(\nu_i m_i).$$

In the approximation that $\omega_e^2/\nu_e^2 \gg 1$, and also that $\omega_e^2\nu_e m_e/\nu_e^2 \gg \omega_i^2\nu_i m_i/\nu_i^2$, we can then write that the ambipolar diffusion coefficient D_{aH} in a magnetic field is given by

$$D_{aH} = D_a/[1+(m_e/m_i)^{\frac{1}{2}}\omega_e^2/\nu_e^2]. \quad (11)$$

[9] Reference 5, p. 61.

If we use the value of the magnetic field ($H=1370$ gauss) at the center line of the microwave cavity, ω_e is equal to $2\pi(3.82\times 10^9)$. If $\nu_e = \bar{v}/l = P_c\bar{v}p$, where $\bar{v}$ is the average thermal electron speed in cm/sec, l the electron mean free path in cm, and $P_c = 20$ cm^{-1} is the collision probability, p the pressure in mm Hg, and if from Biondi and Brown,[10] and Phelps,[11] $D_ap = 540$ for He$^+$,

$$D_{aH}(\text{He}^+) = 540/[p(1+169/p^2)]. \quad (12)$$

This expression for D_ap is valid down to the pressure at which the mean free path of a positive ion becomes comparable with tube dimensions. This limiting pressure is about 0.01 mm Hg for the microwave toroidal cavity employed.

Equation (11) gives a pressure dependency for D_{aH} which is opposite from that observed experimentally (Fig. 7). Equation (11) also predicts $D_{aH}\sim 1/H^2$, and says nothing at all about the observed maximum in τ_{aH} as a function of H (Fig. 8). Furthermore, the magnitudes of $\tau_{aH}(\sim 1/D_{aH})$ computed for $H=1370$ gauss from Eq. (12) at various pressures, (Table I), are considerably larger than those observed (Fig. 7). Thus the values of D_{aH} calculated by orthodox collision theory do not agree with those measured by this experiment. The following arguments will be used in an attempt to explain these discrepancies.

Now, the velocities[9] of electrons and ions in the θ direction are

$$v_{e\theta} = \omega_e v_r/\nu_e,$$

and

$$v_{i\theta} = -\omega_i v_r/\nu_i.$$

It is these velocities in the θ direction which produce a resultant diamagnetic current which decreases the magnetic field in the interior of the toroid to an extent given by the equation

$$\text{grad}[nkT + H^2/(8\pi)] = 0. \quad (13)$$

(The grad$H$ due to the toroidal geometry is neglected here, for the moment.)

Now, in the approximation that at the values of H and p used,

$$\omega_e^2/\nu_e^2 \gg 1 \quad \text{and} \quad \omega_e^2\nu_e m_e/\nu_e^2 \gg \omega_i^2\nu_i m_i/\nu_i^2,$$

$$v_r = 2kT(\text{grad}n)/(n\nu_e m_e\omega_e^2/\nu_e^2), \quad (14)$$

and

$$v_{e\theta} = 2kT(\text{grad}n)/(nm_e\omega_e), \quad (15)$$

and

$$v_{i\theta} = -2kT\nu_e\omega_i(\text{grad}n)/(nm_e\nu_i\omega_e^2). \quad (16)$$

The net momentum of an ion pair in the θ direction is given by

$$\begin{aligned} m_e v_{e\theta} + m_i v_{i\theta} &= (m_e\omega_e/\nu_e - m_i\omega_i/\nu_i)2kT\nu_e(\text{grad}n)/(nm_e\omega_e^2) \\ &= m_e\omega_e[1 - m_i\omega_i\nu_e/(m_e\omega_e\nu_i)]2kT(\text{grad}n)/(nm_e\omega_e) \\ &\cong [1-(m_i/m_e)^{\frac{1}{2}}]2kT(\text{grad}n)/(n\omega_e) \\ &\cong -(m_i/m_e)^{\frac{1}{2}}2kT(\text{grad}n)/(n\omega_e). \end{aligned}$$

[10] M. A. Biondi and S. C. Brown, Phys. Rev. **75**, 1700 (1949).
[11] A. V. Phelps and S. C. Brown, Phys. Rev. **86**, 102 (1952).

