Nuclear Reactions in Stars. II. Protons on Light Nuclei*†

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The rates are calculated for thermonuclear reactions involving the collision of a proton with various light nuclei. These calculations are based directly on low-energy experiments for deuterium, Li, Be, and B, partly on experiments for O^{16}, and purely on theoretical estimates for the other isotopes of O, F, Ne, and Na.

Temperatures are calculated at which D, Li, Be, and B are effectively destroyed by these reactions, both in the interior of young and cool stars which are still contracting gravitationally and in the outer layers of main sequence stars. It is shown that in the interior of even a hot main sequence star only O^{18} and F^{19} (and possibly O^{17}) are destroyed of all the isotopes heavier than nitrogen.

1. INTRODUCTION

THE present paper is a continuation of a previous one[1] (hereafter referred to as I), forming part of a series on the various thermonuclear reactions which might take place in the interior of various stars. We shall deal mainly with the calculation of reaction rates as a function of temperature, density and chemical composition, but a short explanation of their astrophysical significance may be in order.

Hydrogen is the main constituent of most stars and of the gas from which they were presumably formed. Consequently, nuclear reactions involving the capture of protons by various nuclei are most important for stars. The main source of energy in the less luminous main sequence stars (sun and cooler) is the proton-proton chain, involving as its initial step the collision between two protons. This chain of reactions was discussed in detail in I. The main energy source of the more luminous (hotter than sun) main sequence stars is the carbon cycle, involving the capture of protons by the stable isotopes of carbon and nitrogen. A discussion of this reaction cycle should follow I, logically, but Fowler[2] has recently given a detailed discussion of our present knowledge of these reaction rates. We shall only quote Fowler's results briefly in Sec. 4.

The reaction rate for the capture of protons by nuclei of charge Z depends very strongly on Z. This rate is of the right order of magnitude to explain the energy production and lifetime of main sequence stars for $Z=6$ or 7 (carbon-nitrogen cycle). Bethe[3] pointed out long ago that for $Z<6$ the reaction rate is so fast that these elements are burned out in a main sequence star in a time very much shorter than the age of our galaxy[4] (several billion years). For $Z>7$, on the other hand, the rate is so slow that the corresponding rate of energy production is negligible. These conclusions are still essentially correct, but it is nevertheless of some interest to investigate reactions for $Z<6$ and $Z>7$, for the following reasons.

(*a*) $Z<6$: Many astrophysical observations are and will be carried out on star associations and other regions which contain stars formed extremely recently (on an astronomical scale), some as young as a million years. Many such stars have not yet had time for their gravitational contraction from a cold gas cloud to the main sequence, and may have low enough density and temperature for reactions involving D, Li, Be, and B to go on. These reactions would be an important energy source in such stars and, conversely, astrophysical observations may lead to information on the original abundance of these light elements in the gas clouds from which these stars were formed. At present there is some direct data on chemical abundances in stellar atmospheres, but rather little on abundances in interstellar gas.

(*b*) $Z>7$: Since the abundance of oxygen in stars is only of order 1 percent (and the abundance of heavier nuclei much smaller still) and since the reaction rates are very slow, these reactions are certainly negligible as sources of energy production in most stars. But, in connection with detailed theories on the origin of the chemical elements, it may be important to know whether an appreciable fraction of the original content of these elements is burned up in the interior of a hot main sequence star or not. We shall discuss reactions on O, F, Ne, and Na in Secs. 4 and 5.

We shall use the notation of I. Most of this paper will be concerned with the derivation from experiments (or theoretical estimates) of the cross-section factor S, defined by (S and E in laboratory system of measurement)

$$S=\sigma(E)E\exp(2\pi e^2 Z_1 Z_2/\hbar v), \tag{1}$$

extrapolated to the very low energies E of the "stellar energy region" (a few to 50 kev). Once S is known, the mean reaction rate p is obtained very simply (I, Sec. 1) from

$$p=430(\rho x_1)S(A_1{}^2Z_1Z_2)^{-1}\tau^2 e^{-\tau}\ \mathrm{sec}^{-1},$$
$$\tau=42.48[Z_1{}^2Z_2{}^2A_1A_2/(A_1+A_2)]^{\frac{1}{3}}T^{-\frac{1}{3}}. \tag{2}$$

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[1] E. E. Salpeter, Phys. Rev. **88**, 547 (1952).
[2] W. A. Fowler, Mem. Soc. Roy. Sci. Liège **13**, 88 (1954).
[3] H. A. Bethe, Phys. Rev. **55**, 434 (1939).
[4] The p-p reaction is slow enough, even though $Z=1$, because it involves a beta decay instead of γ or α emission.

The cross-section factor S is in units of ev-barns and the temperature T in 10^6 °K.

2. REACTIONS INVOLVING DEUTERONS

In I, Sec. 4, we have already discussed reactions involving deuterons going on in the interior of main sequence stars. Here we are concerned with the interior of young stars, with lower density and temperatures only of the order of 10^6 °K. In such cases, there has not been time to reach the dynamic equilibrium, discussed in I, and the abundance of deuterium may still be appreciable. There are a number of competing reactions and the end result, at these temperatures, of each reaction is the conversion of D into He^3. But the energy release per deuteron is different for the various reactions, discussed below.

