of inertia was made by Bohr,[4] and other evidence for a similar conclusion was presented by Ford.[34] The values of $Q_0$(yield) and $Q_0$(energy) differ by roughly a factor of two for the even-even hafnium isotopes. These are the most highly deformed nuclei studied. The agreement between the values of $Q_0$ calculated from these two sources becomes worse as the nuclei become heavier and approach the closed proton shell at 82 protons and the closed neutron shell at 126 neutrons. For $Pt^{198}$, the least deformed nucleus studied, the ratio of the $Q_0$ from the excitation energy to that calculated from the yield data is about seven. This difference is much greater than the experimental error.

If the rotational model is valid, then in terms of the discussion in Sec. I, the most logical explanation for this phenomenon appears to be that the charge distribution within the nucleus becomes spherical more rapidly than the mass distribution as the closed shell at 82 protons and 126 neutrons is approached.

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[34] K. W. Ford, Phys. Rev. 95, 1250 (1954).

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# Concept of Parentage of Nuclear States and Its Importance in Nuclear Reaction Phenomena

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The concept of fractional parentage of nuclear states is invoked to point out that many dynamical properties of nuclear systems (transition rates and level widths) are controlled by products of the coefficients of fractional parentage for common parents (the "parentage overlap") between initial and final states. Illustrations are given from a range of nuclear reactions including radiative transitions, high- and low-energy stripping, pickup, and photonuclear processes. It is particularly to be emphasized that, because of the occurrence of the coefficients of fractional parentage and certain (vector coupling) weighting factors in the expression for the transition rate, one may find reduced and radiative widths very considerably less than the "single-particle" values even though the states concerned are wholly of an independent-particle character; enhanced transitions are also possible within the same scheme.

## INTRODUCTION

IN recent years, many new types of nuclear reaction have been investigated experimentally with the new techniques and energy ranges available. Some of these reactions have received considerable theoretical attention of a sort in which an attempt is made to present *model-independent* formulas for cross sections by assuming only that certain general mechanisms are responsible for the transitions. As examples one can cite the theory of resonance reactions[1] (which did not, in fact, in its general form even assume a particular mechanism) and the theory of deuteron stripping.[2] It is not our purpose here to review the mass of detailed work that has been done, but rather we wish to show that there is often a considerable underlying similarity in the factors controlling the cross sections of apparently quite different types of reaction. If no nuclear model is assumed, this similarity is of only formal interest, but if a nuclear shell model is held to be valid, this unifying feature is of considerable practical interest and importance. It arises essentially from the fact that a whole range of nuclear reactions can be classified as "one-particle" reactions, by which we mean, not that only one particular particle in the nucleus can make a transition, but rather that a transition causes only a single particle (*any* single particle) to change its state. Since shell-model wave functions consist, rougly speaking, of products of single-particle wave functions, it follows that the differences between the formulas for the cross sections for different types of reactions can be separated out into certain multiplying single-particle

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[1] E. P. Wigner and L. Eisenbud, Phys. Rev. 72, 29 (1947).

[2] S. T. Butler, Proc. Roy. Soc. (London) A208, 559 (1951).

factors, which are the squares of single-particle matrix elements; the other factors, which only take account of the presence of other nucleons in the composition of the total initial and final states, are very similar for all varieties of one-particle reactions. It is this similarity between the formulas for the probabilities of various kinds of transition that we wish to stress here. The remainder of the paper is devoted to showing the usefulness of the fractional parentage concepts in representing this common factor, and to explicitly displaying the factor in a number of examples. In doing this, we shall not repeat the often lengthy formulas established by those workers who have studied the various reactions in detail, but rather we shall point out in each case the particular parameters containing the common factor. The realization of the presence of this factor will be seen to offer explanations of such diverse nuclear properties as the similarity of certain relative $(n,\gamma)$ and $(d,p)$ reaction yields, the spectra of high-energy inelastic proton scattering, and the variation in width of the "giant resonance" peaks of nuclear photodisintegration.