There is then essentially a net mass hydrodynamic motion in the $-\theta$ direction with a velocity

$$v_{\theta\,\mathrm{hydro}}=(m_e v_{e\theta}+m_i v_{i\theta})/(m_e+m_i) \cong -2kT(\mathrm{grad}n)/[n\omega_e(m_e m_i)^{\frac{1}{2}}].$$

In a tube of finite cross-sectional dimensions the velocities $v_{e\theta}$, $v_{i\theta}$, and $v_{\theta\,\mathrm{hydro}}$ may be expected to be of a rotational nature because of the radial electric field set up by the initial free diffusion (non-ambipolar) of the positive ions and electrons, and because of the radial grad$n$. (The free diffusion of the positive ions in the magnetic field is considerably larger than that of the electrons.)

If the $v_{e\theta}$ and $v_{i\theta}$ represent rotational velocities occurring at a radius r from the center, the quantities $v_{e\theta}^2/r$ and $v_{i\theta}^2/r$ represent the centripetal accelerations of the electrons and positive ions, respectively. The centripetal acceleration of an element of volume of plasma is $v_{\theta\,\mathrm{hydro}}^2/r$. In these accelerated frames of reference there is effectively an outward gravitational force acting antiparallel to grad$n$. Such a condition may be expected to be unstable[12] and any small perturbation will grow with an e-folding time constant (time to grow to a value e-fold as great) given by

$$\tau=[\lambda/(2\pi a)]^{\frac{1}{2}}\text{ sec,} \tag{17}$$

where λ is the wavelength associated with a small disturbance or bump, see Figs. 9(a) and 9(b), and a is centripetal acceleration in cm/sec^2. The value of λ may be expected to be of the order of r and is equal to a few centimeters.

Let us take $\lambda=2$ cm. For the electrons

$$a=v_{e\theta}^2/r;$$

and

$$\tau=(r/\pi v_{e\theta}^2)^{\frac{1}{2}}=(r/\pi)^{\frac{1}{2}}/v_{e\theta} =(r/\pi)^{\frac{1}{2}}/[2kT(\mathrm{grad}n)/(nm_e\omega_e)]. \tag{18}$$

If $(r/\pi)^{\frac{1}{2}}\cong 1$, Eq. (18) may be written (19)

$$\tau=[2kT(\mathrm{grad}n)/(nm_e\omega_e)]^{-1}. \tag{20}$$

For $H=1000$ gauss, and for $T=300°$K, the quantity

$$2kT/m_e\omega_e=3.7\times 10^4. \tag{21}$$

Also, since $n\sim\cos(\pi r/2r_0)$ and $(\mathrm{grad}n)=dn/dr\sim(\pi/2r_0)\times\sin(\pi r/2r_0)$,

$$(\mathrm{grad}n)/n\cong(\pi/2r_0)\tan(\pi r/2r_0). \tag{22}$$

Hence, Eq. (18) becomes, in seconds,

$$\tau\cong 2.7\times 10^{-5}[(\pi/2r_0)\tan(\pi r/2r_0)]^{-1} =1.7\times 10^{-5}r_0\cot(\pi r/2r_0). \tag{23}$$

If the approximation of Eq. (19) is not employed, the expression for τ given by Eq. (23) would involve an $r^{\frac{1}{2}}$.

In Eq. (23), r_0 for the circular cross section case is indicated in Fig. 9(b), and for Fig. 9(a), r_0 may be approximated by y_0. It can be seen from Eqs. (18) and (23) that the e-folding time τ for the values $r/r_0=\frac{1}{2}$ and $r_0=2$ cm is approximately 30 μsec, and that for points nearer the periphery, the value of $\tau<30$ μsec is certainly short enough to permit the plasma waves in the microwave toroid to become well established before and during the measured ion-decay time constants of approximately 10^{-3} sec. If in computing the acceleration a used in Eq. (17), $v_{\theta\,\mathrm{hydro}}$ instead of $v_{e\theta}$ is used, the values for τ will be increased in the ratio $(m_i/m_e)^{\frac{1}{2}}$, which is approximately a factor of 80 for He^+ ions. Even this longer τ is short enough to permit the buildup of waves during the period of excitation and observation in the microwave experiment.