(A) The direct conversion of D into He^3 by proton capture,

$$H^1+D^2 \rightarrow He^3+\gamma+5.51 \text{ Mev}, \tag{3}$$

has already been discussed in I, Sec. 5. The cross-section factor derived there (including the correction mentioned in a footnote) is

$$S_p=(11.7\pm4.0)\times10^{-2} \text{ ev barns}. \tag{4}$$

(B) If the abundance of deuterium is at all appreciable we also have to consider reactions involving two deuterons,

$$D^2+D^2 \rightarrow T^3+H^1+4.03 \text{ Mev}, \tag{5a}$$

$$D^2+D^2 \rightarrow He^3+n+3.27 \text{ Mev}. \tag{5b}$$

These reactions have been investigated[5,6] carefully for deuteron energies between 20 and 250 kev. The rates of the two reactions (5a) and (5b) are very nearly the same. The total experimental cross-section factor S, Eq. (1), increases very slowly with energy, being about 2.2 and 2.6 ($\times10^5$ ev barn), respectively, at 25 and 150 kev. We adopt as cross-section factor, for reactions (5a) and (5b) combined,

$$S_D=(2.1\pm0.5)\times10^5 \text{ ev barns}. \tag{6}$$

Substituting the values for S, Eqs. (4) and (6), into Eq. (2) we find for the mean reaction rates for reactions (3) and (4), respectively (with T in 10^6 °K)

$$p_p=(7.0\pm2.5)\times10^4(\rho x_H)\exp(-37.2/T^{\frac{1}{3}})T^{-\frac{2}{3}}, \tag{7}$$

$$p_d=(4.0\pm0.7)\times10^{10}(\rho x_D)\exp(-42.6/T^{\frac{1}{3}})T^{-\frac{2}{3}}. \tag{8}$$

$$p_d=(4.0\pm0.7)\times10^{10}(\rho x_D)\exp(-42.6/T^{\frac{1}{3}})T^{-\frac{2}{3}}$$

Since the value of S is very much larger for the D−D reaction (involving particle emission instead of γ-ray emission), the D−D reaction predominates if the abundance (by mass) of deuterium, x_D, is larger than a small fraction of a percent. The values of (x_D/x_H), for which the reaction rates, Eqs. (7) and (8), are equal, are 2×10^{-3}, 5×10^{-4}, and 2×10^{-4}, respectively, for $T=0.5$, 1, and 2. The value of (x_D/x_H) both in sea water and in meteorites[7] is 1.5×10^{-4}, but could be larger (or smaller) in interstellar gas.

At temperatures T between 0.5 and 2, the He^3 produced in reaction (3) is essentially stable, being converted into He^4 at an appreciable rate only for $T>5$. The tritons produced in reaction (5a) decay into He^3 with a half-life of 12 years. At the low stellar temperatures, we are considering here, all nuclear reactions involving tritons are too slow to compete successfully with the beta decay. We finally have to consider the fate of the neutrons produced in reaction (5b).

The fast neutrons produced in reaction (5b) are quickly slowed down to thermal energies in scattering processes in the stellar gas, mainly by elastic collisions with protons. At a temperature of 10^6 °K the mean thermal energy kT is slightly less than 100 ev. These "thermal" neutrons are finally captured by some of the nuclei present. If the gas consists essentially of pure hydrogen, the only possible process is n-p capture,

$$n+H^1 \rightarrow D^2+\gamma+2.23 \text{ Mev}. \tag{9}$$

Since the branching ratio between reactions (5a) and (5b) is approximately unity, for four deuterons originally taking part in a D−D reaction two He^3-nuclei will be formed and one deuteron reproduced.

If, on the other hand, the stellar interior contains an appreciable abundance of elements other than hydrogen, the neutrons may be absorbed by some isotope with a large capture cross section, instead of by a proton. In particular, if an appreciable amount of He^3 has built up by means of reactions (3) and (5) in the stellar interior, a competing reaction is

$$n+He^3 \rightarrow H^1+T^3+0.76 \text{ Mev}. \tag{10}$$

In this case, the triton again beta-decays to He^3 and for two deuterons taking part in a D−D reaction only one He^3 nucleus and a proton are formed. The cross sections for reactions (9) and (10) are known for thermal neutrons at 300°K (energy of 0.025 ev). Both these cross sections should obey the $(1/v)$ law up to energies of about 50 kev and hence the ratio of their cross sections should be the same at neutron energies of about 100 ev and 0.025 ev. At 100 ev, these two cross sections are 5.2 millibarns and 81 barns, respectively. Thus, reaction (10) will predominate over (11) if the ratio of the abundance (by mass) of He^3 to that of hydrogen is larger than about 2×10^{-4}.

Apart from He^3 there are a large number of isotopes which have neutron-capture cross sections large compared with that of the proton. Amongst the light elements (n,α) reactions take place on Li^6 and B^{10}. At a neutron energy of 100 ev these reaction cross sections should be about 15 and 60 barns, respectively. Amongst the heavy and medium-heavy nuclei, many will have neutron capture resonances at energies comparable with our thermal energy of about 100 ev. We take,

[5] W. A. Wenzel and W. Whaling, Phys. Rev. **88**, 1149 (1952).
[6] W. R. Arnold *et al.*, Phys. Rev. **93**, 483 (1954).
[7] G. Boato, Phys. Rev. **93**, 640 (1954).

merely as an example, the well-known resonance[8] at 120 ev for Co^{59}. The total width of this resonance, Γ, is about 5 ev and the neutron capture cross section (about one tenth the scattering cross section) at resonance is about 1000 barns. The capture cross section, averaged over a Maxwell energy distribution for 10^6 °K, is then about 25 barns. The energy released in the absorption of one neutron by Li^6, B^{10}, and Co^{59} is 4.8, 2.8, and about 7 Mev, respectively. The absorption rate by each of these isotopes equals that by hydrogen when the ratio of its abundance (by mass) to that of hydrogen is about 2×10^{-3}, 8×10^{-4} and 12×10^{-3}, respectively. These abundance ratios are considerably larger than the ones usually found in stellar atmospheres.