## FRACTIONAL PARENTAGE

The concepts of "fractional parentage" and "parent states" were developed for the study of atomic spectroscopy by Racah.[3] He showed that any properly constructed (totally antisymmetric) shell-model state of $n$ particles can be expanded in terms of the states of one particular particle vector-coupled to the "parent states" of the other $(n-1)$ particles. The coefficients in such an expansion are called the "coefficients of fractional parentage" or, in abbreviated form, the "c.f.p." Investigations into the nature of nuclear energy levels based on the quasi-atomic shell model have made direct use of Racah's methods and terminology, apart from changes arising from the existence of an extra quantum number (the isotopic spin). Jahn and Van Wieringen[4] have defined and evaluated the c.f.p. for states of equivalent particles in the $L$-$S$ coupling approximation, and Flowers and Edmonds[5] have done this in $j$-$j$ coupling.

The work of these authors has so far mainly been used to facilitate the calculation of nuclear energy level schemes and magnetic moments, i.e., the prediction of static nuclear properties. However the concept of parent states is also of fundamental importance for the understanding of certain dynamical nuclear properties such as transition rates and level widths. This we now discuss.

The types of transition that we have in mind are those for which the transition operator is either explicitly or effectively a "one-particle" one,[6] i.e., it can be expressed in the form

$$\mathcal{O}=\sum_k O(\mathbf{r}_k,\boldsymbol{\sigma}_k,\boldsymbol{\tau}_k), \tag{1}$$

where each term in the sum only operates on one particular particle $k$ represented by the usual space, spin, and isotopic spin coordinates $\mathbf{r}_k$, $\boldsymbol{\sigma}_k$, and $\boldsymbol{\tau}_k$. The operator $\mathcal{O}$ must always be used when the states concerned possess detailed features; i.e., when they are not extreme "single particle" states. To find a transition probability, the matrix elements of the corresponding operator taken between the initial and final states must be evaluated. If there are present in the system $n$ particles that are concerned in the transition (i.e., that are needed for the specification of the initial and final states), then, since the wave functions of these initial and final states are antisymmetric in all particles, the matrix element for a transition can be written in term of the matrix element of just one particular particle:

$$\langle i|\mathcal{O}|f\rangle = n\langle i|O(\mathbf{r}_n,\boldsymbol{\sigma}_n,\boldsymbol{\tau}_n)|f\rangle. \tag{2}$$

Here we have conventionally chosen the particular particle to be the $n$th one and put $k=n$ in the second matrix element. If now we assume that the fractional parentage expansions of the initial and final states are known, the evaluation of this quantity is quite straightforward. The total matrix element is expressible as a product of three factors:

$$\langle i|\mathcal{O}|f\rangle = n\times\begin{pmatrix}\text{single particle}\\ \text{matrix element}\end{pmatrix}\times(\text{parentage overlap}). \tag{3}$$

This last quantity is a sum of weighted products of the c.f.p. of the initial and final states. The sum is over all the parent states that the initial and final states have in common (the "common parents") and the weighting factors are usually Racah coefficients which take account of the recoupling of the various spins involved in the transition. It is to be noted that these weighting factors depend on the type of transition, so that the definition of parentage overlap is not quite the same for different kinds of transitions. However, we can make some remarks about the values of the parentage overlap which are quite generally true: for instance, its value cannot exceed unity, and can only equal unity in those exceptional cases when the initial and final states are very similar (in the sense of having the same parentage characteristics). In such cases, the

[3] G. Racah, Phys. Rev. **63**, 367 (1943). See earlier R. F. Bacher and S. Goudsmit, Phys. Rev. **46**, 948 (1934).

[4] H. A. Jahn and H. Van Wieringen, Proc. Roy. Soc. (London) **A209**, 502 (1951).

[5] A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) **A214**, 515 (1952).