The concept[12] of an e-folding disturbance leading to Eq. (17) was developed under the assumption that the disturbance is so small that the magnetic lines of force are not appreciably bent but are merely spread. However, as the original disturbance, which is very likely somewhat localized in the magnetic field direction, grows in magnitude, it must necessarily cause appreciable bending of the magnetic lines, and the disturbance may then be expected to be propagated as a wave in the direction of the magnetic field. For lack of a better theory at present, we shall assume that the e-folding of the disturbance continues as an e-folding of the amplitude of a magneto-hydrodynamic or plasma wave. The amplitude build-up time constant τ will, of course, be shorter toward the periphery of the tube.

The build-up rate for energy in the wave may then be expected to be $2/\tau$, and if we use the value of τ corresponding to $v_{\theta\,\mathrm{hydro}}$, we may write that the rate of energy build-up in the wave is equal to

$$2/\tau=4kT(\mathrm{grad}n)/[n\omega_e(m_i m_e)^{\frac{1}{2}}]. \tag{24}$$

We have now postulated and given a rudimentary discussion of a mechanism whereby disturbances can be set up in the plasma. These disturbances, even though they may be propagated as waves, nevertheless constitute a kind of turbulence in the plasma. Although it is rash to attempt to describe a phenomenon as complicated as turbulence, we shall nevertheless attempt to

TABLE I. Theoretical values of D_{aH} for He^+ computed on the basis of standard diffusion theory according to Eqs. (11) and (12) for $H=1370$ gauss.

p mm Hg	$D_a=540/p$	$\tau_a=0.67/D_a$ 10^{-3} sec	$D_{aH}=540/[p(1+169/p^2)]$	$\tau_{aH}=0.67/D_{aH}$ 10^{-3} sec	Expected degree of control $=\tau_{aH}/\tau_a$
2	270	2.5	6.4	105	43
1	540	1.25	3.2	210	168
0.8	676	1.0	2.6	260	260
0.6	900	0.75	1.92	350	466
0.4	1340	0.5	1.28	520	1040
0.2	2700	0.25	0.64	1050	4200
0.1	5400	0.125	0.32	2100	16 000
0.05	10 800	0.06	0.16	4200	70 000

[12] M. Kruskal at the Gaseous Electronics Conference, Princeton, New Jersey, September, 1952 (unpublished).

compare the implications of the foregoing theory with the experimentally observed oscillations in probe and microwave signals.

Presumably the e-folding wave increases in amplitude until twice its reciprocal damping time constant, $2/\tau'$, which represents the proportional rate at which energy is taken out of the wave, is equal to twice its reciprocal e-folding time constant, $2/\tau$, which represents the proportional rate at which energy is fed into the wave.

The damping of the wave can conceivably come about (a) by collisions of the ion pairs participating in the wave with neutral gas atoms, and (b) through loss of the participating ion pairs to the walls of the tube by the diffusion processes resulting from the wave motion. Let us first consider damping mechanism (a).

If $s=s_0 \sin\omega_{MH}t$ represents the lateral displacement of a particle in a magneto-hydrodynamic wave of angular frequency ω_{MH}, the wave energy of an ion pair in the wave is

$$[(m_i+m_e)/2](ds/dt)_{\max}^2=[(m_i+m_e)/2](\omega_{MH}s_0)^2. \quad (25)$$

The rms damping force on an ion pair in the wave may be given by

$$[(m_i+m_e)/2]\omega_{MH}s_0\nu_i/\sqrt{2}, \quad (26)$$

where ν_i is the effective collision frequency of positive ions with neutral atoms, and where it has been assumed that an He^+ ion in colliding with a neutral He atom loses, on the average, one half of its directed wave momentum. The rms damping power is obtained by multiplying Eq. (26) by the rms wave velocity, $\omega_{MH}s_0/\sqrt{2}$. Thus, the rms damping power is given by

$$[(m_i+m_e)/2](\omega_{MH}s_0)^2\nu_i/2. \quad (27)$$

The proportional rate of energy damping $2/\tau'$, (twice the proportional rate of amplitude damping $1/\tau'$) is then given by Eq. (27) divided by Eq. (25), or