The net result of the various reaction chains is then as follows. If the deuterium abundance is low enough (e.g., $x_D/x_H<5\times10^{-4}$ for $T=10^6$ °K) each deuteron results in one He^3 nucleus and releases 5.5 Mev energy, which is 3×10^{-3} of the deuterium restmass energy. If the deuterium abundance is larger than this critical value then (*a*) if the abundance of isotopes other than hydrogen and deuterium is negligible each deuteron results in two thirds of a He^3 nucleus, releasing 3.2 Mev; (*b*) if $x_{He^3}/x_H>2\times10^{-4}$ and heavier elements are negligible, each deuteron results in one half of a He^3 nucleus, releasing 2.0 Mev; (*c*) if the abundance of some heavier isotopes is large enough, each deuteron results in one half a He^3 nucleus, releasing from 3.4 to 5.5 Mev. At much higher temperatures ($T>8$, say) each He^3 nucleus, produced in any of the above reactions, results in one half of a He^4 nucleus and releases an additional energy of 6.4 Mev.

In Table I, we give the mean life of deuterium, and the rate of energy production (proportional to $\rho x_H x_D$) for reaction (3) at various temperatures. The reaction rates given in Eqs. (7) and (8) should be multiplied by a correction factor for the effect of electron screening. This factor[9] is approximately given by $\exp(0.26\rho^{\frac{1}{2}}T^{-\frac{3}{2}})$. For the low densities normally expected in the interior of such cool stars, this correction factor should be unimportant.

3. REACTIONS ON Li, Be, AND B

The proton reactions on the five stable isotopes of these light *p*-shell nuclei have already been discussed briefly by Fowler,[2] but some more recent data is now available and we will discuss the present situation in some detail.

The cross sections for the (p,α) reactions on the two isotopes of lithium,

$$H^1+Li^6\rightarrow He^4+He^3+4.02 \text{ Mev}, \qquad (11a)$$

$$H^1+Li^7\rightarrow He^4+He^4+17.34 \text{ Mev}, \qquad (11b)$$

TABLE I. The mean life of deuterium, t (in years), and the rate of energy production, ϵ (in ergs g^{-1} sec^{-1}), for $\rho x_H=0.1$ and $x_D=10^{-4}$ for various temperatures T (in 10^6 °K).

T	0.35	0.5	0.75	1	1.5
t	2×10^{11}	6×10^8	2×10^6	6×10^4	700
ϵ	5×10^{-5}	1.5×10^{-2}	4	140	1.2×10^4

have been measured recently by Phillips and Sawyer[10] for proton energies of 40 to 250 kev. The cross sections for Li^7 are somewhat low and this reaction is suspected to be due to *P*-state protons, proceeding through a (0+)-state in Be^8 of several Mev width. The measurements of Phillips and Sawyer give cross sections larger by a factor of almost two than some prewar data, but give a cross section factor S, Eq. (1), practically constant between 40 and 240 kev. For these two reactions we adopt the values

$$S_6=(6\pm3)\times10^6 \text{ ev barns}, \quad S_7=(1.2\pm0.5)\times10^5 \text{ ev barns}. \qquad (12)$$

The proton absorption on Be^9 can proceed in two different ways,

$$H^1+Be^9\rightarrow D^2+Be^8+0.56 \text{ Mev}, \qquad (13a)$$

$$H^1+Be^9\rightarrow He^4+Li^6+2.13 \text{ Mev}. \qquad (13b)$$

After reaction (13a), the Be^8 nucleus immediately breaks up into two alpha particles and the deuteron very quickly absorbs a proton. After reaction (13b), the Li^6-nucleus quickly absorbs a proton, reaction (11a). The end result of both reactions (13a) and (13b) is the formation of two alpha particles and one He^3 nucleus from one Be^9 nucleus. For reaction (13), there are two resonances near the stellar energy region, one for a proton energy of 300 kev (width about 150 kev) and one just below threshold (proton energy about -50 kev). Milne[11] has recently measured the cross section for reactions (13a) and (13b) combined for proton energies of 50 to 350 kev. This measured cross section factor, S, is practically constant from 50 to 100 kev. We adopt this value of $S=3\times10^7$ ev barns for S in the stellar energy region. This value may be in error by a factor of two or more, since the position and width of the negative energy resonance is not known very accurately.

The proton reactions on the two isotopes of boron are

$$H^1+B^{10}\rightarrow He^4+Be^7+1.15 \text{ Mev}, \qquad (14a)$$

$$H^1+B^{11}\rightarrow He^4+Be^8+8.58 \text{ Mev}. \qquad (14b)$$

After reaction (14a), the Be^7 nucleus decays to Li^7 in a few weeks and Li^7 quickly absorbs a proton, reaction (11b). The Be^8 nucleus formed in reaction (14b) quickly decays into two alpha particles. The cross

[8] Serge, Morrison, and Feld, *Experimental Nuclear Physics*, pages 291 and 329 (John Wiley and Sons, Inc., New York, 1953).
[9] E. E. Salpeter, Australian J. Phys. **7**, 373 (1954).
[10] J. A. Phillips and G. A. Sawyer, Los Alamos Report 1578, 1953 (unpublished).
[11] E. A. Milne, California Institute of Technology report (unpublished).

TABLE II. Proton-reactions on Li, Be, and B. Q is the net energy release, A and B are the coefficients for Eq. (15). The last four columns give the mean lifetime p^{-1}, in years, for $\rho x_H=10$ and for temperatures of 2, 3, 5, and 8.