[6] It should be noted that we are confining ourselves to one-particle operators in the interests of simplicity and practical importance. One can develop a fractional parentage theory for the expansion of a state of $n$ particles into states of $n-m$ and $m$ particles where $m$ is *any* number less than $n$. Such an expansion would be appropriate for the evaluation of $m$-particle operators. For instance, expansion with $m=2,4$ are appropriate for dealing with deuteron and alpha-particle transitions.

transition rates are larger than the single-particle values by the factor $n^2$. In most instances, the parentage overlap will be less than unity, and often less than $n^{-1}$ so that the transition probability is usually reduced to less than the single-particle value. This happens when the two sets of parent states are different and there are only a few common parents. Since the c.f.p. of any given state satisfy a normalization condition (the sum of the squares is unity), the smaller the number of common parents relative to the total number, the smaller the value of the parentage overlap. It sometimes happens that there are no common parents at all, in which case the parentage overlap is zero and the transition is forbidden. Thus there are selection rules associated with the parentage overlap. We shall later see that these can be quite powerful. It is of great importance to realize that, because of the first and third factors in expression (3), the rate of a transition between pure shell model states needing several particles for their specification may be very different from that of an isolated single particle. Thus the fact that an observed transition rate is very much less than (or, for that matter, greater than) the single-particle value does not imply that other than pure shell-model states are involved.

It is tempting to try to give a simple semiclassical interpretation of the parentage overlap and the associated selection rules. The surface oscillation model of the nucleus[7] seems especially appropriate in this connection. Qualitatively, on this model, one imagines the initial state as consisting of one particle that can make a one-particle transition plus the rest of the nucleus (the core) acting as a hydrodynamical fluid behaving in a certain way (having a certain shape and rotating with a certain angular momentum). The only final states that are accessible by one-particle transitions are those in which the core has the same angular momentum. Even for these states the transition rates may be severely inhibited if a change in shape of the core is implied in the transition. Quantitatively, in the formulas for transition matrix elements, this inhibition factor appears in the form of a shape overlap integral, $\int \varphi_i \varphi_f dV$, between the initial and final core states.[8] This integral, in fact, occurs in the place of the parentage overlap factor (times $n$) in (3). One may associate the two factors, but should not take the analogy too seriously because an essential feature of fractional parentage and the evaluation of the parentage overlap is the antisymmetrization of the wave functions. There is no real classical analog of antisymmetrization and, in particular, its role in the surface oscillation model that we have used in illustration is obscure.

To some extent in the discussion so far, a nuclear shell model has been assumed. However our remarks are relevant to any model provided that the initial and final states are expanded in terms of shell-model states of various configurations, and that the c.f.p. are defined and evaluated. In such cases formula (3) must include a sum over the component configurations of the two states.

If we do indeed assume a shell model with pure configurations for the two states, an interesting situation exists near closed shells. In this region, low-lying states tend to have only few parent states, often only one or two, in contrast to the situation in between shells where there may be several dozen parents or more. Consequently transitions in the closed-shell region are liable either to be very strong (when there are common parents) or very weak (when there are no common parents).

## SOME PRACTICAL ILLUSTRATIONS

To illustrate these rather formal remarks, we briefly consider a number of specific reactions which are of the one-particle type and in which the parentage overlap between the initial and final states therefore plays a dominant role.

### (1) Nucleon Emission from a Compound Nucleus

In this type of transition, the final state is the system "residual nucleus+separating nucleon." By its nature, this total state has only one parent, *viz.* the state of the residual nucleus, and the corresponding c.f.p. is $n^{-\frac{1}{2}}$ where $n$ is the number of nucleons concerned in the specification of the total state. From our general remarks above, the transition rate or reduced width is the single-particle value times $n^2$, times the square of the parentage overlap which now has the form:

$$n^{-\frac{1}{2}}\times(\text{the appropriate c.f.p. of the initial state})\times\left\{\begin{array}{c}\text{a weighting factor}\\ \text{consisting of Racah}\\ \text{coefficients}\end{array}\right\}.$$