$$2/\tau'=\nu_i/2. \quad (28)$$

The Q of the wave, defined as stored wave energy divided by energy lost per radian, may be written as $\tau'\omega_{MH}/2$. The Q of a critically damped wave is $\frac{1}{2}$. If the Q of a wave is somewhat greater than 1, it may be expected that the wave can exist and grow if energy is fed into it with sufficient rapidity. The value of $Q=\tau'\omega_{MH}/2=2\omega_{MH}/\nu_i>1$ therefore sets a lower limit to the frequency of the waves permitted for a given ν_i.

The initial perturbation whose rate of energy growth is $2/\tau$ [see Eq. (20)] will grow if $2/\tau$ is greater than its rate of energy decay $2/\tau'$, given by Eq. (28). On the other hand if $2/\tau<2/\tau'$, the initial perturbation will be damped out.

We must now inquire into the value of ν_i used in Eq. (26). The value of ν_i may be given by

$$\nu_i=v_iP_cp=v_i\sigma n_a, \quad (29)$$

where v_i is the appropriate velocity in cm/sec for the positive ions, P_c, the collision probability in cm^{-1} for positive ions on gas atoms, p, the pressure in mm Hg, σ, the collision cross section in cm^2 for positive ions on gas atoms, and n_a, the atom density in cm^{-3}. The appropriate velocity v_i is presumably given by

$$v_i=(v_{i\text{thermal}}^2+v_{i\text{wave}}^2)^{\frac{1}{2}}, \quad (30)$$

where $v_{i\text{thermal}}$ is the thermal velocity of the positive ions, and $v_{i\text{wave}}$ is the wave velocity of the positive ions.

When the wave is in its initial stages (small amplitude), $v_{i\text{thermal}}\gg v_{i\text{wave}}$, and

$$v_i\cong v_{i\text{thermal}}=(3kT/m_i)^{\frac{1}{2}}$$

in Eq. (29). Then, if the following inequality initially exists [see Eq. (18)],

$$\begin{aligned}2/\tau&=4kT(\text{grad}n)/[n\omega_e(m_im_e)^{\frac{1}{2}}]>\nu_i/2,\\&=(P_cp/2)(v_{i\text{thermal}}^2+v_{i\text{wave}}^2)^{\frac{1}{2}},\end{aligned} \quad (31)$$

the wave will grow in amplitude until $v_{i\text{wave}}$ becomes large enough to increase v_i until $2/\tau$ is no longer greater than $2/\tau'$.

Thus, the equilibrium between the damping of the wave and the growing of the wave yields the equality

$$\begin{aligned}&4kT(\text{grad}n)/[n\omega_e(m_im_e)^{\frac{1}{2}}]\\&\qquad=(P_cp/2)(v_{i\text{thermal}}^2+v_{i\text{wave}}^2)^{\frac{1}{2}},\end{aligned} \quad (32)$$

where $v_{i\text{wave}}$ in Eq. (32) is now the equilibrium value for the ions in the wave at a given r.

From Eq. (32) we can conclude that $v_{i\text{wave}}$ (a) will decrease as p increases, (b) will increase with $(\text{grad}n)/n$, and (c) will decrease as $H(\sim\omega_e)$ increases. In a magneto-hydrodynamic wave of a given frequency,[13] wave perturbations in the magnetic field, wave displacements, and wave electric fields may all be expected to be proportional to $v_{i\text{wave}}$; wave perturbation in the ion density may be expected to be proportional to $v_{i\text{wave}}^2$. The probe measurements (which can detect variations in ion density) described in this paper showed oscillations in probe current whose amplitudes are therefore presumably proportional to $v_{i\text{wave}}^2$.

We are now prepared to draw the following qualitative comparisons between the conclusions drawn from Eq. (32) and the probe measurements performed in the induction excited toroid in an annular magnetic field. As has already been described, these measurements show (1) that the magnitude of fluctuations in probe current, (presumably proportional to the square of the wave amplitude) decreases with pressure, thereby confirming conclusion (a); (2) that the wave amplitude increases markedly toward the periphery of the tube, thereby confirming (b)—this same experimental result is reported by Bohm;[1] and (3) that the wave amplitude decreases with increasing H, thereby confirming (c).