Net reaction	Q (Mev)	A	B	$T=$ 2	3	5	8
$Li^6+H^1 \rightarrow He^4+He^3$	4.0	6×10^{12}	84.1	1.7×10^8	5×10^4	3	
$Li^7+H^1 \rightarrow 2He^4$	17.3	1.2×10^{11}	84.7	1.5×10^{10}	4×10^6	250	
$Be^9+2H^1 \rightarrow 2He^4+He^3$	6.2	3×10^{13}	103.6		7×10^9	6×10^4	10
$B^{10}+2H^1 \rightarrow 3He^4$	19.3	2.5×10^{13}	120.6			1.5×10^9	5×10^4
$B^{11}+H^1 \rightarrow 3He^4$	8.7	1.2×10^{14}	121.0			4×10^8	1×10^4

section for reaction (14a) has been measured by Burcham and Freeman[12] for proton energies of 200 to 500 kev. In this region, the cross-section factor S decreases slowly with increasing energy, being about 1.2×10^7 ev barns at 200 kev. We adopt a value of $S=2\times10^7$ ev barns for the stellar energy region, but this value could possibly be in error by quite a large factor, since little is known about possible resonances below 200 kev. The cross section for reaction (14b) has been measured by Schardt[13] in the 100-kev region and gives a value of $S=10^8$ ev barns, which is probably accurate to within a factor of three or four.

The results of this section are summarized in Table II. We can write the mean reaction rate p in the form

$$p=A(\rho x_H)T^{-\frac{2}{3}}\exp(-B/T^{\frac{1}{3}})\ \text{sec}^{-1}, \qquad (15)$$

with T in 10^6 °K and ρ in g/cc. The coefficients A and B are given in Table II, as are also the mean reaction times (p^{-1}) for a few typical temperatures.

4. REACTIONS ON O^{16}

We have so far discussed proton reactions on nuclei of $Z<6$. We should discuss next reactions on the stable isotopes of carbon and nitrogen. It was pointed out long ago by Bethe[3] and others that these reactions form the so-called carbon-nitrogen cycle, the net result of the cycle being the conversion of four hydrogen atoms into one helium atom without changing the total abundance of carbon and nitrogen. The individual steps in the cycle are:

$$C^{12}(p,\gamma)N^{13};\quad N^{13}(\beta^+\nu)C^{13};\quad C^{13}(p,\gamma)N^{14};$$
$$N^{14}(p,\gamma)O^{15};\quad O^{15}(\beta^+\nu)N^{15};\quad N^{15}(p,\alpha)C^{12}.$$

This cycle has been discussed quantitatively by Fowler[2] in the light of the most recent experimental information and we shall merely quote some of his results:

The reaction $N^{15}(p,\gamma)O^{16}$ competes, in principle, with the last reaction of the cycle, but its cross section is smaller by a factor of about 10^4 and hence only one part in 10^4 of the carbon and nitrogen is converted into oxygen in one complete cycle. It is now very probable (but by no means certain) that there is no resonance in the stellar energy region for the reaction $N^{14}(p,\gamma)O^{15}$. This reaction is then the slowest of the cycle reactions and determines the over-all rate of the cycle, at least for temperatures well below $40(\times10^6$ °K). The extrapolated cross-section factor for this reaction is $(3.3\pm1.5)\times10^4$ ev barns. If we write the mean rate p of this reaction (and hence of the cycle) in the form of Eq. (15), we have

$$A=(5\pm2)\times10^{10},\quad B=152.3. \qquad (16)$$

This rate p has to be multiplied by a factor correcting for the effect of electron shielding, which is about 1.5 for the central conditions in the sun. When dynamic equilibrium has been established, the abundance ratios of the four carbon and nitrogen isotopes are then given by the rates of the various reactions. These ratios are

$$x_{12}/x_{13}\approx4,\quad x_N/x_C\approx20(13/T)^2,\quad x_{14}/x_{15}\approx3000. \qquad (17)$$

We discuss next proton reactions on the various isotopes of oxygen, fluorine, neon, and sodium. Of these, by far the most important is O^{16}, which is likely to be much more abundant than any other isotope heavier than nitrogen (of the order of a percent, by mass, for the sun). The situation regarding the absorption of low energy protons by O^{16} is somewhat unclear at the moment and we discuss it in detail.

Rough measurements of the absolute total cross section for a proton energy $E_p=2$ Mev have been made by various authors for the reaction

$$H^1+O^{16}\rightarrow F^{17}+\gamma+0.59\ \text{Mev}. \qquad (18)$$

The values for this cross section[14] range from 3×10^{-6} barn to 3×10^{-5} barn. We adopt for this cross section, and hence for the factor S,

$$\sigma(2\ \text{Mev})\approx1\times10^{-5}\ \text{barn},$$
$$S(2\ \text{Mev})\approx6\times10^3\ \text{ev barns}. \qquad (19)$$

These values should be accurate to within a factor of about 5. The relative cross section as a function of proton energy has been measured[15] in the range of $E_p=1.4$ to 4 Mev. This cross section is a smoothly increasing function of E_p in this range, except for a sharp resonance at 3.5 Mev.

To extrapolate the cross section factor S, Eq. (19), to zero proton energy, we have to know the energy position, angular momentum, etc. of the resonance levels contributing most to this reaction. A resonance

[12] W. E. Burcham and J. M. Freeman, Phil. Mag. 41, 337 (1950).
[13] A. W. Schardt, Ph.D. thesis, California Institute of Technology, 1951 (unpublished).
[14] F. J. Eppling (private communication).
[15] R. A. Laubenstein *et al.*, Phys. Rev. 84, 12 (1951).

level in F^{17}, for which the proton is only just bound with $E_p \approx -60$ kev, has been observed[16] in a $O^{16}(d,n)F^{17}$ reaction. The effect of this level was also found in an experiment[15] on the elastic scattering of low-energy protons from O^{16}. An analysis[17] of the scattering data indicates a rather large nuclear radius $R(\sim 6\times 10^{-13}$ cm), a very large reduced width $\gamma^2(\sim 6\times 10^{-13}$ cm Mev) and $J=\frac{1}{2}$, even parity (S-state proton) for this level. The quantitative features of this analysis are not very accurate, since the experimental data do not extend to very low proton energies and since the variation with energy of the level shift is quite appreciable for such a large reduced width.