The explicit formula has been given in a previous publication,[9] where four experimentally known reduced widths of $C^{13}$ and $N^{13}$ were cited as examples of its application. Two of the compound states involved belonged to the configuration $1p^9$, and the other two belonged to $1p^8\ 2s$ and $1p^8\ 1d$. For the first two, the above formula could be applied directly using the previously mentioned work[4,5] on the parentage of states of equivalent particles. For the last two states, this could not be done because the compound states belong to mixed configurations of the type $l^{n-1}l'$ and there is considerable choice in the precise way such states may be constructed. Usually, for formal mathematical purposes, states of such mixed configurations are constructed by simply vector-coupling the $l'$

[7] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 27, No. 16 (1953).

[8] E.g., this factor appears in formulas (VIII. 14) and (VII. 14) of reference 8; these apply to beta and gamma transitions, respectively.

[9] A. M. Lane, Phys. Rev. 92, 839 (1953).

particle on to definite proper states of $l^{n-1}$ and then antisymmetrizing. Such states are said to be defined "genealogically." As far as a particle being in the $l'$ orbit is concerned, each total state has only one parent state of $l^{n-1}$ and the corresponding c.f.p. for this state is $n^{-\frac{1}{2}}$. If the actual physical states of mixed configurations are approximated by these simply-constructed states, the reduced width for nucleon emission to the "unique parent" state is then seen to be just the single-particle one (apart from possible Racah coefficients in the weighting factor).

It has been pointed out[10] that there is considerable experimental evidence that states of mixed configurations have single-particle reduced widths, at least in light nuclei. Thus one gathers that the "unique parent" approximation to these states is a good one. The theoretical reason for this is certainly not obvious, but detailed investigation[10] provides some justification for the approximation in particular cases. We do not propose to discuss this matter here, but merely to point out that the experimental facts give support to the approximation. Assuming the approximation enables us to evaluate the parentage overlap for states of mixed configurations.

### (2) Beta Decay

Since the beta-decay operator is a one-particle operator, this comes into the class of one-particle reactions. However, since beta decay usually takes place between low-lying states of nuclei (which almost always have common parents), the *selection rules* arising from parentage are of little practical use, although the parentage overlap directly controls the *ft* value. Explicit expressions for the beta-decay matrix elements in *L-S* and *j-j* coupling have been given,[11] and can be seen to be of the form (3).

### (3) Radiative Transitions

These are one-particle transitions as an immediate consequence of the one-particle nature of the electromagnetic transition operators. The precise manner in which a transition probability is controlled by the parentage overlap between the initial and final states has been shown already.[12] The formulas for $E1$, $M1$, and $E2$ transition matrix elements all have a form identical to (3).

An interesting situation exists when one state (say the emitting state) has a mixed configuration of the type $l^{n-1}l'$ discussed in example (1). If the 'unique parent' state is the ground state of $l^{n-1}$, it follows that the intrinsically strongest transitions will be to those states of $l^n$ which also have this ground state as a parent. Such states will be those near the ground state of $l^n$, and consequently one expects radiative transitions from the upper state to favor high-energy transitions more strongly than is suggested by their greater energy alone.

The preference for ground-state transitions has been noted[13] in the capture of thermal neutrons in a number of cases: $F^{20}$, $Al^{28}$, $Pb^{207}$, $Pb^{208}$ (final nuclei). Considerations similar to those above indicate a possible simple explanation for these phenomena.

The recent unpublished work of P. C. Gugelot on the radiative capture of 18-Mev protons by copper also shows a rather considerable favoring of transitions to the lower-lying excited states (an apparently too-slowly increasing level density with excitation energy), suggesting that the target nucleus may not be too violently disturbed even by the addition of so fast a particle.