[13] Reference 5, p. 80.

Probe measurements on the induction-excited toroid also show that the lower limit of the frequency of the waves increases with pressure. This result confirms the prediction that, if the Q of the wave must be greater than 1,

$$Q=\tau'\omega_{MH}/2=2/\nu_i=2\omega_{MH}/(v_iP_cp)>1.$$

Hence the lower limit of the allowed values of ω_{MH} must increase with pressure. As has already been stated in this paper, probe measurements and oscillations on the microwave rf envelope also show that the frequency of the oscillations increases with magnetic field. This result may be explained by recognizing that the wave propagation velocity in cm/sec is

$$V=H/[4\pi(m_i+m_e)n]^{\frac{1}{2}},$$

and that whether eigenfunctions of standing waves can be set up around the toroid or whether the disturbances are strictly running waves, the observed frequency of probe signal should increase with H.

In view of the foregoing qualitative agreement between theoretical description of these waves and their experimentally observed properties, it seems reasonable to accept at least some of the elements of this rudimentary theory. At any rate, one should seriously consider the possible mechanism here described for the spontaneous generation of these waves.

Although in the microwave experiment the oscillations on the signal were not observed directly during the measuring period of the afterglow, it is possible that plasma waves were nevertheless present. They may not have been observed because of the small net integrated effect their reduced amplitude would produce on the cavity resonance frequency.

Indeed, it would be very rash to attempt to describe how turbulence in the form of plasma waves might influence D_{aH}, the effective ambipolar diffusion coefficient of ions and electrons across a magnetic field. In a quiescent plasma, the theoretical value of D_{aH} varies as does $1/H^2$ [see Eqs. (10) and (11)]. Bohm[14] explains the large discrepancy between his measured and calculated values of D_{aH} by saying that plasma oscillations bring about electric fields which are perpendicular to the magnetic field, and that the resultant outward drift velocities are therefore proportional to $1/H$. We have some arguments[15] which indicate that if the damping of the plasma waves comes about by collisions with neutral gas atoms, the diffusion coefficient should vary somewhere between $1/H$ and $1/H^2$, but, should be considerably larger than that given by Eq. (11) which assumes a quiescent plasma. These arguments further indicate that if the damping of the waves is produced by collisions of the waves with the wall, the diffusion coefficient should vary as $1/H$. Either or both of these arguments might explain the large discrepancy in general magnitude of our measured values of diffusion rate compared with the values calculated for a quiescent plasma. The peculiar pressure dependency, [Fig. 7], of the measured rates of diffusion might be explained by Eq. (32) which shows that the velocity (and amplitude) of ions in the waves, and hence the rate of diffusion perpendicular to H, increase when the pressure decreases.

There is another consideration concerning the influence of the magnetic field in our toroids which we have not as yet discussed in detail, namely, the effect of grad$H$ which results from the toroidal geometry. This variation in H across the toroid, which is 40 percent in the microwave toroid, represents a far greater variation in $H^2/(8\pi)$ than can possibly be compensated for by nkT, i.e., the pressure balance equation cannot be satisfied. In an attempt to satisfy Eq. (14) the plasma should move to the region of lower magnetic field and build up nkT in this region. The manner in which the plasma crosses the magnetic field to the region of lower field is probably influenced by the turbulence in the form of waves. The rate ($\sim D_{aH}$) at which the plasma crosses the field in an attempt to set up an equilibrium condition one would expect to increase with H. It may, therefore, be this outward motion produced by grad$H$ which produces the high-field portion of the experimental curves of Fig. 8. The low-field portion of the curves of Fig. 8 presumably indicates $D_{aH}\sim 1/H$ or $1/H^2$, and the minimum in D_{aH} (maximum in τ_{aH}) comes where the $\sim 1/H$ process and the $\sim H$ process are equal. We therefore have a possible explanation of the maxima in the experimental curves of Fig. 8.

It is planned to repeat the microwave experiment in a straight tube which has no grad$H$ and in which probes will be inserted for the possible detection of plasma waves.