If we assume that the entire experimental cross section[15] for the $O^{16}(p,\gamma)F^{17}$ reaction for $E_p=1.4$ to 4 Mev is due to this low-lying resonance level, we can calculate Γ_γ, the partial width for the gamma ray transition to the F^{17} ground state, as a function of energy. Using a single-level Breit-Wigner formula and the values for γ^2 and R given by the scattering data, one finds[17] that Γ_γ is approximately proportional to E_γ^5, where $E_\gamma=(E_p+0.6$ Mev) is the energy of the γ ray. This is consistent with the theoretical energy variation of Γ_γ for electric quadrupole radiation, rather than electric dipole ($\Gamma_\gamma \propto E_\gamma^3$). Electric quadrupole radiation, in turn, is consistent with the assumption that the F^{17} ground state is $[(5/2)+]$, as is suggested by the shell model and by the mirror nucleus O^{17}. Under these assumptions we can then calculate the cross-section factor S for the stellar energy region, $E_p^{st}\approx 40$ kev, from Eq. (19) and dispersion theory. Using $\gamma^2=6\times 10^{-13}$ cm Mev and $R=6\times 10^{-13}$ cm, the level shift can be calculated as a function of energy.[18] For the stellar energy region the apparent position of the resonance level is shifted from -60 kev to $E_r\approx -140$ kev. Assuming $\Gamma_\gamma \propto E_\gamma^5$, we then find for S in the stellar energy region approximately 2×10^3 ev barns.

However, the above analysis is made very doubtful by the following fact. From the above assumptions and the experimental cross section, Eq. (19) it follows that, for $E_p=2$ Mev, the gamma-ray partial width Γ_γ is about 5 ev. This value is orders of magnitude larger than any theoretically expected value for quadrupole radiation, but of the right order of magnitude for dipole radiation. The suspicion that the (p,γ) cross section for $E_p=1.4$ to 3 Mev cannot largely be due to quadrupole radiation is also strengthened by the following fact. The very marked resonance for S-state protons[15] found for elastic proton scattering from O^{16} at $E_p=2.66$ Mev is missing completely in the $O^{16}(p,\gamma)$ cross section,[15] this resonance also requiring electric quadrupole radiation. On the other hand, the evidence for the resonance level at -60 kev being $(\frac{1}{2}+)$ seems very good and the evidence for the F^{17} ground state being $[(5/2)+]$ at least fairly good (apart from the shell model and the analogy with O^{17}, the strong γ-ray transition to the ground state from the $[(7/2)+]$ level at $E_p=3.47$ Mev favors $[(5/2)+]$ for the ground state). These arguments make it seem unlikely that the proton capture is due to S-state protons, which require an angular momentum change $\Delta J=2$.

We shall now investigate the possibility that the bulk of the proton capture cross section is due to P-state protons on O^{16} which form a $(\frac{3}{2}-)$ state in F^{17} and hence give rise to electric dipole radiation to the groundstate. The ratio of nuclear radius R to "effective Bohr radius" is reasonably large for the $(O^{16}+p)$ system, about 2.7 for $R=5\times 10^{-13}$ cm. In this case the barrier penetration factor for P-state protons is comparable with that for S-state protons for $E_p \gtrsim 2$ Mev and is smaller by a factor of only about 5 even for very low values of E_p. Let us assume the existence of a $(\frac{3}{2}+)$ resonance level for E_p of the order of magnitude of 4 Mev. If this level has a very large reduced width γ^2, comparable with Wigner's upper limit of $(3\hbar^2/2_m R)=10^{-12}$ cm Mev, its proton width at resonance will be several Mev. Such a broad resonance level would not necessarily have been noticed in $O^{16}(p,p)$ and other reactions to date. The resonance denominator in the Breit-Wigner formula for such a state varies very little from $E_p=1.4$ to 4 Mev. One can then calculate the $O^{16}(p,\gamma)F^{17}$ cross section σ in this energy range due to such a state, assuming $\Gamma_\gamma \propto E_\gamma^3$. An approximate calculation of σ gave a function of energy which was at least not in bad disagreement with the experimental data. The partial γ-ray width required for $E_p=2$ Mev was about 10 ev, which is not unreasonable for electric dipole radiation. Under the assumption of such a broad P-state resonance, S was again calculated for the stellar energy region and found to be about 100-ev barns.‡

We provisionally adopt this value of $S=100$-ev barns, which should be accurate to within a factor of about 10 or 20, if the above assumptions are correct. More accurate experiments on the elastic scattering of low-energy protons from O^{16} are now in progress.[14] An analysis of these experiments should clarify the situation somewhat and more accurate measurements of the $O^{16}(p,\gamma)F^{17}$ cross section, especially at proton energies well below 2 Mev and above 4 Mev, would be very useful. Expressing the reaction rate in the form of Eq. (15), we have

$$A \sim 1.5\times 10^8, \quad B=166.7. \tag{20}$$

[16] F. Ajzenberg, Phys. Rev. 83, 875 (1951).

[17] R. A. Laubenstein and M. J. Laubenstein, Phys. Rev. 83, 18 (1951).

[18] R. G. Thomas, Phys. Rev. 88, 1109 (1952).