Of course, the electric excitation of nuclei under charged particle bombardment (Coulomb excitation) is directly governed by the radiative transition probabilities from the ground state, and so, to this extent, is included in the present considerations. Detailed theoretical studies revealing the relation between cross section and transition probability have been reported for both positive[14,15] and negative[16] particle bombardment. Experimentally the $(e,e')$ reaction on $C^{12}$ has been recently investigated with high-energy electrons.[17] It is found that the 7.68-Mev level is excited only relatively weakly compared with the 4.45-Mev level. The latter state is 2+ and, on any shell model, has a strong parentage overlap with the ground state. The 7.68-Mev state is believed to be 0+ and does not appear in the shell-model spectrum.[18] Its parentage overlap with the ground state is thus presumably small since it differs from it by the excitation of more than one nucleon (though note that it is incorrect to attribute the observed difference in excitation probability to overlap factors alone since the single particle probabilities are different for the two excitations).

### (4) High-Energy Stripping and Pickup Reactions

In high-energy pickup reactions (for example) one imagines that the incident particle "snatches" a nucleon from the target nucleus. A treatment of this phenomenon in the impulse approximation essentially assumes that the condition of all the other nucleons is unchanged. Since the squares of the c.f.p. of the target nucleus determine the relative times that these other nucleons spend in their various parent states,

[10] A. M. Lane, Proc. Phys. Soc. (London) (to be published), and Atomic Research Establishment, Harwell Report T/R 1289, 1954 (unpublished).
[11] A. M. Lane, Proc. Phys. Soc. (London) (to be published).
[12] A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954).
[13] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **93**, 1260 (1954).
[14] K. A. Ter-Martirosyan, J. Exptl. Theoret. Phys. (U.S.S.R.) **22**, 284 (1952).
[15] K. Alder and A. Winther, Phys. Rev. **91**, 1578 (1953).
[16] Thie, Mullin, and Guth, Phys. Rev. **87**, 962 (1952); L. I. Schiff, Phys. Rev. **96**, 765 (1954).
[17] R. Hofstadter (private communication).
[18] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

it is clear that these quantities determine the relative cross sections for the pickup production of the various possible residual (parent) states. When the target nucleus is a closed shell, since any component $(lj)$ shell has only one parent state (the state of one hole in $jj$ coupling), if a nucleon is removed from the last shell only one final state (the ground state) of the residual nucleus is allowed. When the target is not a closed shell, there are, in general, several parent states and so several final states that are open to the residual nucleus. This means that when individual product groups are not resolved, one expects the spectrum of product particles to be sharp at closed shells and to broaden on going away from closed shells. The results of Selove[19] with 90-Mev protons on $Si^{28}(p,d)Si^{27}$ (closed $1d_{5/2}$ shell, spectrum width $\sim$5 Mev) and $Al^{27}(p,d)Al^{26}$ (nonclosed shell, spectrum width $\sim$10 Mev) exemplify this idea. It should be mentioned that in the discussion of such work, Selove concludes that the shell model is not a satisfactory model on the grounds that no resolved product groups are observed to correspond to the removal of nucleons from the inner closed shells; Selove expects such a group to occur at about 8 Mev (the presumptive single-particle spacing) from the observed group due to pickup from the outer shell. However, apart from the fact that this energy should be increased to allow for the excess pairing energies of the last nucleons in the inner shells, one may expect considerable configuration interaction at such high excitations in the residual nuclei. For instance, if a $1s_{\frac{1}{2}}$ neutron is "snatched" from $Si^{28}$, the simple $1s_{\frac{1}{2}}$ hole state of $Si^{27}$ is probably immediately "dissolved" and shared amongst proper eigenstates perhaps over a wide energy range. Thus no sharp product groups should necessarily be expected for the removal of nucleons from the inner shells. In contrast to the one-peak spectra from $Si^{28}$ and $Al^{27}$ that we have mentioned, Selove (private communication) finds that the spectrum of deuterons in the $Be^9(pd)Be^8$ reaction has two well-defined peaks separated by about 15 Mev. On an alpha-particle model of $Be^9$ these two peaks would be said to correspond to the removed neutron being in the one case the odd neutron and in the other a neutron from an alpha particle. However we again find that a shell model gives an equally good explanation because, for instance, in $L$-$S$ coupling, the ground state of $Be^9$, which has partition [441] has parent states in $Be^8$ in two distinct groups, namely [44] and [431] which are known to be separated by about 17 Mev. The difference behavior that Selove finds for target nuclei of the $4n$, $4n+1$, and $4n+3$ types certainly reflects the existence of 4-groups in the nucleus of the "saturated spin-isotopic spin" type. However it seems to us that there is no evidence for any actual spatial separation between such saturated groups of four particles as is implied in the alpha-particle model.