ACKNOWLEDGMENTS

The authors wish to thank O. T. Fundingsland and G. E. Austin of the Geophysics Research Division of the Air Force Cambridge Research Center who planned and supported the experiment. The authors also wish to acknowledge the skillful and painstaking work of George W. Woodsum and Professor I. Koncius in the construction of the apparatus and the taking of data.

[14] Reference 1, p. 12.

[15] It is not felt that these arguments are in a sufficiently satisfactory state to take up space in the Physical Review. The reader is referred to Scientific Report No. 8, Contract AF 19(122)-89 which will be sent by the authors on request.

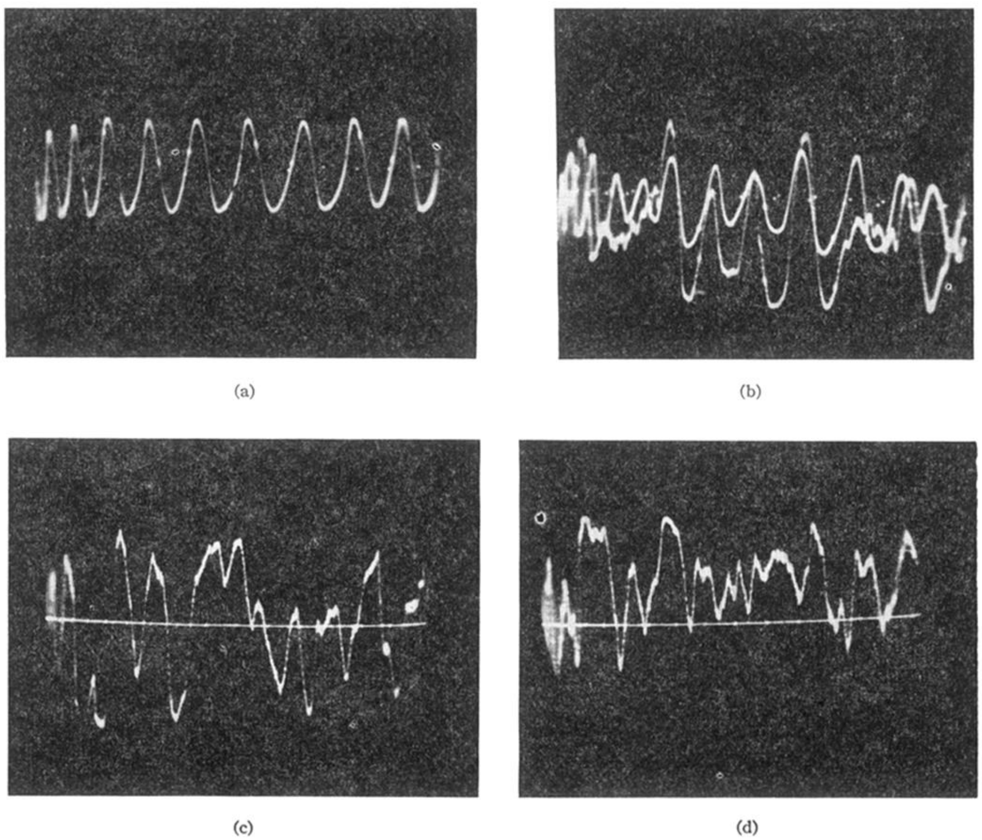


FIG. 2. (a) Argon at 0.5 mm Hg, $H=350$ gauss; frequency of oscillations is 6500 cps. (b) Argon at 0.5 mm Hg, $H=460$ gauss, $H=530$ gauss. (c) Argon at 0.2 mm Hg, $H=530$ gauss. (d) Argon at 0.2 mm Hg, $H=690$ gauss.

Current wave forms picked up by a probe in a plasma produced in the toroid shown in Fig. 1 at the indicated values of pressure and magnetic field in argon gas. The gas is excited by a dc potential of 600 volts between two probes which are diametrically opposite across the toroid. These wave forms are attributed to plasma waves of the magneto-hydrodynamic type. Note increase in higher-frequency components with increase in the magnetic field. Time is the abscissa and time marker dots are 100 μsec apart.