‡ *Note added in proof.*—Recent work [Warren *et al.*, Can. J. Phys. 32, 563 (1954)] indicates that the $O^{16}(p,\gamma)F^{17}$ reaction for $E_p=0.8$ to 2 Mev proceeds mainly by radiative capture into the state of $E_p \sim -60$ kev of F^{17}, followed by a gamma-ray transition to the ground state. This also seems to indicate capture of P-state protons (if the -60-kev state is really $J=\frac{1}{2}+$), but the reasons for the small probability of transitions directly to the ground state and for the energy dependence of the cross section are not at all clear. Our adopted value for S is thus highly tentative.

The F^{17} produced in reaction (18) beta decays to O^{17} with the emission of a positron of end-point energy 1.72 Mev with a half-life of about 1 minute.

5. REACTIONS ON O, F AND THE NE-NA CYCLE

For the three remaining isotopes of oxygen and fluorine, (p,α) reactions are energetically allowed. The energy releases per reaction for $O^{17}(p,\alpha)N^{14}$, $O^{18}(p,\alpha)N^{15}$, and $F^{19}(p,\alpha)O^{16}$ are 1.20, 3.97, and 8.12 Mev, respectively. No cross sections for these reactions have as yet been measured for reasonably low-proton energies and we have to estimate them theoretically.

In the $O^{17}(p,\alpha)$ reaction (for protons of about 40 kev, the "stellar energy region") the energy of the emerging alpha particles is low and their barrier penetration factor P quite small. Assuming a radius of the compound nucleus of about 5×10^{-13} cm, P is of the order of 10^{-4} for S-state alpha particles, slightly smaller for P state and about 10 times smaller for D state. If we assume resonance levels of reduced widths of about 10^{-13} cm Mev, the alpha-particle widths will be of the order of 100 ev. A resonance level in F^{18} has been observed[19] corresponding to a proton energy of (zero ± 100 kev). The spin and parity of this state is not yet known, but any spin and parity can be reached from both the $(O^{17}+p)$ and from the $(N^{14}+\alpha)$ channel. Thus a (2 or 3+) state can be reached by S-state protons and D-state α particles, a (1, 2, 3 or 4−) state by P-state proton and P or F α particles, etc. If we assume the exact position of the resonance level to be 50 kev away from the stellar energy, its spin and parity to be (2 or 3+) or (1 or 2−) and the above-mentioned values for values for reduced width and nuclear radius, then the Breit-Wigner single-level formula gives a value of 2×10^{5} ev barns for the cross section factor S. We adopt this value for S, but the correct value could be smaller by a factor of up to 100, if the spin and parity of the level is less favorable, larger by a factor of up to 10^{4} if the level is right in the stellar energy region.

Little is known of resonance levels in F^{19} and Ne^{20} in the energy region relevant to the $O^{18}(p,\alpha)N^{15}$ and $F^{19}(p,\alpha)O^{16}$ reactions for low energy protons. But the energy of the emerging alpha particle is high enough so that its barrier penetration factor is about unity for $(O^{16}+\alpha)$ and not very much smaller for $(N^{15}+\alpha)$. The widths of resonance levels in the compound nucleus which can be reached by the alpha particles should therefore be quite large, the levels almost overlapping and the exact positions of the resonance levels should therefore not affect the reaction rates extremely strongly. We adopt for the cross section factor S the following estimates, based on barrier penetration factors and density of resonance levels in the relevant energy region: $S=10^{8}$ and 10^{9} ev barns, respectively, for $O^{18}(p,\alpha)$ and $F^{19}(p,\alpha)$.

Neon-sodium cycle. The proton reactions on the four stable isotopes of neon and sodium form a cycle somewhat similar to the carbon cycle. For all except the heaviest of the four isotopes only (p,γ) reactions are energetically possible, followed by beta-decays in two of the three cases. For Na^{23} the (p,α) reaction is energetically possible, leading back to Ne^{20}. The net result of this cycle would again be the conversion of four protons into one alpha particle without destroying Ne or Na. We shall see however that $Na^{23}(p,\gamma)Mg^{24}$ is probably not much slower than the (p,α) reaction. The detailed steps in this "cycle" are[19,20]:

$$H^1+Ne^{20}\rightarrow Na^{21}+\gamma+2.45\text{ Mev},\qquad Na^{21}\xrightarrow{23\text{ sec}}Ne^{21}+e^{+}+\nu+2.50\text{ Mev}, \tag{21}$$

$$H^1+Ne^{21}\rightarrow Na^{22}+\gamma+6.74\text{ Mev},\qquad Na^{22}\xrightarrow{3\text{ yr}}Ne^{22}+e^{2}+\nu+\gamma+2.84\text{ Mev}, \tag{22}$$

$$H^1+Ne^{22}\rightarrow Na^{23}+\gamma+8.80\text{ Mev}, \tag{23}$$

$$H^1+Na^{23}\begin{cases}\nearrow Ne^{20}+\alpha+2.37\text{ Mev},\\ \searrow Mg^{24}+\gamma+11.70\text{ Mev}.\end{cases} \tag{24}$$

No cross sections are known for these reactions and very little about the relevant resonance levels in the compound nuclei, except the order of magnitude of the level density. To get (p,γ) transitions to the ground state of the final nucleus by means of electric dipole radiation, an incident proton in a P-state is required in all four cases (parity of all initial and final nuclei positive). We estimate the cross-section factors S by making the following (very crude) approximations and assumptions: (i) The reduced particle widths of the resonance levels, in energy units (γ^2/R), are of the order of the level spacing, as is the energy distance of the nearest level to the stellar energy region. (ii) The gamma-ray widths are given by Weisskopf's estimate,[21] for nuclear radii of about $\sigma\times 10^{-13}$ cm, $\Gamma_\gamma\sim(E_\gamma{}^3)$ ev, where E_γ is the gamma-ray energy in Mev.