[19] W. Selove, Phys. Rev. **92**, 1328 (1953).

### (5) Low-Energy ($\leq$15 Mev) Stripping and Pickup Reactions

According to the stripping theory,[2] one imagines stripping and pickup reactions simply to involve the exchange of a nucleon between the colliding nuclei outside the surface of the target nucleus. In this region, the exchanged nucleon is temporarily dissociated from the others in the target, and so the latter cannot change their state. This means that just as in the example (1), the transition rate (for the case of pickup) is determined by the reduced-width of the initial (target) state for dissociation into a nucleon and the final (residual) state. This reduced width is, in turn, simply proportional to the square of the corresponding c.f.p. The explicit relation between stripping cross section and reduced width has been demonstrated and used[20] to extract values of the latter from experimental data.

It has been remarked[21] that one sometimes finds considerable similarity between the relative intensities of gamma rays (of given multipolarity and parity change) emitted in the radiative capture of thermal neutrons to various states of the residual nucleus and the relative intensities of the proton groups that give the same residual states by $(d,p)$ stripping. This seemingly curious relationship follows immediately from the present considerations and those of item (3) above if we may assume that the ground state of the target nucleus for the $(n,\gamma)$ reaction is the chief parent of the radiating state—for then the radiative widths of the gamma transitions will be proportional to the squares of the c.f.p. of the residual states for the target nucleus and so will the intensities of the proton groups. The fact that the intermediate systems are wholly different for the two processes is irrelevant.

One interesting experimental result[22] is the absence of a proton group to the 7.68-Mev state in $C^{12}$ in the reaction $B^{11}(d,n)C^{12}$ using deuterons of 8 Mev. This suggests that (since this is presumably a stripping reaction) the ground state of $B^{11}$ is not a parent of the 7.68-Mev state of $C^{12}$. Since the ground state of $B^{11}$ is the only parent of the ground state of $C^{12}$ (strictly true in $j$-$j$ coupling and approximately so in $L$-$S$ coupling), it follows that the parentage overlap of the 7.68-Mev and ground states of $C^{12}$ must be small or zero. We saw in example (3) that the fact that the inelastic electron scattering to the 7.68-Mev state was so small could be explained on these same grounds.

### (6) Moderate and High-Energy ($\geq$10 Mev) $(n,p)$ and $(p,p'$ Reactions)

Arguments for treating these reactions in the impulse approximation have been put forward in a paper[23]

[20] Fuyimoto, Kikuchi, and Yoshida, Proc. Theoret. Phys. (Japan) **11**, 264 (1954).
[21] B. B. Kinsey (unpublished).
[22] V. R. Johnson, Phys. Rev. **86**, 302 (1952).
[23] Austern, Butler, and McManus, Phys. Rev. **92**, 350 (1953).