For $Ne^{20}(p,\gamma)$ the calculated value for S is rather small, $S\sim 2\times 10^{3}$ ev barns, due to the low Q-value of the reaction which leads to wide level spacing (of order 1 Mev) and small gamma-ray width (order 10 ev). For the other two isotopes of neon, the level spacing is appreciably smaller and Γ_γ larger and we estimate $S\sim 1\times 10^{5}$ and 3×10^{5} ev barns, respectively, for Ne^{21} and Ne^{22}. For $Na^{23}(p,\alpha)$, the barrier penetration factor for the emerging alpha particle is less than 10^{-2} and the estimated α width of the order of 1 kev, giving $S\sim 10^{7}$ ev barns. For $Na^{23}(p,\gamma)$ the estimated Γ_γ is a few hundred ev and this reaction is not very much slower than the (p,α) reactions.

The above estimates are of course very crude. The correct values of S could be smaller than the estimates

[19] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **24**, 321 (1952).

[20] P. M. Endt and J. C. Kluyver, Revs. Modern Phys. **26**, 95 (1954).

[21] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. XII.

by factors of 10 or 100 and they could be very much larger still if some resonances fall in or very near the stellar energy region. It should also be pointed out that resonance levels in Mg^{24} of different spin and parity are required for the two different modes of decay, ($Mg^{24}+\gamma$) and ($Ne^{20}+\alpha$). Hence the branching ratio of these two modes will also depend quite strongly on the details of the nearest levels.

We can again express the reaction rate p in the form of Eq. (15). The coefficients A and B for this equation are given in Table III. The mean life (p^{-1}) of the various nuclei is given in Table IV, together with the reaction time for the conversion of hydrogen into helium.

6. DISCUSSION

We shall now summarize the significance of the reaction rates, calculated in this paper, to two problems. First, the possible sources of energy production in various stars. Second, the extent to which the nuclear reactions which have taken place in the interior of various stars have altered the relative abundance of the chemical elements and their isotopes. The most important variable for these considerations is the temperature in the stellar interior. For most of the stars which lie on the main sequence (referred to as *MS*) the central temperature, T_c (in 10^6 °K), lies in the range of about 6 to 25. A star with T_c less than about 15 lives mainly on the proton-proton chain, with $T_c>15$ mainly on the carbon cycle. For *Castor C*, the sun, and *Sirius A*, T_c is about 8, 13, and 20, respectively. For the very brightest *MS* stars and some stars which have a "burned out core" (all He) and are burning hydrogen only in a thin shell, T_c may be about 30 or even slightly larger.

$Z<6$.—Astrophysical observations are now becoming available on very young star groups. One famous example is the star aggregate in *ζ-Perseus* which is known to be only about 1.3 million years old.[22] An open star cluster which may be even younger is at present being investigated by Walker.[23] He finds that most of the fainter stars in this cluster have appreciably lower surface temperature, and hence larger radii, than *MS* stars of the same luminosity. These stars probably are still contracting gravitationally and have much lower central temperatures than equivalent *MS* stars. Some of these stars are probably still burning whatever D, Li, Be, and B they contained initially. For such special stars the burning of these elements may be an important source of energy production.[24] For instance, if the original abundance ratio of D/H had the terrestrial value, then the total energy content of deuterium burning is about the same as that of the gravitational contraction. The observations and calculations on the interior of such stars are too incomplete to draw any conclusions about the original abundance of such light elements.

Table III. The coefficients A and B for Eq. (15) for the reaction rate of the absorption of protons on the following nuclei.

	O^{17}	O^{18}	F^{19}	Ne^{20}	$Ne^{21,22}$	Na^{23}
A	3×10^{11}	1.5×10^{14}	1.5×10^{15}	3×10^{9}	3×10^{11}	2×10^{13}
B	167		181	194		208

Table IV. $\log_{10}$ of the mean reaction times (in years) for $\rho x_H=100$ for the absorption of protons on various nuclei for various temperatures T (in 10^6 °K). The first two columns give $\log_{10}$ of the mean life of hydrogen for (i) proton-proton chain alone, (ii) for p-p chain plus C,N-cycle for $x_{C,N}=0.01$.

T	"H" with $x_{C,N}=0$	"H" with $x_{C,N}=0.01$	O^{16}	O^{17}	O^{18}	F^{19}	Ne^{20}	$Ne^{21,22}$	Na^{23}
10	10.4	10.4	17	14	11	13	21	19	20
15	9.6	9.4	$12._5$	$9._5$	$6._5$	8	16	14	15
30	8.6	4.7	$7._5$	$4._5$	$1._5$	$1._5$	9	7	$7._5$
100	7.3	−1.5	0	−3	−6	−7	0	−2	−2

As Tables I and II show, the central temperatures of almost all *MS* stars are sufficiently high so that all of the original D, Li, Be, and B will have absorbed protons long ago. It should be noted that these light elements do not undergo any cycles, but are irrevocably destroyed, the end products consisting entirely of He^3 and He^4. Hence, in the interior of *MS* stars there can exist no isotopes lighter than carbon, except H^1, He^3, and He^4. In the hotter *MS* stars ($T_c>10$) the He^3 is further converted into He^4. It is of some interest, however, to discuss the abundance of these light elements in the outer parts of a *MS* star, where the temperatures are appreciably lower than T_c.

The age of the sun, and of many other *MS* stars, is known to be about 5×10^9 years. Fairly reliable calculations of the temperature and density, as a function of distance from the center r, are now available for *MS* stars and especially for the sun. We can then calculate the radial distance r and temperature T_r of the shell in a *MS* star, where the mean lifetime of a light element is equal to 5×10^9 years. The results of such a calculation for the sun are given in Table V, based on the reaction rates of Secs. 2 and 3 and on a calculation by Naur[25] giving temperature and density as a function of r ("Model 1," $T_c=13.5$). The fraction M_r/M of the solar mass contained inside radius r is also given in Table V.

If there is no mixing between various layers in the sun, then each element will be essentially absent in the solar matter inside the appropriate radial distance r, since the temperature increases with decreasing r and since the reaction rates increase very rapidly with temperature. Conversely, the element would have essentially its original abundance in the layers outside the radius r. In reality the sun most probably contains a fairly thick outer layer which is under convection and

[22] A. Blaauw, Bull. Astron. Netherlands **11**, 405 (1952).
[23] R. Walker (private communication).
[24] E. E. Salpeter, Mem. Roy. Soc. Liège **13**, 116 (1954).
[25] P. Naur, Astrophys. J. **119**, 365 (1954).

TABLE V. Position in the sun for which the mean life of various nuclei is 5×10^9 years. r is distance from the center, R the solar radius, T_r, the temperature in 10^6 °K, M_r the mass contained inside radius r and M the total solar mass.

	D	Li^6	Li^7	Be^9	B^{10}	B^{11}
r/R	0.86	0.62	0.57	0.49	0.37	0.39
T_r	0.54	2.0	2.4	3.2	4.9	4.7
M_r/M	1.00	0.98	0.97	0.93	0.79	0.83

hence thoroughly mixed. If the lower boundary of this convective zone lies deeper than the critical radius r for an element, then atoms of this element will be destroyed when convection carried them though the lower parts of the zone. Hence, the abundance of such an element should be very low even in the solar atmosphere at the top of the zone.[26]

The abundance of lithium (normally consisting mainly of Li^7) and of beryllium in the solar atmosphere has been observed spectroscopically.[26] The abundance of Be is about the same as on the earth, whereas that of Li is lower by a factor of about 50. One is tempted to assume that the lower boundary of the outer convective zone lies between the critical values of r for Li and Be, say, at a temperature of 2.6 to 3. No conclusive observations on the abundance of D and B in the solar atmosphere are as yet available. If the above assumptions on the convective zone are correct, B should have its original abundance in the atmosphere and D should be completely missing (unless it is produced continuously by some process taking place in the atmosphere). Note that, even for B, the fractional mass of the sun which can still contain this element is only about 0.2.

We finally should keep in mind the fact that, if the original abundance of D was high enough, some neutrons were produced in the D—D reaction. The number of neutrons produced was probably not very large and (see Sec. 2) most of them probably were absorbed by H^1 or He^3. But it is possible that the abundance of some very rare heavy elements with suitable resonances was altered by neutron absorption during the deuterium burning.

$Z>7$.—If temperatures were high enough for proton absorption on all isotopes of O, F, Ne, and Na to go on, the net result would be as follows. All oxygen and fluorine would eventually be converted into He^4 and into nitrogen. The neon and sodium isotopes would not be destroyed at all (but "catalyze" the H—He conversion) if the (p,γ) reaction on Na^{23} is very much slower than (p,α). If (p,γ) can compete, sodium, and neon would eventually be converted into magnesium. But under most stellar conditions temperatures are not high enough for any but a few of these reactions to go on.

Central temperatures of *MS* stars may vary over a wide range of temperatures (from 5 to 30, say), depending on the luminosity and other factors. Whether a particular isotope will absorb protons to an appreciable extent or not at a particular temperature depends on whether the mean reaction time is shorter or longer than the corresponding reaction time for the conversion of H into He. Such a comparison of reaction rates is given in Table IV.

For stars appreciably cooler than the sun ($T_c<10$), all the isotopes heavier than nitrogen are essentially unaffected. In the hotter *MS* stars which burn hydrogen on the carbon-cycle ($T_c>15$), both O^{18} and F^{19} (and possibly, but not certainly, O^{17}) are destroyed much more rapidly than hydrogen, forming nitrogen and O^{16}. Unless the assumptions of Sec. 4 are violently wrong or the abundance of C and N orders of magnitude less than the expected percent or so, O^{16} will not be depleted appreciably during the mean life of hydrogen. Since the original abundance in stars of O^{17}, O^{18}, and F^{19} most probably was rather low, the depletion of these isotopes in the hotter *MS* stars is not likely to be a very important source of energy.

Unless resonances increase the cross sections for the Ne—Na cycle greatly, these isotopes are unaffected during the lifetime of hydrogen. If a star happened to contain Ne or Na, but no carbon or nitrogen at all ($x_{C,N}<10^{-6}$) and had central temperatures higher than 30, the Ne—Na cycle would be a more important energy source than the proton chain, but such abnormal abundances are extremely unlikely. Proton reactions on Mg, Al, and heavier nuclei can be neglected completely. The cross-section factors S for reactions on Mg and Al are comparable with those for Ne and Na, but the barrier penetration factors for the proton very much smaller.

To summarize: Hydrogen in the core of main sequence stars will be converted into He^4 at temperatures below 30×10^6 °K. When this conversion has practically gone to completion, neon and all heavier elements will still have their original abundance. Of the lighter elements, besides He^4 which comprises most of the matter, only carbon, nitrogen and O^{16} (and possibly O^{17}) remain. After the H—He conversion, the stellar core may contract again and the temperature may rise to very much more than 30×10^6 °K. At these high temperatures the reaction rates for proton absorption from heavier nuclei would be quite high, but there simply is not any hydrogen left. Nuclear reactions will set in again when temperatures are high enough for alpha particles to be captured. These reactions will be discussed in a sequel to this paper.

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[26] J. L. Greenstein and E. Tandberg-Hannsen, Astrophys. J. **119**, 113 (1954).