mainly concerned with their angular distributions. Acceptance of this approximation places these reactions in the "one-particle" category; one imagines the incident particle to collide with just one target nucleon. In inelastic scattering, the struck nucleon may receive energy which is eventually shared, but, during the time of collision, the other target nucleons are not disturbed. Thus, as in the other examples we have given, the cross sections for such reactions are determined by the squares of the parentage overlaps, which are defined with weighting factors peculiar to the process. (The parentage overlaps can be written as sums over products of reduced-width amplitudes). Explicit demonstration of this has been given by Austern *et al.*,[23] in their formula (14), in which their quantity $\sum_s A_{st}(j_s,m_s,j_p,\mu_p) \times B_{st'}(j_s,m_s,j_n,\mu_n)$ our parentage overlap. In inelastic scattering reactions on closed-shell nuclei, this result implies that only those states sharing the (unique) parent of the ground state can be excited. In the $C^{12}(p,p')C^{12}$ reaction with 90-Mev protons,[24] the 7.68-Mev level appears to be less readily excited than the 4.45-Mev level. This is precisely the same result as that obtained with inelastic electron scattering mentioned in example (3) and with stripping in example (5). Again we see that apparently very different reaction mechanisms can yield very similar results and that the parentage overlap factors may well provide the correct explanation of these similarities.

There is some evidence from the unpublished results of P. C. Gugelot on the inelastic scattering of 18-Mev protons that the impulse approximation[23] may have some validity in this energy range. He finds an apparent rate of increase of level density with excitation that is somewhat less than expected, on the basis of the usual thermodynamic expressions; this reflects the worsening parentage overlap between ground and excited states as the excitation increases and is just the result to be expected on the basis of the present considerations. Of course, a direct inelastic scattering process such as that which we have in mind will, in general, give an entirely different spectrum of product particles from that predicted by the compound nucleus theory where most particles emerge with low energies. Especially when the target nucleus is well approximated by a shell-model description, the parentage overlap factor implies that the direct process favors high-energy particle groups leaving behind those residual states whose overlap with the ground state is appreciable. If no allowance is made for the presence of the direct process in interpreting a spectrum of inelastically scattered particles, there will be an apparently anomalously low rate of increase of level density with energy for excitations of up to 5 Mev or more in the residual nucleus.

[24] K. Strauch and W. F. Titus, Phys. Rev. **95**, 854 (1954).

### (7) Photonuclear Reactions

The present ideas have one of their most interesting applications in the study of photonuclear reactions, where they offer a possible explanation of the sharpness of the giant electric dipole photonuclear peaks at closed shells.[25] Owing to the one-particle nature of the electric dipole operator, provided that the ground state of the target nucleus is a reasonably good shell-model state, photoexcitation can take place solely to a few shell-model component states differing in configuration by one particle from the ground state, and having at least one parent state in common with it. Of course, since they occur at such high energies, these states are almost never good eigenstates, but are dissolved by configuration interaction over some energy range. Nevertheless, unless this interaction is strong (and the complex potential model[26] suggests that it is not), one expects that the width of the photonuclear peak is roughly equal to the energy spread of the contributing shell-model states. At closed shells, there are only very few such states because the ground state of a closed shell has only one parent state (in $j$-$j$ coupling); on the other hand, the matrix elements are especially strong[27] [essentially due to the factor $n$ in expression (3)]. Thus the photonuclear peaks are expected to be considerably sharper at closed shells than in between shells, where there are many more (and widely-spread) contributing states with weaker matrix elements. Details of quantitative calculations will be published elsewhere.[27]

## CONCLUSIONS

Several of the types of one-particle reactions mentioned above have in the past been discussed from a purely single-particle viewpoint in which the initial and final wave functions are represented by the wave functions of a single particle. We have indicated that, in general, this approach is invalid and that one must take into account in a nuclear reaction the presence of all nucleons needed to specify the states concerned: this leads to a factor in the transition matrix elements that we have called the parentage overlap. The parentage overlap appears in the cross sections for all reactions in the one-particle category, and consequently provides an explicit unifying feature of these reactions. In the examples, the factor has been shown to be able to explain a number of phenomena that cannot be understood from the elementary single-particle viewpoint.

[25] D. H. Wilkinson, Philadelphia Conference on Photonuclear Reactions, 1954 (unpublished), p. 5.
[26] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).
[27] D. H. Wilkinson, Phil. Mag. (to be published).